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**THESIS**

**SEARCH PARAMETER OPTIMIZATION FOR DISCRETE,  
BAYESIAN, AND CONTINUOUS SEARCH ALGORITHMS**

by

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September 2017

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**SEARCH PARAMETER OPTIMIZATION FOR DISCRETE, BAYESIAN, AND  
CONTINUOUS SEARCH ALGORITHMS**

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## ABSTRACT

Search and Detection Theory is the overarching field of study that covers many scenarios. These range from simple search and rescue acts to prosecuting aerial/surface/submersible targets on mission. This research looks at varying the known discrete and Bayesian algorithm parameters to analyze the optimization. It also expands on previous research of two searchers with search radii coupled to their speed, executing three search patterns: inline spiral search, inline ladder search, and a multipath ladder search. Analysis reveals that the Bayesian search and discrete search work similarly, but the Bayesian search algorithm provides a more useful output in location probability. Results from the continuous search were similar to previous research, but variance in time to detection became more complex than basic increasing or decreasing ranges.

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## List of Acronyms and Abbreviations

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<b>AAW</b>	Anti-Air Warfare
<b>ASW</b>	Anti-Submarine Warfare
<b>ASWORG</b>	Anti-Submarine Warfare (ASW) Operations Research Group
<b>ASUW</b>	Anti-Surface Warfare
<b>CASP</b>	computer-aided search program
<b>CNA</b>	Center for Naval Analyses
<b>ISR</b>	Intelligence, Surveillance and Reconnaissance
<b>JW</b>	Joint Warfare
<b>MIW</b>	Mine Warfare
<b>NPS</b>	Naval Postgraduate School
<b>SDT</b>	Search and Detection Theory
<b>SOF</b>	Special Operation Forces
<b>SW</b>	Strike Warfare
<b>USCG</b>	United States Coast Guard
<b>USN</b>	U.S. Navy
<b>USV</b>	unmanned surface vehicle
<b>UUV</b>	unmanned underwater vehicle
<b>WWII</b>	World War II

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# CHAPTER 1:

## Introduction

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The process of finding the location of one or more unknown subjects is known as Search and Detection Theory (SDT) and is of great interest to U.S. Navy (USN). While there are many applications, both military and non-military, SDT is of particular import to Anti-Submarine Warfare (ASW). This is due to the desire to locate adversarial submarines because the nature of submarines is to use their covert operating patterns to conduct Intelligence, Surveillance and Reconnaissance (ISR), Anti-Surface Warfare (ASUW), Strike Warfare (SW) and Special Operation Forces (SOF) missions.

With the advent of high end diesel-electric submarines, increased tensions and activity from capable nuclear submarines (Russia and China), and the proliferation of submarine technologies to previously non-capable navies, the need for better approaches to SDT is evident. The proliferation of these submarine technologies, and the rise of autonomous vehicles such as unmanned underwater vehicles (UUV) and unmanned surface vehicles (USV), both as preprogrammed searchers and potential targets, dictate that the parameters of SDT algorithms must be investigated.

### 1.1 History

The Operations Evaluation Group (now known as the Center for Naval Analyses (CNA)) [1] was created during World War II (WWII) to enlist the help of mathematicians and scientists to assist in developing new ASW tactics and strategies. This small group of MIT scientists worked with ASW Operations Research Group (ASWORG) directly in the field for optimum observations and real-time solutions [2]. These scientists used the burgeoning search theory to improve U-boat search and neutralization, especially with favorable patterns and operating characteristics for patrol aircraft.

After the war, search theory was extended to other naval operations such as searching for the *USS Scorpion* (SSN-589), lost ordnance, and tracking Russian ballistic missile submarines [3]. The United States Coast Guard (USCG) developed a computer-aided search program (CASP) [4] for missing boats and mariners lost at sea.

Dr. Bernard Koopman of the WWII ASWORG developed the historical basis for search theory by dictating the following basic principles.

1. Quantify the probability of the sensor capability to detect the target.
2. Classify the movement of the searchers and target.
3. Analyze the constraints on the searcher's forces and resources
4. Develop the highest probability of detection with the greatest economy of forces [5].

## **1.2 Applications and Previous Research**

The applications for SDT are numerous, so that while the context of this research is limited to an ASW submersible search problem, the basics can be extended to various problem sets and models. Research conducted at Naval Postgraduate School (NPS) has focused on naval, non-naval, and other Joint Warfare (JW) applications. Research has also been conducted at non-military institutions for civilian applications.

### **1.2.1 Some Previous Research**

1. Zachary Lukens's 2016 thesis [6] investigated SDT applied to a single target in the South China Sea and two searchers using cookie cutter sensors utilizing inline spiral, inline ladder, and multipath ladder searches. His research found that increasing either the detection radii or speeds decreased the time to detection and that spiral search was faster than ladder searches at large radii, but both ladder searches were faster than spiral search for small radii.
2. Volkan Sozen's 2014 thesis [7] used SDT applied to Unmanned Aerial Vehicles (UAV) patrolling a border. He modeled the search as one UAV searching the area, two UAVs searching the same area, and two UAVs independently searching two equally split sub-areas. His research found that detection increased with the second searcher and splitting the searchers into independent sub-areas, the border was covered more efficiently.
3. Dr. Michael Atkinson [8] (together with Moshe Kress and Rutger-Jan Lange) researched searching for a target discrete location based on receiving of intelligence of varying reliability and the time cost incurred based on searching a certain area. His research became the problem of looking at the probability of detection based on

amount of intelligence gathered weighed against the target escaping due to a delayed search. This has direct applications to military and non-military problems based on the information quality. The study concluded that high intelligence credibility had better results in waiting to attack, while low credibility had better results for immediate attack.

4. H. Zhou and H. Wang in [9] examined optimum paths for a solitary searcher looking for a solitary target using spiral, ladder, loop and square spiral paths. They showed that a stationary searcher is optimally placed in the center of the search area. It also showed that, with a moving searcher, the ladder pattern gives the most likely detection. In [10], they examined optimum searcher placement seeking a target travelling back and forth between a detectable operating area and an undetectable hiding area. They showed closed form solutions for determining mean time to detection based on the placement of the searcher in a given path from hiding to operating area or within the operating area itself. Their results showed how to distribute sensors ("searchers") among the routes and the operating area. They extended their study to include sensors in detecting a target moving between a hiding area and an operating area [11]. Based on the average time to detection, they were able to evaluate the performance of placing the sensor(s) to monitor various travel pathway(s) or to scan the operating area.

### **1.2.2 Applications**

The USN has utilized SDT for

- ASW
- ASUW
- Anti-Air Warfare (AAW)
- SW
- Mine Warfare (MIW)
- other operations.

Other branches and JW applications include

- ISR
- Counter-drug operations
- Counter-terrorism operations

- Ballistic Missile Defense
- SW and Artillery Fires operations
- Search and Rescue.

Multiple civilian applications are also prevalent in research and development, to include:

- Search and Rescue
- Quality Control
- Resource exploration and search (oil, minerals, fishing, etc.)
- Microeconomics (resource to market matching).

---

## CHAPTER 2: Discrete Search

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### 2.1 Theory

The following sections outline the basic concepts and theory behind the discrete search theory.

#### 2.1.1 Terms

$A$  - Total search area (no searches conducted outside of  $A$ )

$n$  - Total number of equal area subdivisions (boxes) of  $A$  : ( $i \in \{1..n\}$ )

$p_i$  -  $P[\text{tgt in } i\text{th box}]$  probability of target present in  $i$ th box

$q_i$  -  $P[\text{miss detection} | \text{tgt in } i\text{th box}]$  probability of failing to detect target in  $i$ th box  
given it is present in  $i$ th box

$k_i$  - number of *glimpses* (discrete time interval) with search was conducted in  $i$ th box

$m$  - total number of glimpses:  $m = \sum_{i=1}^n k_i$

By the concept of total probability,  $\sum_{i=1}^n p_i = 1$ .

#### 2.1.2 General Concepts

The basic assumptions for the discrete search model are [12]:

- Single stationary target
- Single searcher with imperfect sensor ( $q_i$ )
- Total search time limited ( $m$  glimpses)
- No time lost transitioning from one box to another
- Each search conducted independent of other glimpses
- No false positives
- Non-adaptive search.

The probability of the target not being detected in the  $i$ th box on a given glimpse is

$P[\text{miss detection}] = P[\text{miss detection} \mid \text{tgt in } i\text{th box}]P[\text{tgt in } i\text{th box}] = P_M^i = p_i q_i$  and the detection probability on a given glimpse for the  $i$ th box is  $P[\text{detection}] = P[\text{detection} \mid \text{tgt in } i\text{th box}]P[\text{tgt in } i\text{th box}] = P_D^i = p_i(1 - q_i)$ . After  $k_i$  glimpses, the probabilities of missing or detecting in the  $i$ th box are  $P_M^{i,k_i} = p_i q_i^{k_i}$  and  $P_D^{i,k_i} = p_i(1 - q_i^{k_i})$ , respectively. This leads to [13], by the total probability concept, the likelihood of total failure and detection as

$$P_M = \sum_{i=1}^n p_i q_i^{k_i} \quad (2.1)$$

and

$$P_D = \sum_{i=1}^n p_i(1 - q_i^{k_i}). \quad (2.2)$$

### 2.1.3 Model Overview

The greedy algorithm is a myopic search model that is designed to attempt to optimize Equation 2.1 in order to streamline the detection based on a limited number of resources (time, searchers, etc.). For each glimpse, the probability of detection is calculated given previous glimpses have failed to detect the target. This change in detection probability [13]

$$\Delta = (1 - q_i^{k_i+1})p_i - (1 - q_i^{k_i})p_i = (1 - q_i)q_i^{k_i}p_i \quad (2.3)$$

is used to select the maximum detection probability and the search is conducted in that box, increasing  $k_i$  by 1 for that box. This updates the probability for that box alone. If the target is not found, the search proceeds to the next glimpse using the same formula. This will be repeated until the target is found or  $\sum_{i=1}^n k_i = m$ . The cumulative detection probability is the sum of the maximum  $\Delta$  for at every step or Equation 2.3.

### 2.1.4 Example

The example in Table 2.1 from [12], outlines the "greedy" search algorithm. The target is in one of three boxes, with associated location and conditional miss probabilities. The probability of detection for each cell is calculated based on finding the target on the first glimpse with no previous looks ( $\vec{k} = (0, 0, 0)$ ) so that  $q_i^{k_i} = 1$ . The highest probability is in the first cell making the  $q_i^{k_i} < 1$  reducing the probability on the next round of glimpses.

The cumulative probability after six glimpses is the sum of the maximum probabilities at each round,  $P_D = .25 + .125 + .1 + .08 + .064 + .0625 = .6815$ .

Table 2.1. Optimal Sequence of Discrete Looks. Adapted from [12].

$(k_1, k_2, k_3)$			$p_1(1 - q_1)q_1^{k_1}$	$p_2(1 - q_2)q_2^{k_2}$	$p_3(1 - q_3)q_3^{k_3}$
$p_1$	$p_2$	$p_3$	(0,0,0)	<b>.25</b>	.1
			(1,0,0)	<b>.125</b>	.1
	$q_1$	$q_2$	(2,0,0)	.0625	<b>.1</b>
			(2,0,1)	.0625	<b>.08</b>
			(2,1,1)	.0625	<b>.064</b>
			(2,2,1)	<b>.0625</b>	.0512

## 2.2 Problem and Model

This basic model for discrete search is based solely on the conditional miss probabilities ( $q_i$ ) and the location probabilities ( $p_i$ ). These parameters will be varied and results shown.

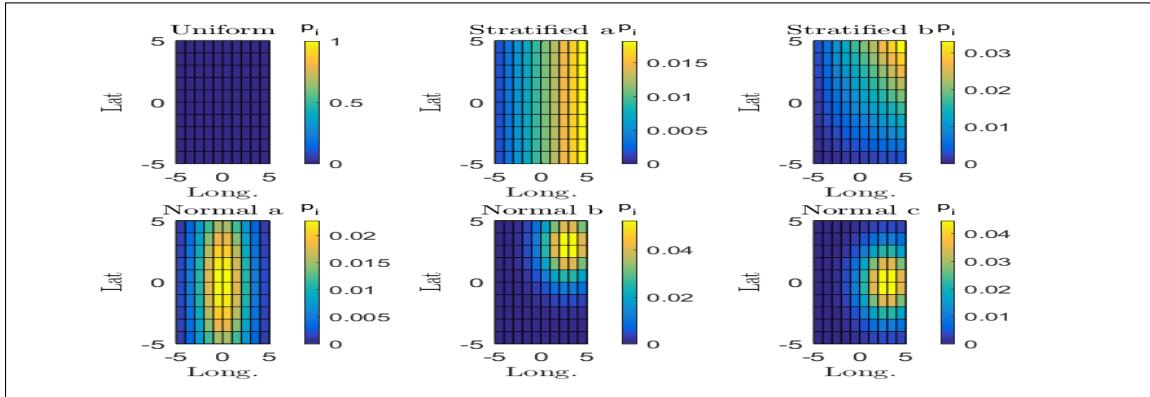
### 2.2.1 Parameter

The location probabilities will be varied according to Figure 2.1 and the detection probabilities will be varied according to Figure 2.2. The uniform location and conditional detection probabilities are to set up the base case to compare other distribution combinations too.

The base location probability per box,  $p_0 = \frac{1}{n} = \frac{1}{100}$  as shown in the uniform case. Two linear stratified location probability densities are used to simulate the target being placed preferentially along some line of longitude (small scale) for the first distribution or some line of constant longitude/latitude line. Each of these increase by a factor of 10 over the search area in the direction of greatest change. Three Gaussian distributions are used to simulate the likelihood of the target having a well-localized location probability prior. They follow a distribution function  $p(x, y) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-x_0)^2}{2\sigma_x^2}-\frac{(y-y_0)^2}{2\sigma_y^2}}$ . The centered Gaussian has different  $\sigma_{lat}$  and  $\sigma_{long}$ . The other two Gaussian plots are centered on a corner and side with  $\sigma_{lat} = \sigma_{long}$ .

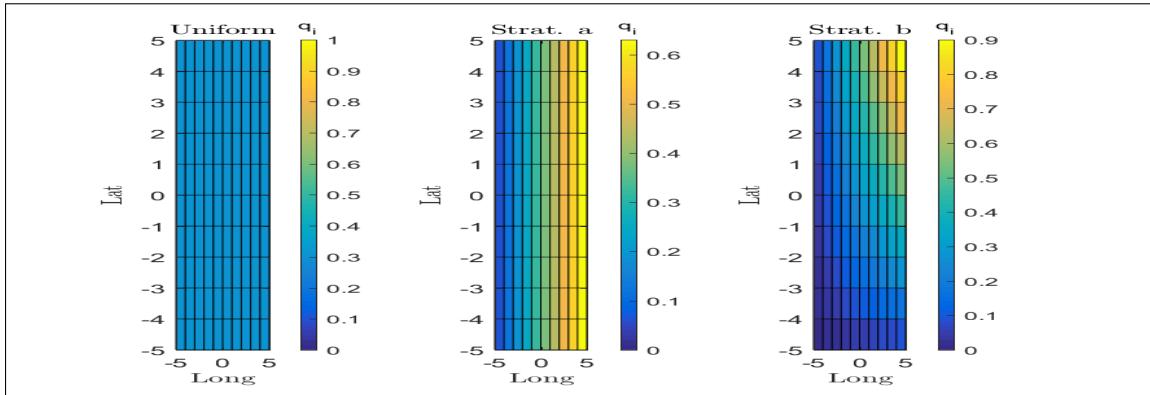
The conditional detection probability densities are uniform as a base sensor that works the same in all locations  $q_i = .3$ , giving a 70% of detection at each location. The two stratified detection probability densities are linear stratified along latitude or longitude from  $q_i = (.063, .63)$ . This simulates the sensor performance based on some geographic profile (i.e., change in bottom type for SONAR).

Figure 2.1. Location Probability Distributions  $\mathbf{p}_i$



From top left clockwise: Uniform dist., Linear stratified along longitude, Linear stratified along Long/Lat, Gaussian dist., Gaussian centered in corner, Gaussian centered on side

Figure 2.2. Conditional Miss Probability Distributions  $\mathbf{q}_i$



From left to right: Uniform dist., Linear stratified along longitude, Linear stratified along latitude

## 2.3 Results

The following sections show the results for six different location probability densities and the three conditional search probability densities. Each combination was examined for  $n = 50$  glimpses (half the number of available boxes).

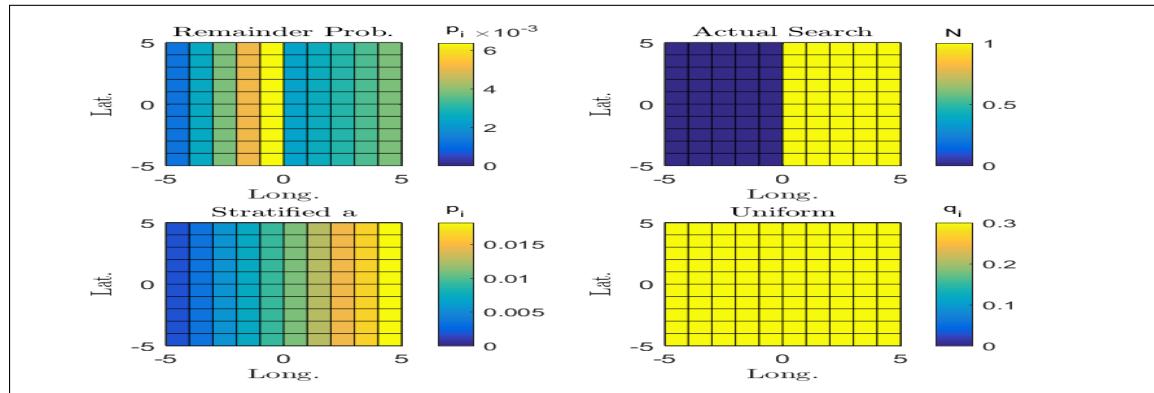
Since there are 18 possible combinations each with various tables and figures, the following will show a small sample of the results. All of the figures and tables will be presented in Appendices A and B, respectively.

### 2.3.1 Linear Stratified Location

Figures 2.3, 2.4, and 2.5 show the remaining location probability density for the three conditional search probability densities after 50 glimpses. Also shown are the boxes that were selected to search by the greedy algorithm.

For the uniform conditional search probability in Figure 2.3, unsurprisingly, the selected boxes start with the highest  $p_i$  and slowly work to the left as  $p_i$  lowers.

Figure 2.3. Stratified Location and Uniform Search Density Results

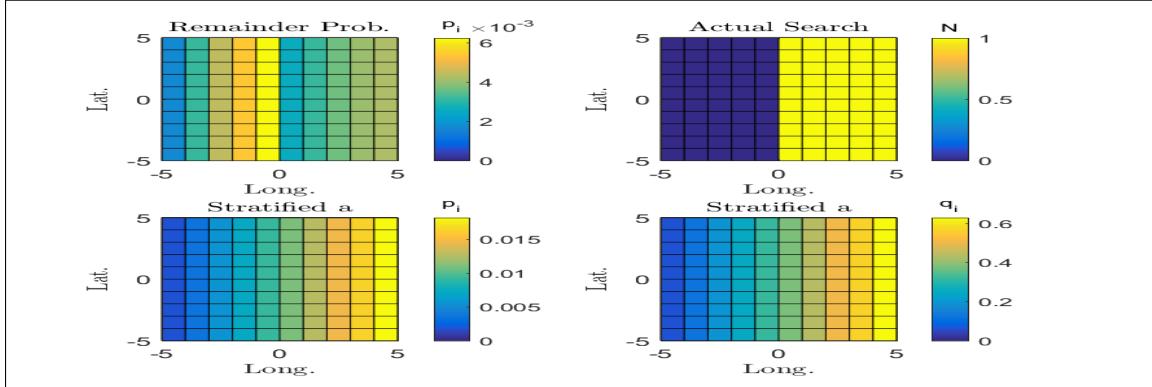


Remaining probability of detection and boxes search based on stratified  $p_i$  and uniform  $q_i$

For the longitudinal stratified conditional search probability in Figure 2.4, the selected boxes also started with highest  $p_i$  and slowly work to the left as  $p_i$  lowers. This is slightly unexpected since a better probability of detection exists in areas of lower target location probability. But the factor of  $1 - q_i$  is 93.7% in the western side and 37% in the east. This

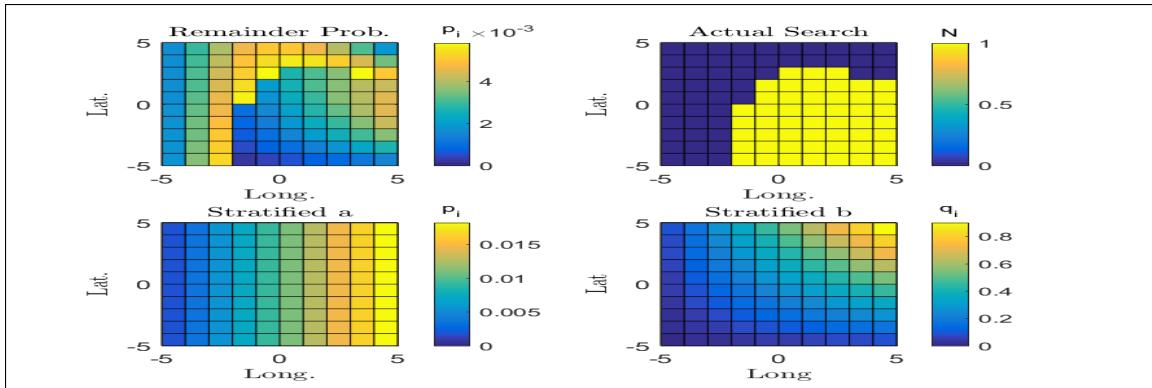
coupled with the  $p_i$  being a factor of 10 higher in the east leads it to be a higher probability of detection even though the sensor performs worse there.

Figure 2.4. Stratified Location and Longitudinal Stratified Search Results



Remaining probability of detection and boxes search based on stratified  $p_i$  and linear stratified  $q_i$

Figure 2.5. Stratified Location and Corner Stratified Search Density Results



Remaining probability of detection and boxes search based on stratified  $p_i$  and corner stratified  $q_i$

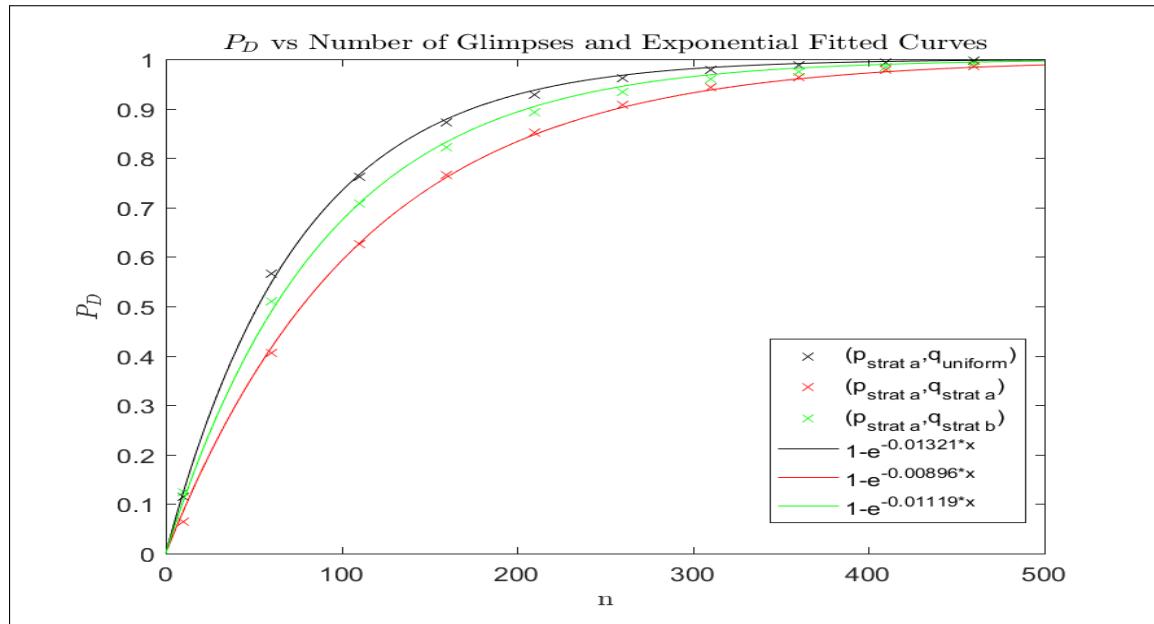
For the corner stratified conditional search probability in Figure 2.5, the selected boxes start in the southeast due to the juxtaposition of high  $p_i$  and low  $q_i$ . The path moves mostly along discretized radial arcs centered on the southeast corner. Which box is chosen via the greedy algorithm is dependent on when  $1 - q_i$  will push a probability of detection higher for a lower  $p_i$ .

For the longitudinal stratified  $p_i$  location probability density, the following results were found based on running the search for up to 500 glimpses. A data set using a step size of 5 is shown in Figure 2.6 along with associated fit lines as calculated from MATLAB's fit algorithm based on:

$$f(x) = 1 - e^{-bx}.$$

Each of the fit lines are color coded to the data set with black for uniform, red for longitudinal stratification, and green for corner stratification. As the number of glimpses increases, the probability approaches 1, with the uniform case increasing faster than the other two.

Figure 2.6. Stratified Location Cumulative Probability and Fit Lines



Cumulative probabilities of detection based on the number of glimpses. Included are the fit best fit line

Table 2.2. Parameter Variation for  $p_i$  for Longitudinal Stratification

Parameter	$b$	$r^2$	SSE	RMSE
$p_{strat}, q_{uniform}$	0.013208	0.99908	0.00065853	0.008554
$p_{strat}, q_{long}$	0.0089597	0.99920	0.00066179	0.0085751
$p_{strat}, q_{corner}$	0.011187	0.99808	0.0013515	0.012254

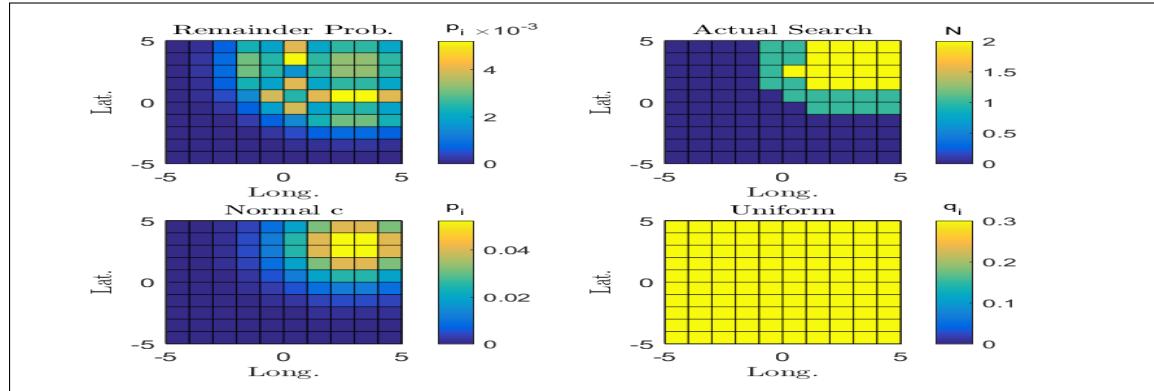
Best fit line parameters for a longitudinally stratified location probability density of the form  $f(x) = 1 - e^{-bx}$

### 2.3.2 Gaussian Corner Location

The remaining location probability densities for the three conditional search probability densities after 50 glimpses are shown in Figures 2.7, 2.8, and 2.9. Also shown are the boxes that were selected to search by the greedy algorithm.

For the uniform conditional search probability in Figure 2.7, unsurprisingly, the selected boxes start with highest  $p_i$  and slowly work outward as  $p_i$  depletes. A second look begins occurring in those higher probability boxes prior to searching all boxes.

Figure 2.7. Gaussian Corner Location and Uniform Search Density Results

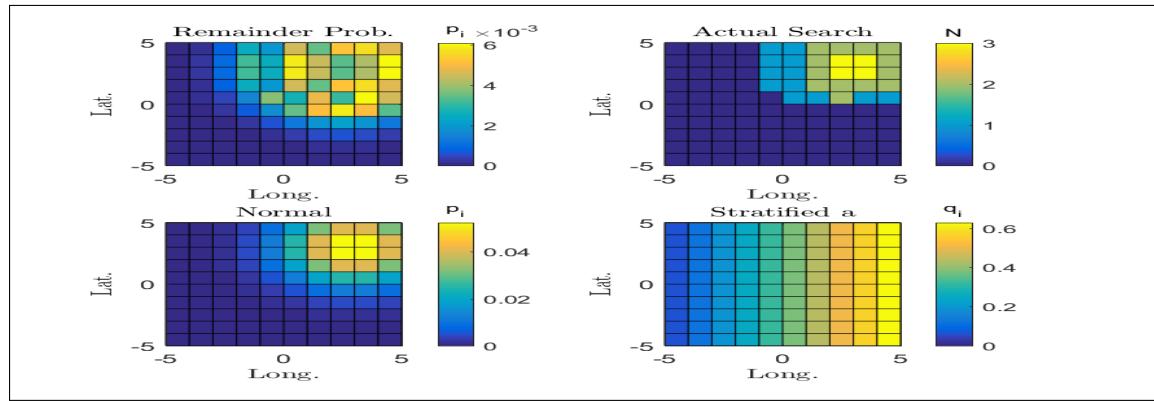


Remaining probability of detection and boxes search based on Gaussian  $p_i$  and uniform  $q_i$

For the longitudinal stratified conditional search probability in Figure 2.8, the selected boxes also started with highest  $p_i$  and slowly works outward as  $p_i$  lowers. This is slightly

unexpected since there is better probability of detection in some areas of lower target location probability. As the probability of detection decreases on subsequent looks, the lower  $q_i$  increases the likelihood of finding the target to the west of the initial location locus. From the Remainder Probability graph in Figure 2.8, the higher location probability remains dispersed in the northwest corner albeit non-uniformly.

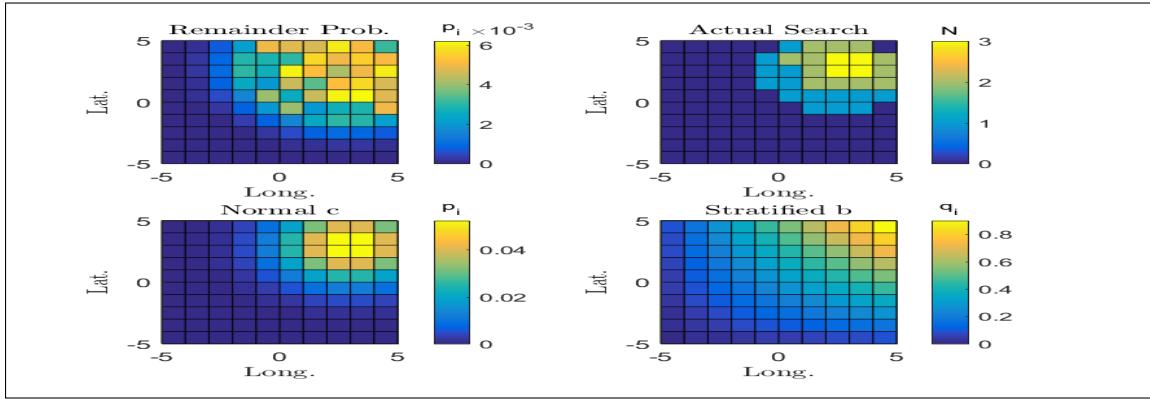
Figure 2.8. Gaussian Corner and Longitudinal Stratified Search Density Results



Remaining probability of detection and boxes search based on stratified  $p_i$  and linear stratified  $q_i$

For the corner stratified conditional search probability in Figure 2.9, the selected boxes start in the northeast due to the high  $p_i$  overriding the high  $q_i$ . The Remainder Probability graph in Figure 2.9 shows some areas 2-3 times greater than other areas, especially around the original locus. Some areas will be searched multiple times before every box is searched once.

Figure 2.9. Gaussian Corner and Corner Stratified Search Density Results



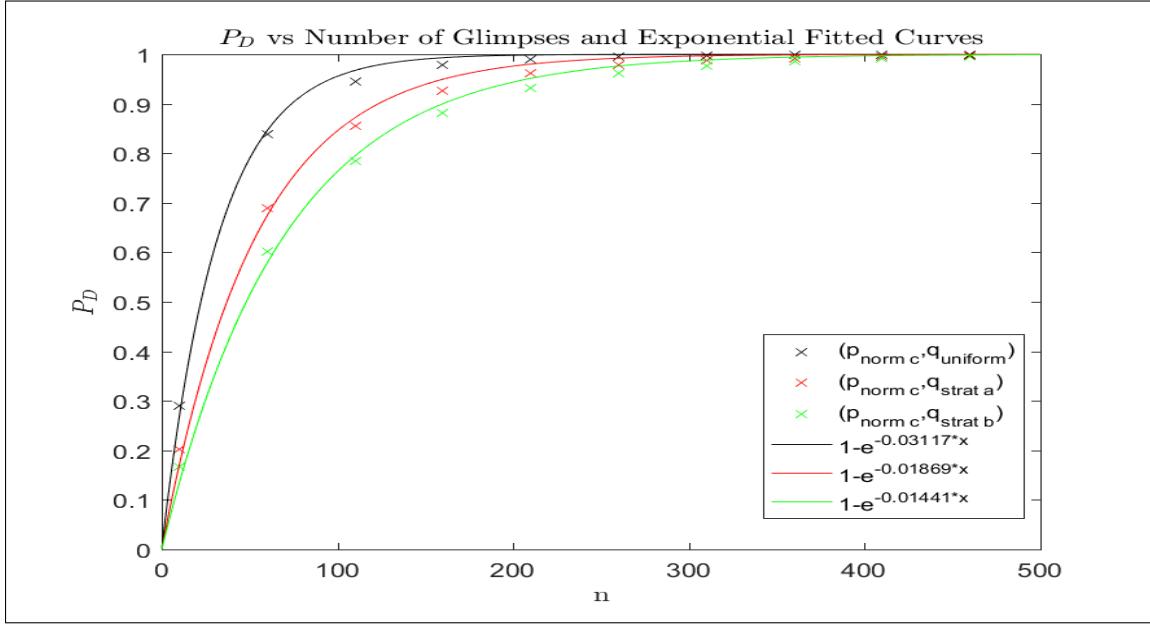
Remaining probability of detection and boxes search based on stratified  $p_i$  and corner stratified  $q_i$

For the Gaussian Corner  $p_i$  location probability density, the following results were found based on running the search for up to 500 glimpses. A data set using a step size of 5 is plotted in Figure 2.10 along with associated fit lines as calculated from MATLAB's fit algorithm based on:

$$f(x) = 1 - e^{-bx}.$$

Each of the fit lines are color coded to the data set with black for uniform, red for longitudinal stratification, and green for corner stratification. As the number of glimpses increase, the probability approaches 1, with the uniform case increasing faster than the other two.

Figure 2.10. Gaussian Corner Cumulative Probability and Fit Lines



Cumulative probabilities of detection based on the number of glimpses. Included are the fit best fit lines

Table 2.3. Parameter Variation for  $p_i$  for Corner Gaussian Location

Parameter	$b$	$r^2$	SSE	RMSE
$p_{GC}, q_{uniform}$	0.03117	0.99683	0.0013941	0.012446
$p_{GC}, q_{long}$	0.01869	0.9952	0.0027014	0.017325
$p_{GC}, q_{corner}$	0.01441	0.99523	0.0029724	0.018173

Best fit line parameters for a Gaussian Corner location probability density of the form  $f(x) = 1 - e^{-bx}$

### 2.3.3 Cumulative Detection Probability

The total probability of detection for the after 50 glimpses is shown in Table 2.4. The Gaussian  $p_i$ 's have higher probabilities, which is not unexpected since they have a much higher concentration on a few points. This allows for multiple searches over a smaller number of points instead of 50 different boxes. The other result seems to be that higher

probability results when  $p_i$  matches up to  $1 - q_i$  as it does when  $p_{stratified}$  has a higher probability of detection when paired with  $q_{corner,strat}$  instead of  $q_{long,strat}$ .

Table 2.4.  $P_D$  after 50 glimpses

$p_i \backslash q_i$	Uniform	Long. Stratified	Corner Stratified
Uniform	0.35	0.4055	0.44906
Long. Stratified	0.50909	0.34927	0.46119
Corner Stratified	0.57157	0.43958	0.40031
Gaussian	0.55932	0.52965	0.5919
Gaussian Side	0.71881	0.55738	0.65176
Gaussian Corner	0.79975	0.63655	0.54999

Total probability of detection after 50 glimpses based on various combinations

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# CHAPTER 3:

## Bayesian Search

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### 3.1 Theory

The following sections outline the basic concepts and theory behind the Bayesian approach discrete search theory.

#### 3.1.1 Terms

The same terms from Chapter 2 apply along with the following random variables:

$L_i$  - Target is located in box  $i$

$M_i$  - Target is not detected in box  $i$

$p_0$  -  $P[\text{tgt not in } A]$  probability of target not in search area

By the concept of total probability,  $\sum_{i=1}^n p_i = P_A$  and  $1 - P_A = p_0$ .

#### 3.1.2 General Concepts

We use the same basic assumptions of Section 2.1.2 with an added assumption that the target may not be in the search area. The basics of Bayesian search theory is to update the probability of the target being located in the  $i$ th ( $p_i$ ) box based on previous failures to detect. Searching the  $i$ th box and failing to find it will decrease its likelihood of being located there and increase the probability of its being located in another box. This posterior probability comes from Bayes' Theorem.

$$P(L_i | M_i) = \frac{P(M_i | L_i)P(L_i)}{P(M_i)} \quad (3.1)$$

The probability of not detecting the target in the  $i$ th box ( $P(M_i)$ ) is currently the only uncalculated quantity. It is, by total probability, the sum of the conditional detection failures based on the location probabilities.  $P(M_i) = P(M_i | L_i)P(L_i) + P(M_i | \neg L_i)P(\neg L_i)$ .  $P(L_i)$  and  $\neg P(L_i)$  are  $p_i$  and  $1 - p_i$ , respectively.  $P(M_i | L_i) = q_i$  and  $P(M_i | \neg L_i) = 1$  based

on the assumption of now false positives. This updates Equation 3.1 to

$$p'_i = P(L_i \mid M_i) = \frac{q_i p_i}{(1 - p_i) + p_i q_i}. \quad (3.2)$$

The probability that the target is located in  $j$ th box ( $j \neq i$ ) must also be updated since it is more likely that it is not located in the  $i$ th box,

$$P(L_j \mid M_i) = \frac{P(M_i \mid L_j)P(L_j)}{P(M_i)}, \quad (3.3)$$

where  $P(L_i) = p_j$  and  $P(M_i) = (1 - p_i) + p_i q_i$  as found above.  $P(M_i \mid L_j)$  is the conditional probability of failing to detect the target in box  $i$  given it is in box  $j$ . Since there are no false positives, the searcher will always fail to detect when searching in a box where the target is not located, making  $P(M_i \mid L_j) = 1$ . This updates the location probability by

$$p'_{j \neq i} = P(L_j \mid M_i) = \frac{p_j}{(1 - p_i) + p_i q_i}. \quad (3.4)$$

This updates all  $p_{i \neq j}$  including  $p_0$ .

### 3.1.3 Model Overview

In the Bayesian search model, the conditional probabilities of detection failure only factor into updating the new location probabilities. As such, the search algorithm searches the box with the highest likelihood of containing the target and searches. Upon failure to find, it updates all location probabilities and repeats until the target is found or the number of glimpses has been reached.

This still follows the Bayesian search algorithm from Chapter 2 in that at each step the probability to detect  $P_{Di} = p_i(1 - q_i)$  is calculated for all boxes. The search is conducted in the box with the highest detection probability. After a failure to detect, the  $p_i$ 's are updated via Equation 3.2 and Equation 3.4 [14]. The Bayesian search algorithm then treats the next glimpse as the first with the updated location probabilities [14].

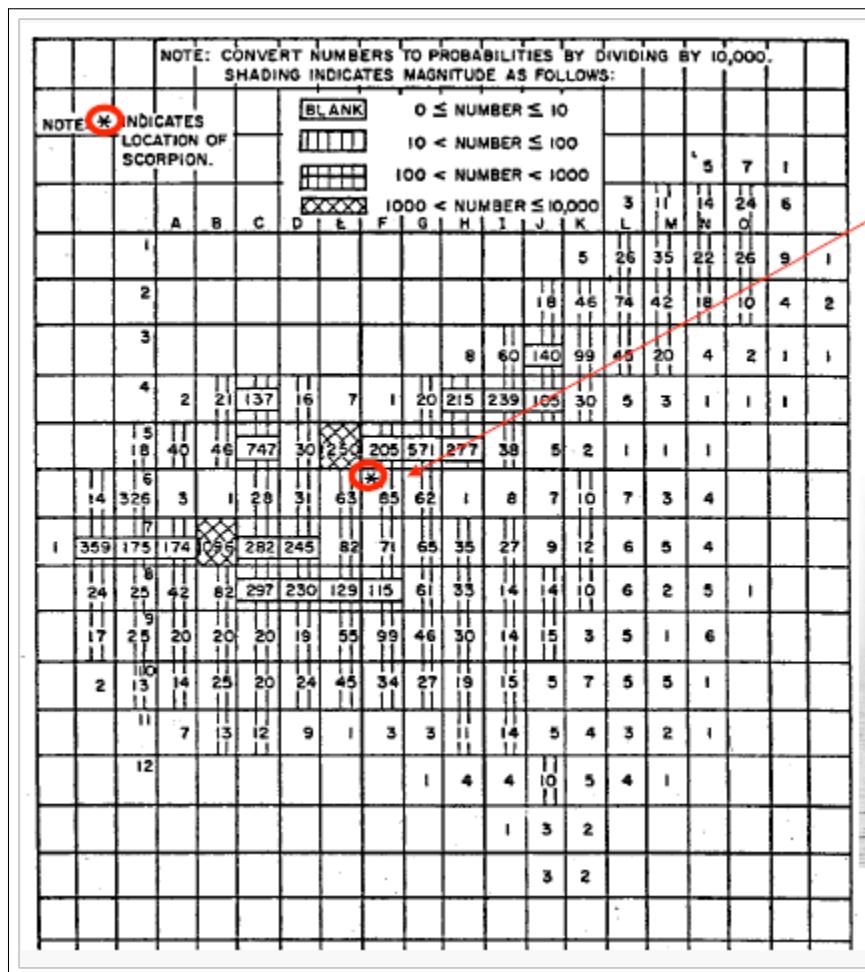
If  $p_0 = 0$ , this will provide the same output as the Discrete Search Method. For example, the updated location probabilities are just  $\Delta_i$  from the Discrete Method divided by  $1 - q_i$ 's and then normalized so that the sum is equal to 1. Multiplying  $1 - q_i$ 's back in after

normalization will make the same evaluation as in Table 2.1 [14].

### 3.1.4 Examples from Other Research

The Bayesian method has been used for several real-world searches. Famously used in *USS Scorpion*, a submarine lost in 1968, it was also used in *MV Derbyshire*, the *SS Central America*, the lost hydrogen bomb from a B-52 crash, Air France flight 447, and most recently flight MH370. Figure 3.1 is the location probability density originally constructed for the *USS Scorpion* [15].

Figure 3.1. *USS Scorpion* Location Probability. Source: [15].



The basic steps outlined in previous examples are:

1. Construct plausible conjectures as to the cause of the loss of the object.
2. For each conjecture, construct a location probability density map ( $P(L_i) = p_i$ ).
3. Build the conditional detection probability maps for every location covered under the location probability map ( $P(M_i | L_i) = q_i$ ). This will be a function of sensor's ability to detect in the given environment and must be built individually based on current conditions and sensor dynamics.
4. Combine the maps in Step 2 and 3 to create an overall detection probability density map ( $p_i(1 - q_i)$ ).
5. Develop a search path that scans the highest to lowest probability areas.
6. Revise the location probabilities continuously during the search. If multiple conjectures are being tested simultaneously, each should be updated independently and if the search fails to corroborate aspects of a particular conjecture the likelihood of that scenario will need to be downgraded [15].

One last aspect of the Bayesian search, that is not being explored in this research, is the concept of the cost of a search. If, for example, searching one particular box takes longer time, requires more resources, or is more dangerous, then that should be taken into account as a weight on the search. If the cost function can be estimated as  $c_i$ , then the new detection probabilities come out as  $\frac{p_i(1 - q_i)}{c_i}$  [16].

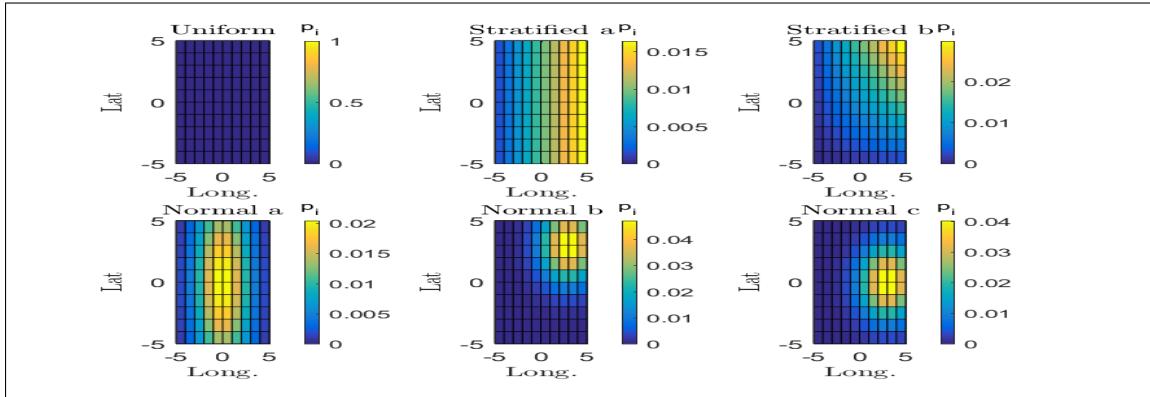
## 3.2 Problem and Model

This basic model for Bayesian search is based solely on the conditional miss probabilities ( $q_i$ ) and the initial location probabilities ( $p_{oi}$ ). These parameters will be varied and results shown.

### 3.2.1 Parameters

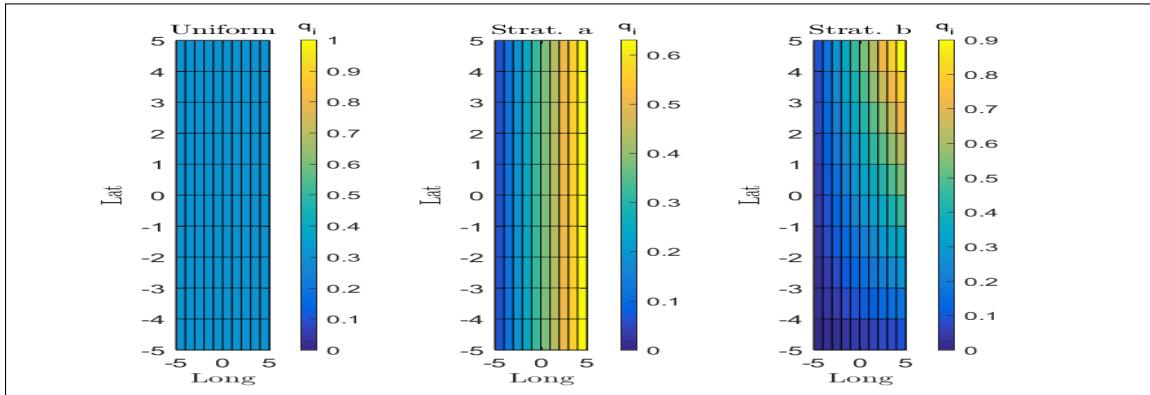
The location and conditional miss will have the same variations as presented in Chapter 2 (Figures 2.1 and 2.2) and they are repeated in Figures 3.2 and 3.3 for the reader's convenience with  $p_0 = .1$  and the detection probabilities will be varied according to Figure 2.2, which are shown again here for clarity.

Figure 3.2. Bayesian Initial Location Probabilities  $p_{oi}$



From top left clockwise: Uniform dist., Linear stratified along longitude, Linear stratified along Long/Lat, Gaussian dist., Gaussian centered in corner, Gaussian centered on side

Figure 3.3. Bayesian Miss Probabilities  $q_i$



From left to right: Uniform dist., Linear stratified along longitude, Linear stratified along latitude

### 3.3 Results

The following sections show the results for six different location probability densities and the three conditional search probability densities. Each combination was examined for  $n = 50$  glimpses (half the number of available boxes).

The tables and figures showing all 18 possible combinations are presented in Appendix A

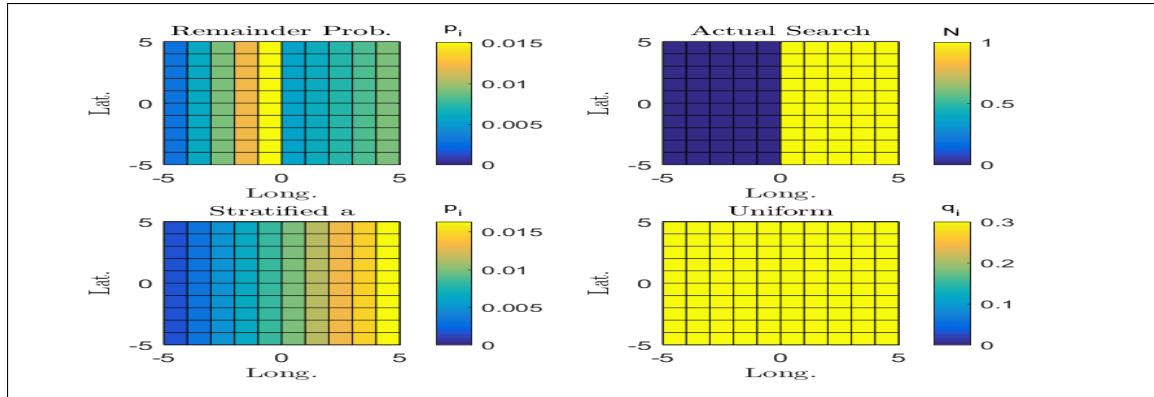
and Appendix B, respectively.

### 3.3.1 Linear Stratified Location

Figures 3.4, 3.5, and 3.6 show the remaining location probability densities for the three conditional search probability densities after 50 glimpses. Also shown are the boxes that were selected to search by the Bayesian search algorithm.

For the uniform conditional search probability in Figure 3.4, unsurprisingly, the selected search boxes start with highest  $p_i$  and slowly work to the left as  $p_i$  lowers. This follows the same pattern as the discrete search from Chapter 2. The exception being that the upper-leftmost graph of Figure 3.4 now shows the updated location probabilities in Bayes' algorithm instead of merely the next search step as in the discrete search method. This follows as the Bayesian method and Discrete method calculate roughly the same probability of detection with the exception that the Bayesian search modifies  $p_0 = .1 \rightarrow 0.1846$ .

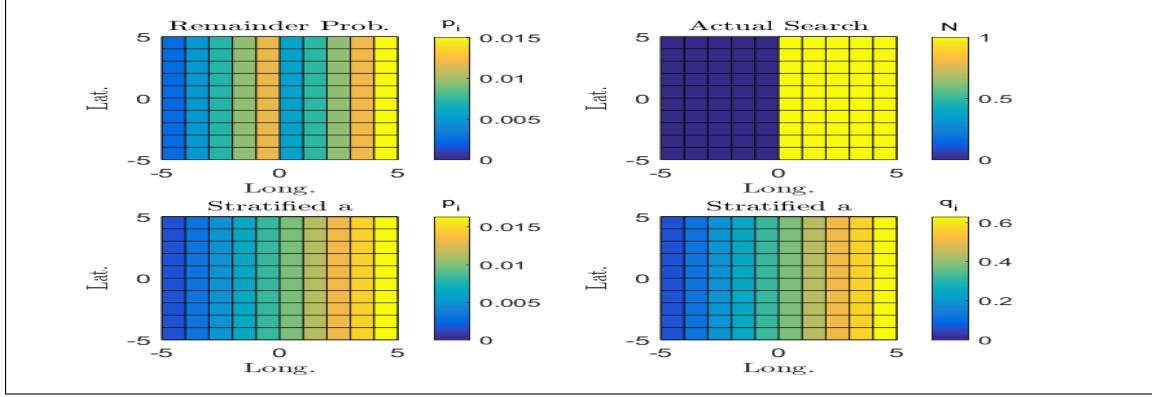
Figure 3.4. Stratified Location and Uniform Search Density Results



Remaining probability of detection and boxes search based on stratified  $p_i$  and uniform  $q_i$

For the longitudinal stratified conditional search probability in Figure 3.5, the selected search boxes also start with highest  $p_i$  and slowly work to the left as  $p_i$  lowers. This follows the slightly unexpected results from the discrete search of Chapter 2 possibly from the factor of  $1 - q_i$  being 93.7% in the western side and 37% in the east. This coupled with the  $p_i$  being a factor of 10 higher in the east than the west leads it to be a higher probability of detection even though the sensor performs worse there. The probability of the target not being in the search area  $p_0 = .1 \rightarrow 0.1458$ .

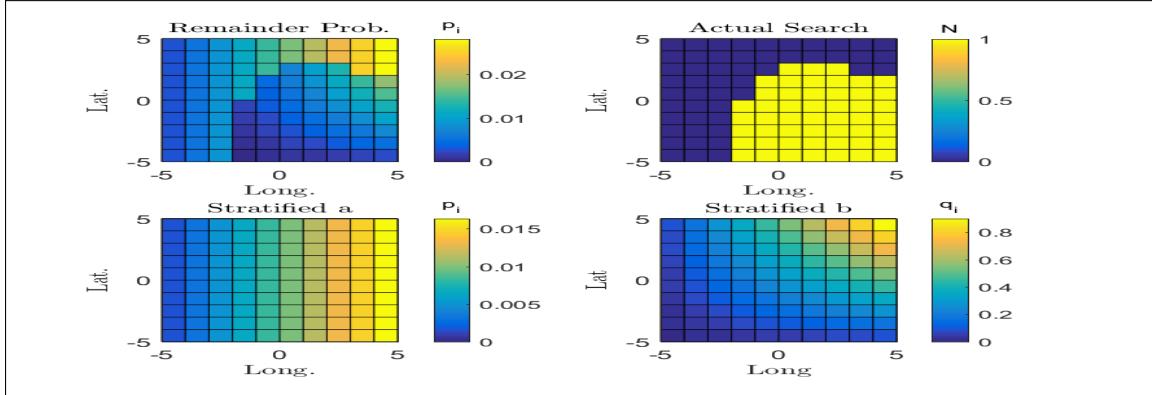
Figure 3.5. Stratified Location and Longitudinal Stratified Search Results



Remaining probability of detection and boxes search based on stratified  $p_i$  and linear stratified  $q_i$

For the corner stratified conditional search probability in Figure 3.6, the selected boxes start in the southeast due to the juxtaposition of high  $p_i$  and low  $q_i$ . The path moves mostly along discretized radial arcs centered on the southeast corner. Which box is chosen via the Bayesian search algorithm is dependent on when  $1 - q_i$  will push a probability of detection higher for a lower  $p_i$ . The probability of the target not being in the search area  $p_0 = .1 \rightarrow 0.1710$ .

Figure 3.6. Stratified Location and Corner Stratified Search Density Results



Remaining probability of detection and boxes search based on stratified  $p_i$  and corner stratified  $q_i$

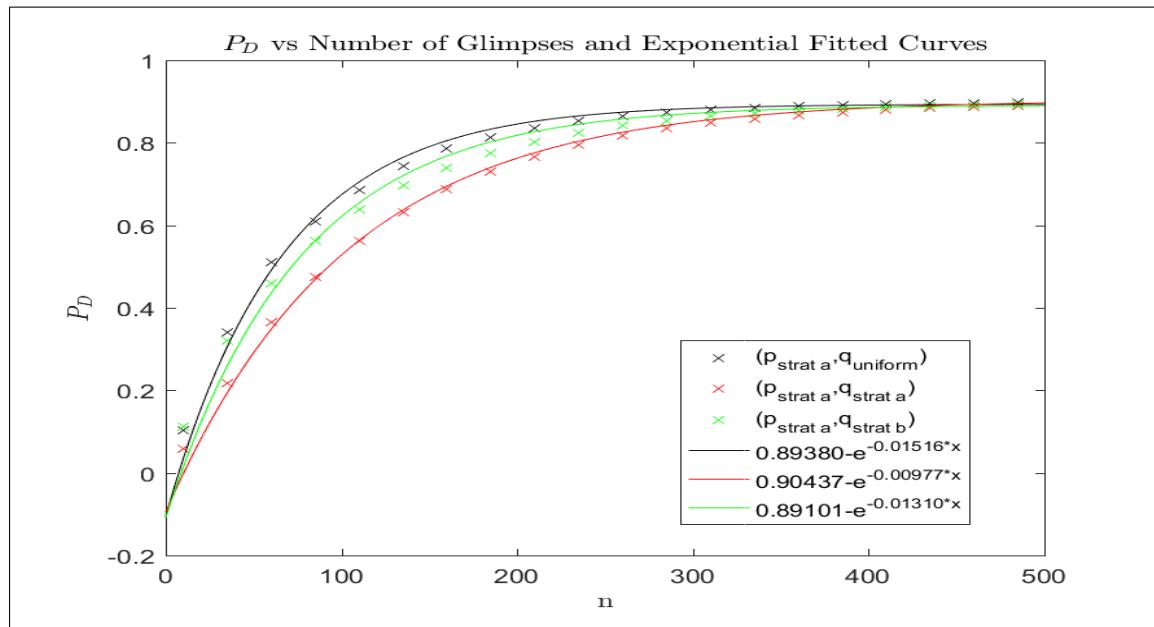
For the longitudinal stratified  $p_i$  location probability density, the following results were

found based on running the search for up to 500 glimpses or a total probability of detection at 95%. A data set using a step size of 5% is plotted in Figure 3.7 along with associated fit lines as calculated from MATLAB's fit algorithm based on

$$f(x) = a - e^{-bx}.$$

Each of the fit lines is color coded to the data set with black for uniform, red for longitudinal stratification, and green for corner stratification. As the number of glimpses increases, the probability approaches  $P_A = .9$ , with the uniform case increasing faster than the other two. The associated parameters for the best fit lines are presented in Table 3.1.

Figure 3.7. Stratified Location Cumulative Probability and Fit Lines



Cumulative probabilities of detection based on the number of glimpses. Included are the fit best fit lines

The results are very similar to results from Chapter 2 as the same decision algorithm is used; however, the exponential growth approaches  $P_A = .9$  the initial probability that the target is in the search area  $A$ .

Table 3.1. Parameter Variation for  $p_i$  for Longitudinal Stratification

Parameter	$a$	$b$	$r^2$	SSE	RMSE
$p_{strat}, q_{uniform}$	0.8938	-0.015155	0.99013	0.0086678	0.0093465
$p_{strat}, q_{long}$	0.90437	-0.0097729	0.99496	0.0053481	0.017237
$p_{strat}, q_{corner}$	0.89101	-0.0131	0.97996	0.017876	0.031514

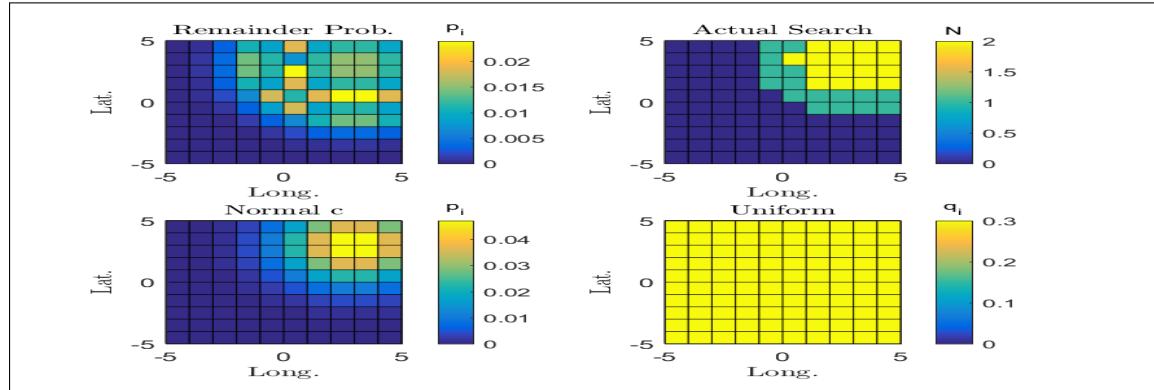
Best fit line parameters for a longitudinally stratified location probability density of the form  $f(x) = a - e^{-bx}$

### 3.3.2 Gaussian Corner Location

Figures 3.8, 3.9, and 3.10 show the remaining location probability densities for the three conditional search probability densities after 50 glimpses. Also shown are the boxes that were selected to search by the Bayesian search algorithm.

For the uniform conditional search probability in Figure 3.8, unsurprisingly, the selected boxes started with highest  $p_i$  and slowly works outward as  $p_i$  shifts higher probability to other boxes. A second look begins occurring in those higher probability boxes prior to searching all boxes.

Figure 3.8. Gaussian Corner Location and Uniform Search Density Results

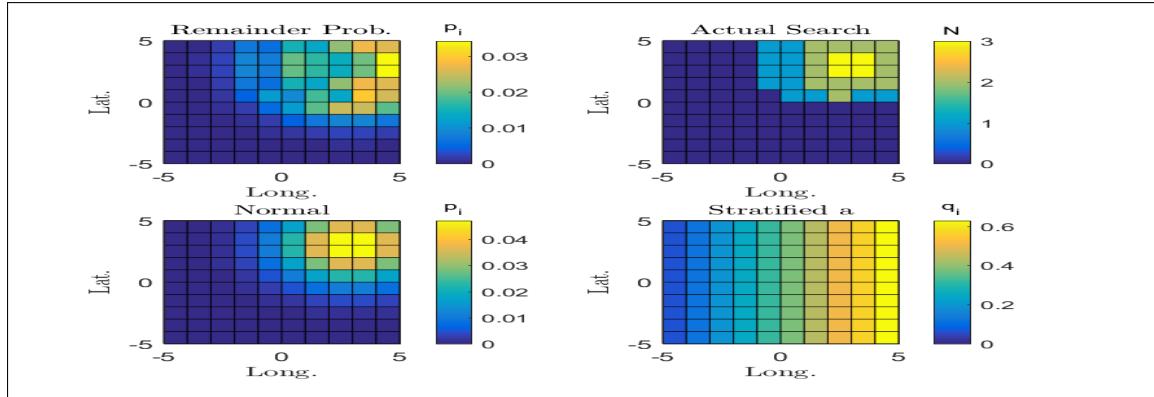


Remaining probability of detection and boxes search based on Gaussian  $p_i$  and uniform  $q_i$

For the longitudinal stratified conditional search probability in Figure 3.9, the selected boxes also started with highest  $p_i$  and slowly work outward as  $p_i$  lowers. This follows the slightly unexpected result from Chapter 2 since there are better probabilities of detection in some

areas of lower target location probability. As the location probability shifts to other boxes, the higher  $p_i$  increases the likelihood of the target being located to the west of the initial location locus. From the Remainder Probability graph in Figure 3.9, the higher location probability remains dispersed in the northwest corner albeit non-uniformly.

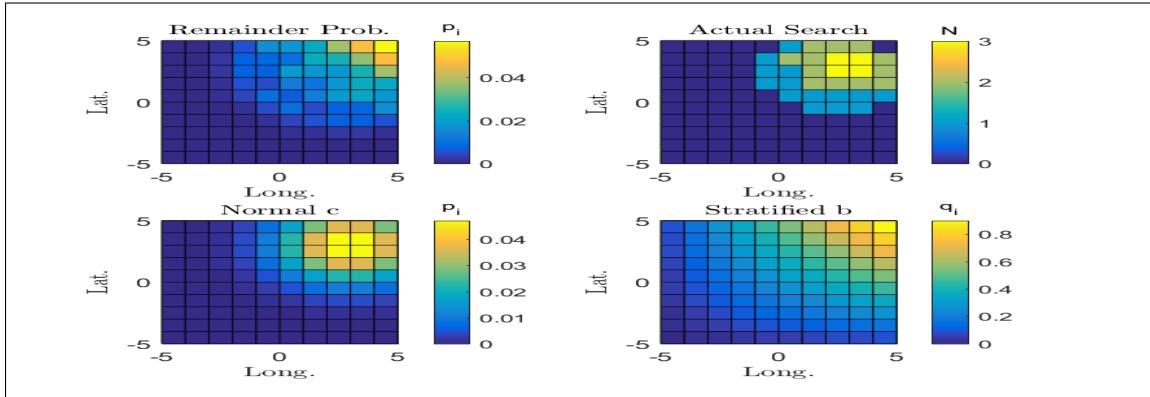
Figure 3.9. Gaussian Corner and Longitudinal Stratified Search Density Results



Remaining probability of detection and boxes search based on stratified  $p_i$  and linear stratified  $q_i$

For the corner stratified conditional search probability in Figure 3.10, the selected boxes start in the northeast due to the high  $p_i$  overriding the high  $q_i$ . The Remainder Probability graph in Figure 3.10 shows some areas 2-3 times greater than other areas, especially around the original locus. Some boxes have been searched twice or three times before every box has been searched once. Since the location probability distribution  $p_i$  remains identical to that from Chapter 2 (other than the being 90% of the original), the overall patterns of the search are identical to the results from Chapter 2.

Figure 3.10. Gaussian Corner and Corner Stratified Search Density Results



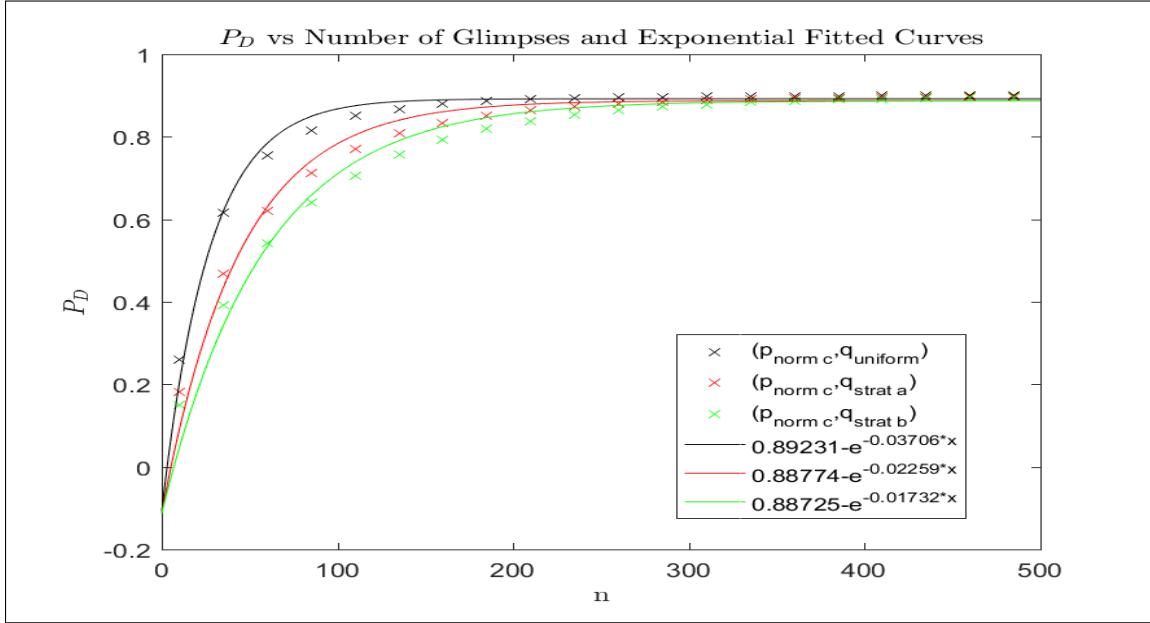
Remaining probability of detection and boxes search based on stratified  $p_i$  and corner stratified  $q_i$

For the Gaussian corner  $p_i$  location probability density, the following results were found based on running the search for up to 500 glimpses or a total probability of detection at 95%. A data set using a step size of 5% is plotted in Figure 3.11 along with associated fit lines as calculated from MATLAB's fit algorithm based on

$$f(x) = a - e^{-bx}.$$

Each of the fit lines are color coded to the data set with black for uniform, red for longitudinal stratification, and green for corner stratification. As the number of glimpses increase, the probability approaches  $P_A = .9$ , with the uniform case increasing faster than the other two. The associated parameters for the best fit lines are presented in Table 3.2.

Figure 3.11. Gaussian Corner Cumulative Probability and Fit Lines



Cumulative probabilities of detection based on the number of glimpses. Included are the fit best fit lines

Table 3.2. Parameter Variation for  $p_i$  for Corner Gaussian Distribution

Parameter	$a$	$b$	$r^2$	SSE	RMSE
$PGC, q_{uniform}$	0.89231	-0.03706	0.98424	0.0068884	0.019563
$PGC, q_{long}$	0.88774	-0.022588	0.97697	0.014581	0.028462
$PGC, q_{corner}$	0.88725	-0.01732	0.9747	0.018924	0.032424

Best fit line parameters for a corner Gaussian location probability density of the form  $f(x) = a - e^{-bx}$

The results are very similar to results from Chapter 2 as the same decision algorithm is used.

### 3.3.3 Cumulative Detection Probability

The total probability of detection produced by the Bayesian search algorithm after 50 glimpses is shown in Table 3.3. The Gaussian  $p_i$ 's have higher probabilities, which is not

unexpected since they have a much higher concentration on a few points. This allows for multiple searches over a smaller number of points instead of 50 different boxes. The other result seems to be that higher probability results when  $p_i$  matches up to  $1 - q_i$  as it does when  $p_{stratified}$  has a higher probability of detection when paired with  $q_{corner,strat}$  instead of  $q_{long,strat}$ . This follows the same pattern as the Discrete Search Method but lower overall since there is some probability that the target is outside of  $A$ . The increase to  $p_0$  after 50 glimpses is shown in Table 3.4.

Table 3.3.  $P_D$  after 50 glimpses

$p_i \backslash q_i$	Uniform	Long. Stratified	Corner Stratified
Uniform	0.315	0.36495	0.40415
Long. Stratified	0.45818	0.31435	0.41507
Corner Stratified	0.51441	0.39562	0.36028
Gaussian	0.50339	0.47669	0.53271
Gaussian Side	0.64693	0.50164	0.58658
Gaussian Corner	0.71978	0.57289	0.49499

Total probability of detection after 50 glimpses based on various combinations

Table 3.4.  $p_0$  after 50 glimpses

$p_i \backslash q_i$	Uniform	Long. Stratified	Corner Stratified
Uniform	0.1460	0.1575	0.1678
Long. Stratified	0.1846	0.1458	0.1710
Corner Stratified	0.2059	0.1655	0.1563
Gaussian	0.2014	0.1911	0.2140
Gaussian Side	0.2832	0.2007	0.2419
Gaussian Corner	0.3569	0.2341	0.1980

Probability of target not in  $A$  after 50 glimpses based on various combinations

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## CHAPTER 4: Continuous Search

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### 4.1 Theory

The following section outlines the basic concepts and theory behind a continuous exhaustive search.

#### Terms

$A$  - Total search area (no searches conducted outside of  $A$ )

$v_S$  - Searcher's velocity

$v_T$  - Target's velocity

$R$  - Range of the sensor. Cookie cutter sensor that will detect the target if the range is less than  $R$ .  $R = R(v_S)$

$w$  - Sweep width.  $w = 2R$

The sensor is a "cookie cutter" sensor that has 100% chance of detecting the target if the distance from the searcher to the target is less than  $R$ . In [6] both  $R$  and both  $v_S$  were independent of each other and varied accordingly. Here,  $R$  is a function of  $v_S$  which more closely resembles real-world sensor performance, especially in SONAR sensors.

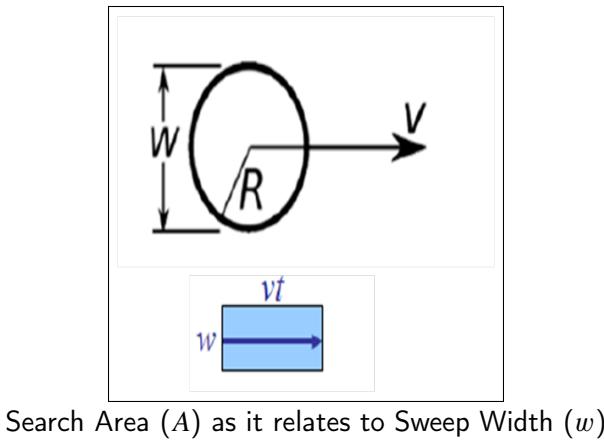
#### 4.1.1 General Concepts

The basic assumptions for this continuous search model are [6] [9] [13]:

- Single target undergoing Brownian diffusion
- Two searchers with "cookie-cutter" sensors
- Exhaustive search using one of three search patterns
- Sensors radius is velocity dependent  $R(v_S)$
- Each searcher results independent of the other
- No false positives
- Non-adaptive search.

As shown in Figure 4.1 the sweep width for a moving searcher ( $w = 2R$ ) will cover an area searched  $A(t) = wvt$ . The exhaustive search covers all of  $A$  without overhanging any previously searched area or any area outside of  $A$ . The total time to coverage then is  $t^* = \frac{A}{vw}$  [6].

Figure 4.1. Search Area and Sweep Width. Source: [9].



#### 4.1.2 Model Overview

Based on the work done by H. Wang and H. Zhou [9] and Z. Lukens's thesis [6], the two searchers will move along one of three set paths: the inline ladder, the multipath ladder and the inline spiral search. For each path setup, 100,000 independent Monte Carlo simulations are conducted with the target uniformly distributed over  $A$ . The independent targets move randomly with a Brownian motion with a given diffusion rate.

The searchers will move along the given path independently of each other without coordination. An input parameter of the initial separation of the two searcher is given so that the searchers don't overlap throughout the search. The separation in the inline searches was estimated at  $4L_x$  where  $L_x$  is half the width of  $A$  [6].

## 4.2 Problem and Model

The model is thus two searchers (aircraft, surface vessels, subs, etc.) of similar capability performing the inline spiral in Figure 4.2, the inline ladder search in Figure 4.3, and the

multipath ladder search in Figure 4.4.

Figure 4.2. Spiral Search. Source: [9].    Figure 4.3. Inline Ladder Search. Source: [9].

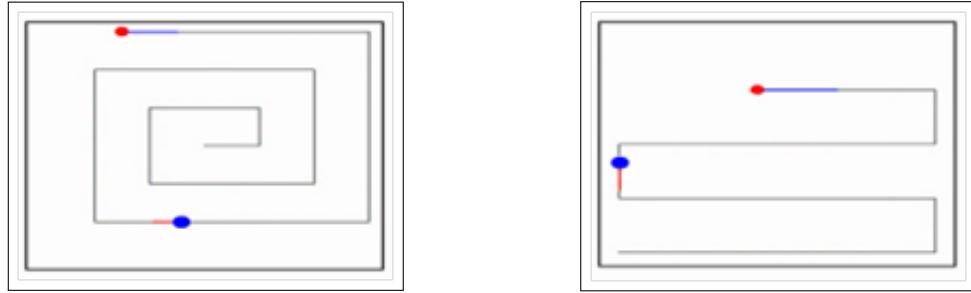
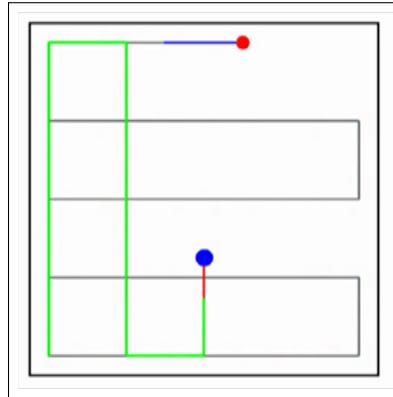


Figure 4.4. Multipath Ladder Search Pattern. Source: [9].



#### 4.2.1 Parameter

As an extension of previous work done [6], this chapter will explore the search patterns with a sensor that obeys Equation 4.1.

$$R(V_S) = R_0 e^{-V_S/V} \quad (4.1)$$

$R_0$  is the performance of the sensor at  $v_S = 0$  and  $V$  is the exponential decay factor based on the sensor performance at speed. Each sensor, in reality, will behave differently based on its characteristics. For instance, a towed SONAR array requires a minimum speed in order to be drooped correctly and at the proper depth, whereas a hull-mounted SONAR does not have this minimum effect and is only limited at higher speed by self-noise. Further

compounding the real life sensors is that, at higher speeds, the self noise has non-linear effects based on cavitation above a threshold. To simplify the issue, the sensor is assumed to be operating in the range where Equation 4.1 holds. Each search velocity will be varied as  $v_S \in (8, 9, 10, 11, 12)$  and the velocity decay factor from  $V \in (5, 6, 7, 8, 9, 10)$  giving 150 different combinations for each search pattern. The initial search radius,  $R_0 = 2.5$  nm, will be set based on the largest search radius in [6].

### 4.3 Results

In Tables 4.1, 4.2, and 4.3, the time in hours to achieve a probability of detection at 90% ( $P_D = 90\%$ ) are shown for  $V = 9$  the inline ladder search, the multipath ladder, and the spiral search, respectively.

Table 4.1. Time to  $P_D = 90\%$  ( $V = 9$ )

Velocities	Ship 1					
	8	9	10	11	12	
Ship 2	8	6.8662	6.9782	7.1117	7.5250	7.7083
	9	7.5069	7.8241	7.7043	7.8639	7.9255
	10	9.7362	9.0589	8.7202	8.4741	8.3054
	11	10.1011	10.0333	9.2198	8.9486	8.6854
	12	10.0425	10.1511	10.0968	9.2826	8.9569

Inline ladder search

Table 4.2. Time to  $P_D = 90\% (V = 9)$

Velocities		Ship 1				
		8	9	10	11	12
Ship 2	8	8.1866	8.2470	8.2123	8.1351	8.0883
	9	9.3043	8.9871	8.7202	8.3385	8.1969
	10	10.3288	9.5669	9.2282	8.7452	8.4140
	11	10.7790	10.0333	9.4231	9.0164	8.6854
	12	10.4768	10.5311	9.7168	9.1197	8.7940

Multipath ladder search

Table 4.3. Time to  $P_D = 90\% (V = 9)$

Velocities		Ship 1				
		8	9	10	11	12
Ship 2	8	9.6390	9.3043	9.6515	10.2367	10.3139
	9	9.8330	9.4100	9.3129	9.7621	9.8254
	10	10.4135	9.4822	9.3129	9.5587	9.6625
	11	10.5078	10.0333	9.5587	9.6943	9.7168
	12	10.3682	11.6710	11.5625	10.6397	10.5854

Spiral search

In a marked difference from the results found in [6], faster search speed does not result in a quicker time to reach  $P_D = .9$  in all cases. For instance, the lowest time to detection for the inline ladder search is with  $v_1 = 8$  and  $v_2 = 8$ , for multipath ladder  $v_1 = 12$  and  $v_2 = 8$ , and for spiral search  $v_1 = 9$  and  $v_2 = 8$ . In fact, keeping  $v_2 = 8$  and varying  $v_1$  over the range causes an increase and then decrease. Similar results happen on some other variations but as can be seen it is not consistent from one speed to another. Another noteworthy result is that varying the speed of ship 1 has less of an effect normally than varying ship 2's speed. An example of this can be seen in the multipath ladder. Raising  $v_1$  from 8 to 12 knots keeps

the time to detection relatively constant around 8 hours, while raising  $v_1$  from 8 to 12 knots increase the time to detect to approximately 8.5 hours.

Similar to the results found in [6], the inline ladder search has the lowest time to detect, followed by the multipath ladder, and finally the spiral search for a given set of parameters. As expected, increasing  $V$  only lowers the time to detection. This follows [6] as increasing  $V$  raises the search radii without changing the speed. Direct comparison to other results from [6] is problematic. This work varies three of the four parameters from Equation 4.1, directly varying  $v_1$  and  $v_2$  and indirectly  $R_1$  and  $R_2$ . In [6],  $R_1$  and  $R_2$  are varied directly. If Equation 4.1 is constructed with a  $V$  and  $R_0$  so that  $R_1$  and  $R_2$  equal values from [6], then results will be the same as the search algorithm work in the same manner.

A similar presentation for the various searches is outlined below in Figures 4.5-4.10.

### Spiral Search

Figure 4.5.  $P_D$  vs Time

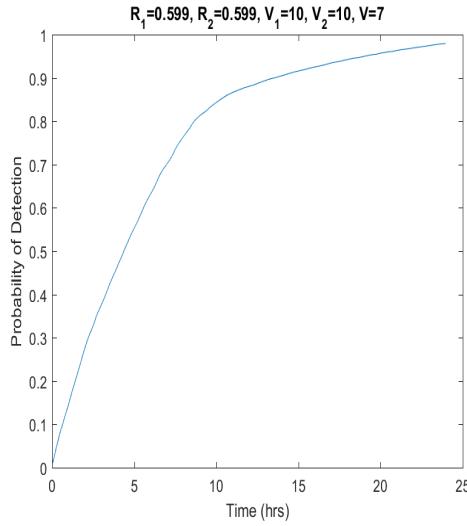
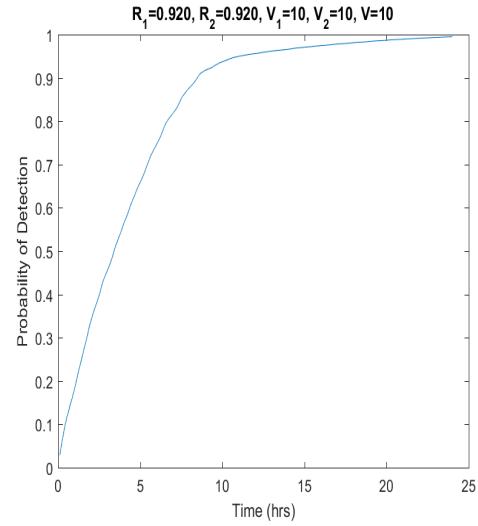


Figure 4.6.  $P_D$  vs Time



### Multipath Ladder

Figure 4.7.  $P_D$  vs Time

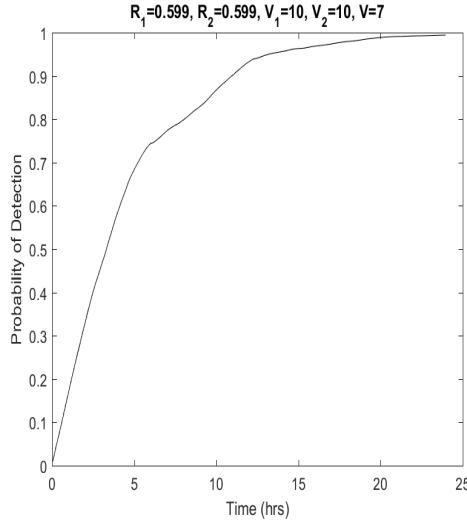
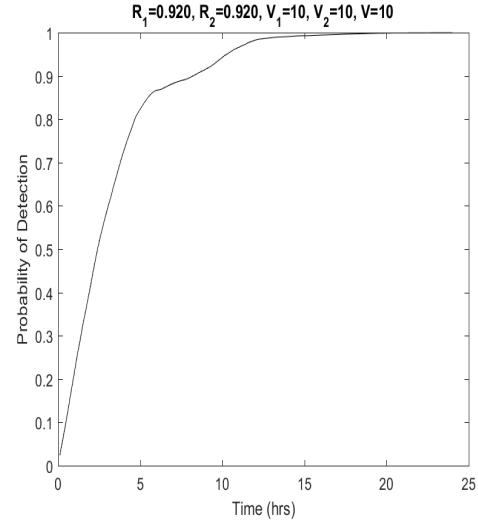


Figure 4.8.  $P_D$  vs Time



### Inline Ladder Search

Figure 4.9.  $P_D$  vs Time

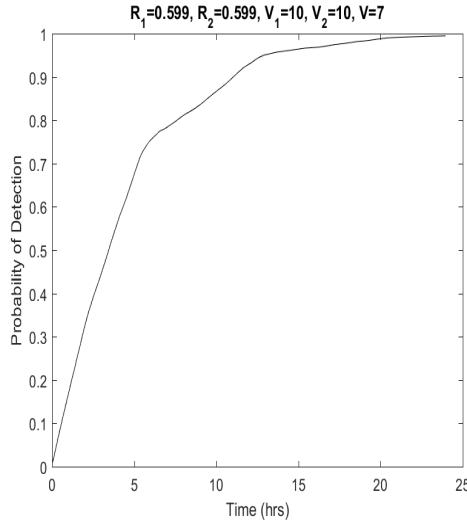
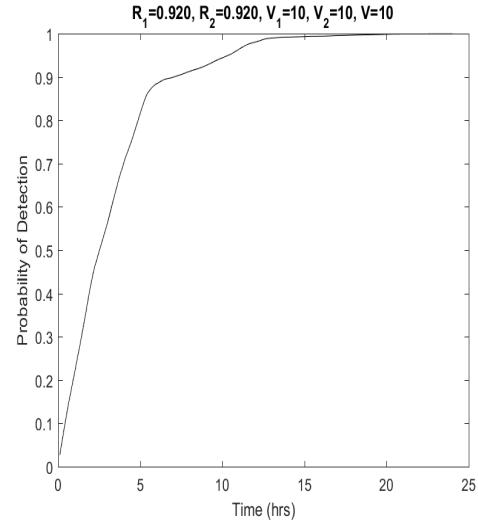


Figure 4.10.  $P_D$  vs Time



As can be seen from the figures, there is an approach to approximately  $P_D = 1 - e^{-bt}$ . An increase in  $V$ , as stated before increases  $R(v_s)$  for a given  $v_s$ . This obviously hastens  $P_D$  to higher level more quickly.

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## CHAPTER 5: Conclusions and Future Work

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Based on the results from Chapters 2-4 the following sections summarize the conclusions drawn.

### 5.1 Results and Conclusions

#### 5.1.1 Discrete and Bayesian Searches

Both the Discrete and Bayesian searches follow the greedy algorithm and give similar results. In fact, if  $p_0 = 0$ , then the results will be identical to within rounding error. As expected, the more concentrated  $p_i$  density, the higher the probability to detect  $P_D$  and the lower number of glimpse needed to reach it. This even has the capacity to override higher conditional miss probability in that same concentrated area. Both search systems provide predictable, repeatable results, but the Bayesian search provides a more useful output. By modifying the  $p_i$  at each step, not only can  $p_0 > 0$ , but faulty conjectures that created the  $p_i$  be identified by scrutinizing the changing  $p_i$  and increasing  $p_0$ . This provides a useful product by the algorithm at any point, the  $p_i$  have been calculated if the number of glimpses have been exceeded and can be used at a later date. For a discrete search, the output from the algorithm is just the failure to detect on previous searches.

#### 5.1.2 Continuous Searches

As  $v_s$  increases,  $P_D$  has a non-intuitive result. It varies in a non-linear, way some times increasing or decreasing as  $v_s$  changes. Another non-intuitive result is that  $v_1$  has a different change in  $P_D$  than  $v_2$ . The last result, an expected one, is that increasing  $V$  increases the  $P_D$  by increasing  $R$ .

### 5.2 Future Work

#### 5.2.1 Discrete and Bayesian Searches

Improvements to the Discrete and Bayesian model in future work could include:

- Multiple  $p_i$  tested simultaneously updating all based on multiple conjectures
- Multiple  $q_i$  tested simultaneously updating all based on multiple exclusive sensors
- Time dependent  $q_i$  such as  $q_i = e^{-t/\tau}$  as greedy algorithm not the optimal solution
- Time dependent  $p_i$  for a moving target.

### 5.2.2 Continuous Searches

Improvements to the continuous search in future work could include:

- Different search patterns
- Non-cookie cutter sensor with some non-uniform probability of detection
- Game theory approach with a target counter-detecting and evading.

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## APPENDIX A: Figures

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### A.1 Discrete

#### Uniform Location and Uniform Search Density Results

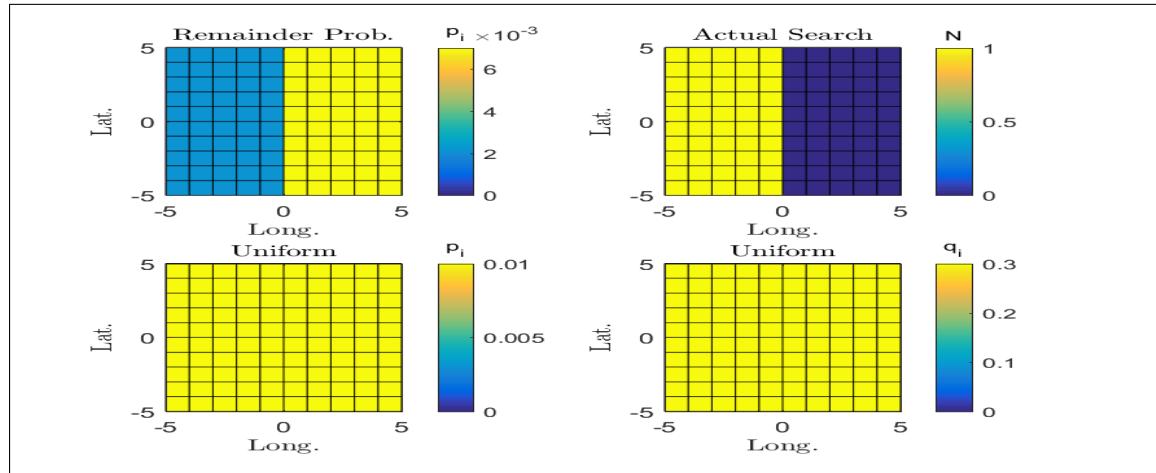


Figure A.1. Remaining probability of detection and boxes search based on uniform  $p_i$  and uniform  $q_i$

#### Uniform Location and Longitudinal Stratified Search Density Results

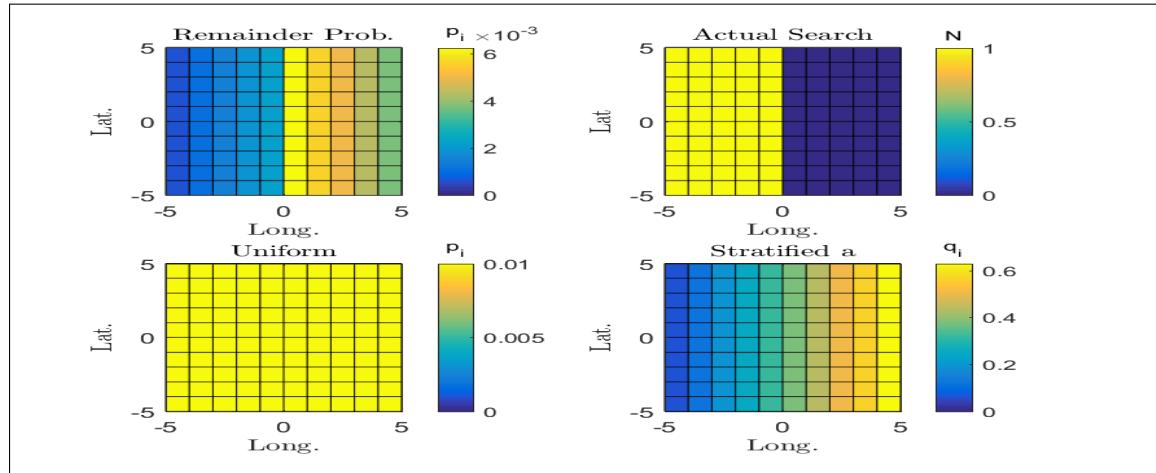


Figure A.2. Remaining probability of detection and boxes search based on uniform  $p_i$  and linear stratified  $q_i$

### Uniform Location and Corner Stratified Search Density Results

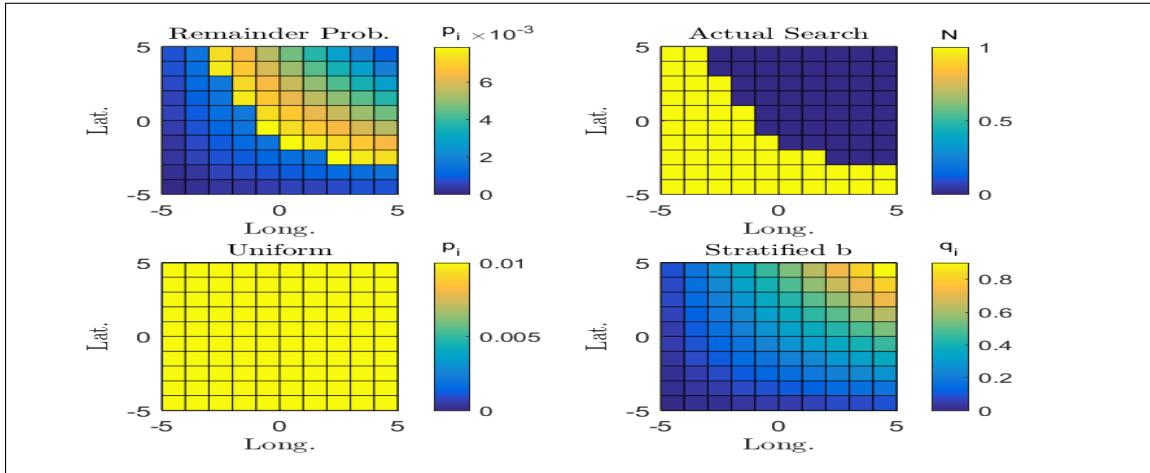


Figure A.3. Remaining probability of detection and boxes search based on uniform  $p_i$  and corner stratified  $q_i$

### Uniform Location Cumulative Probability and Fit Lines

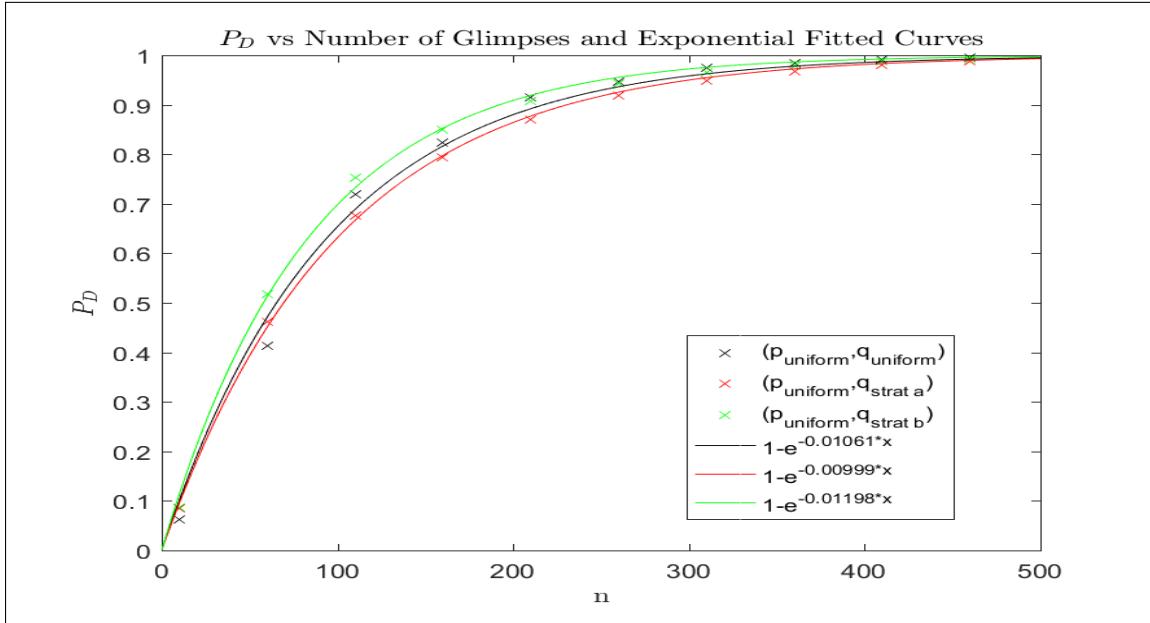


Figure A.4. Cumulative probabilities of detection based on the number of glimpses. Included are the fit best fit lines

### Corner Stratified Location and Uniform Search Density Results

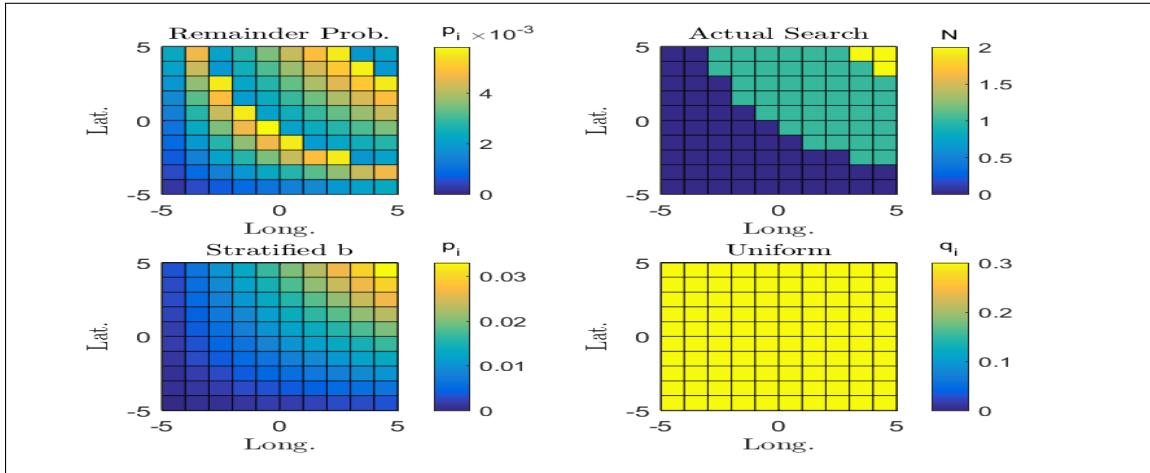


Figure A.5. Remaining probability of detection and boxes search based on corner stratified  $p_i$  and uniform  $q_i$

### Corner Stratified Location and Longitudinal Stratified Search Density Results

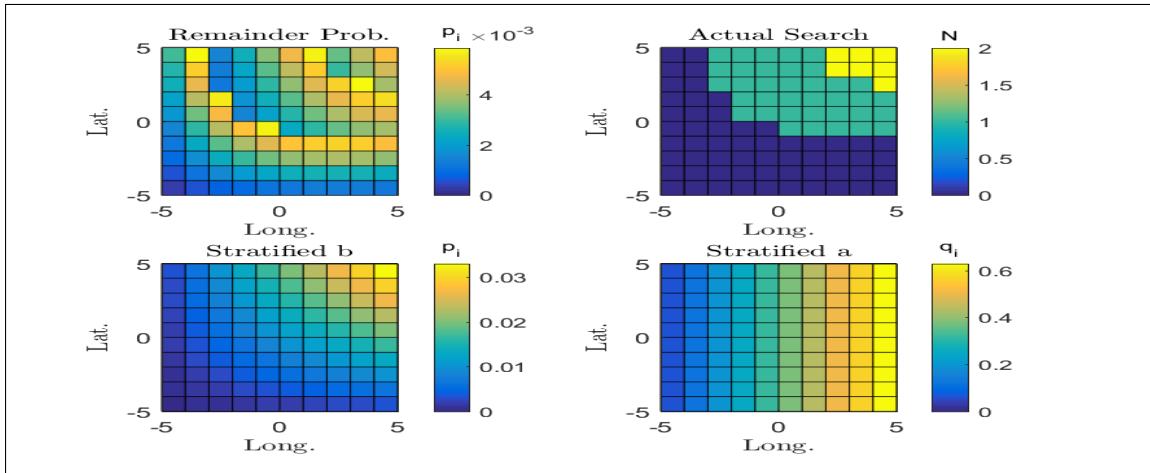


Figure A.6. Remaining probability of detection and boxes search based on corner stratified  $p_i$  and linear stratified  $q_i$

### Corner Stratified Location and Corner Stratified Search Density Results

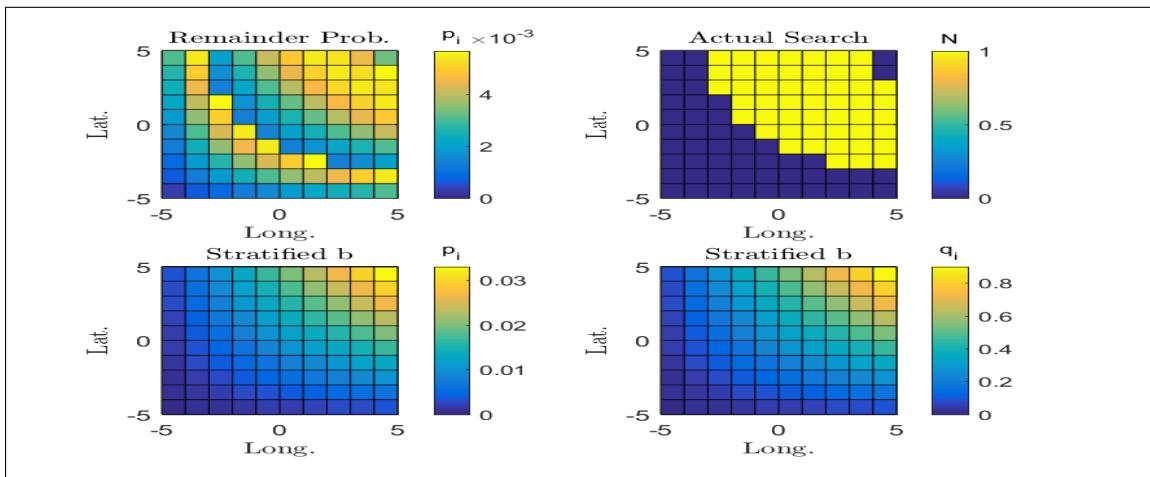


Figure A.7. Remaining probability of detection and boxes search based on corner stratified  $p_i$  and corner stratified  $q_i$

### Corner Stratified Location Cumulative Probability and Fit Lines

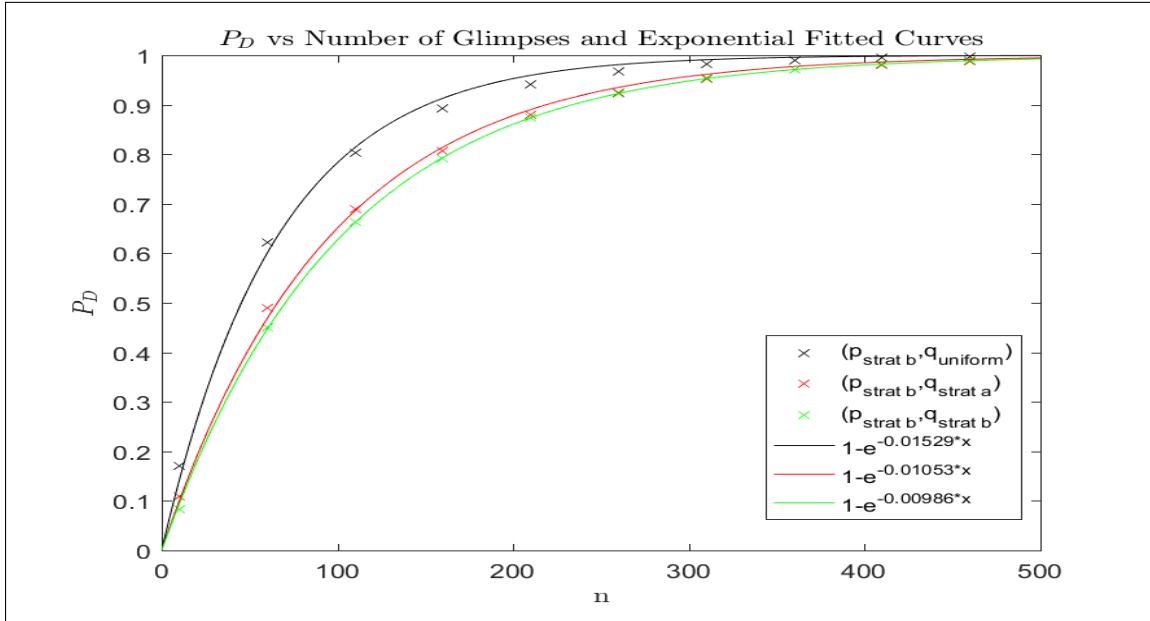


Figure A.8. Cumulative probabilities of detection based on the number of glimpses. Included are the fit best fit lines

### Gaussian Location and Uniform Search Density Results

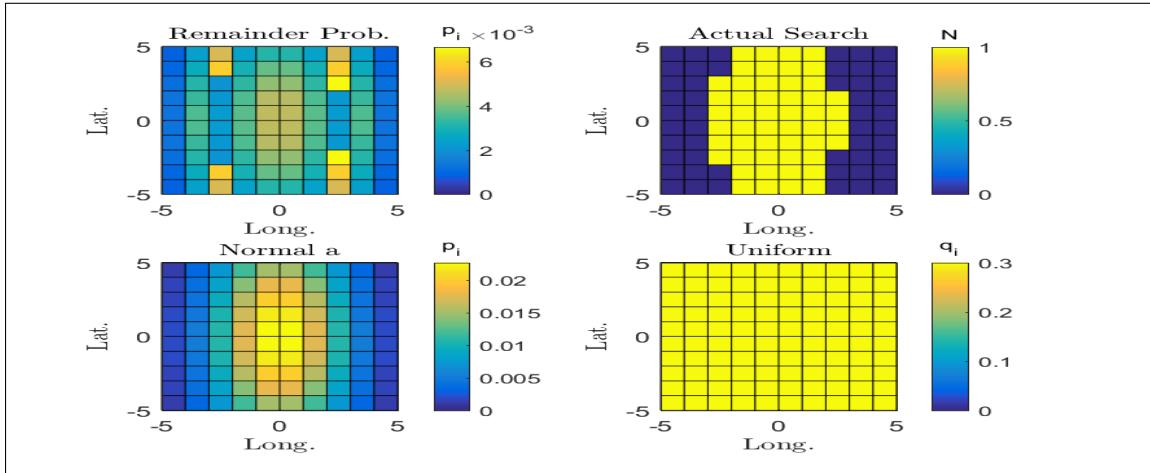


Figure A.9. Remaining probability of detection and boxes search based on Gaussian  $p_i$  and uniform  $q_i$

### Gaussian Location and Longitudinal Stratified Search Density Results

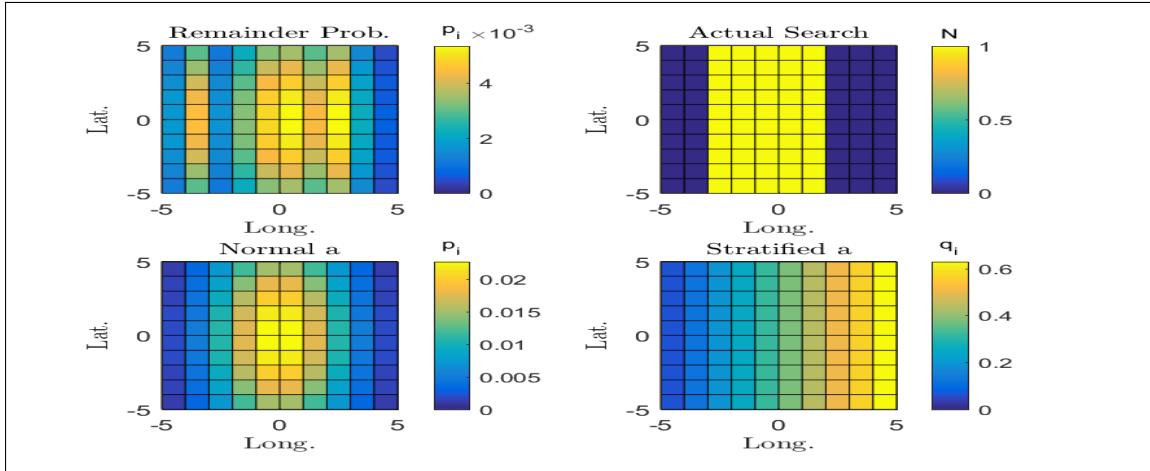


Figure A.10. Remaining probability of detection and boxes search based on Gaussian  $p_i$  and linear stratified  $q_i$

### Gaussian Location and Corner Stratified Search Density Results

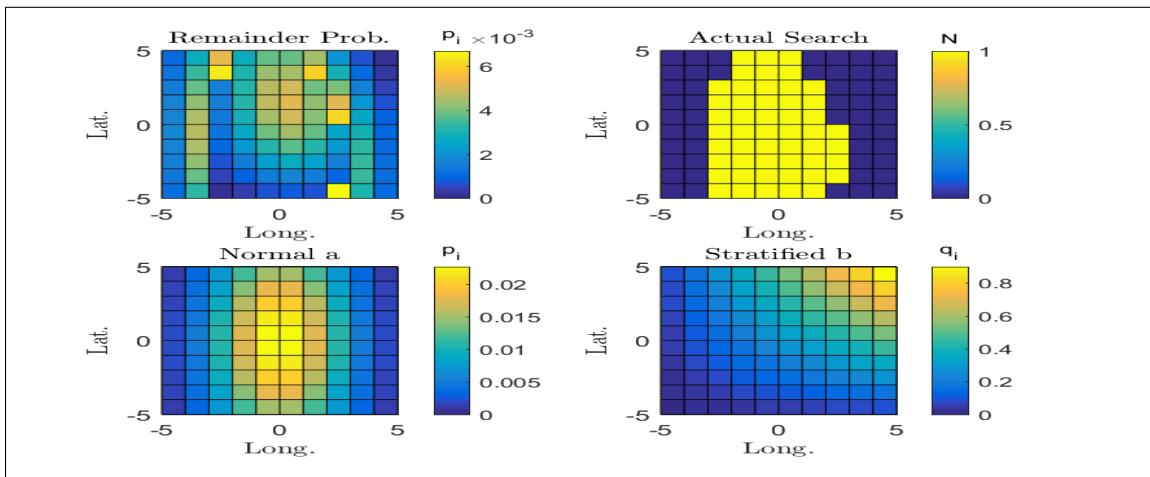


Figure A.11. Remaining probability of detection and boxes search based on Gaussian  $p_i$  and corner stratified  $q_i$

### Gaussian Location Cumulative Probability and Fit Lines

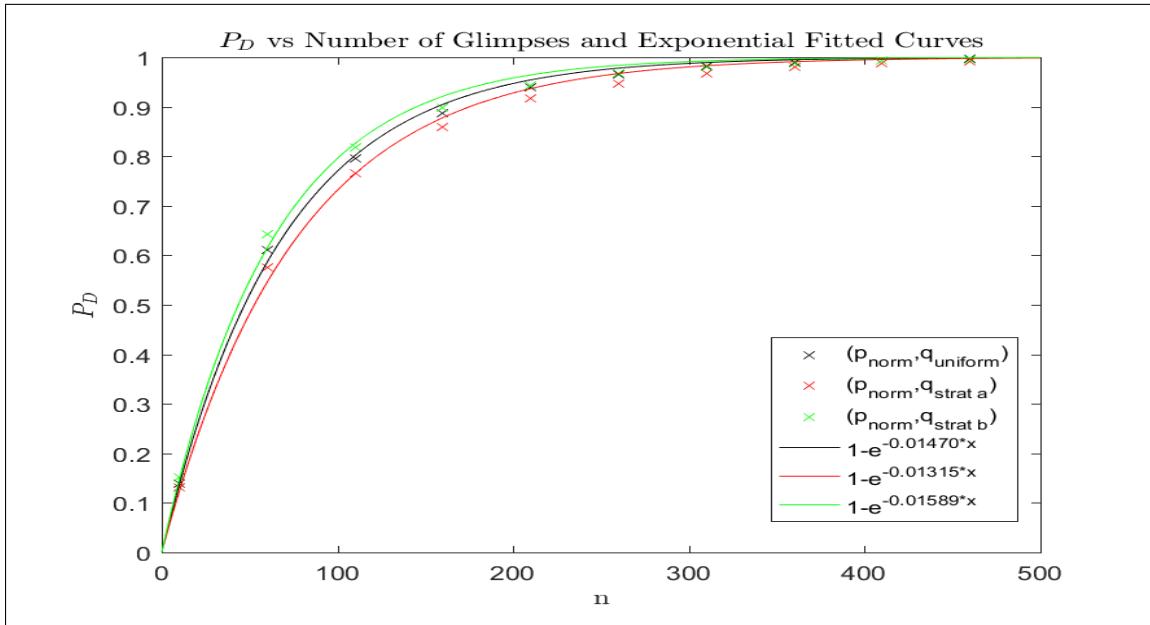


Figure A.12. Cumulative probabilities of detection based on the number of glimpses. Included are the fit best fit lines

### Side Gaussian Location and Uniform Search Density Results

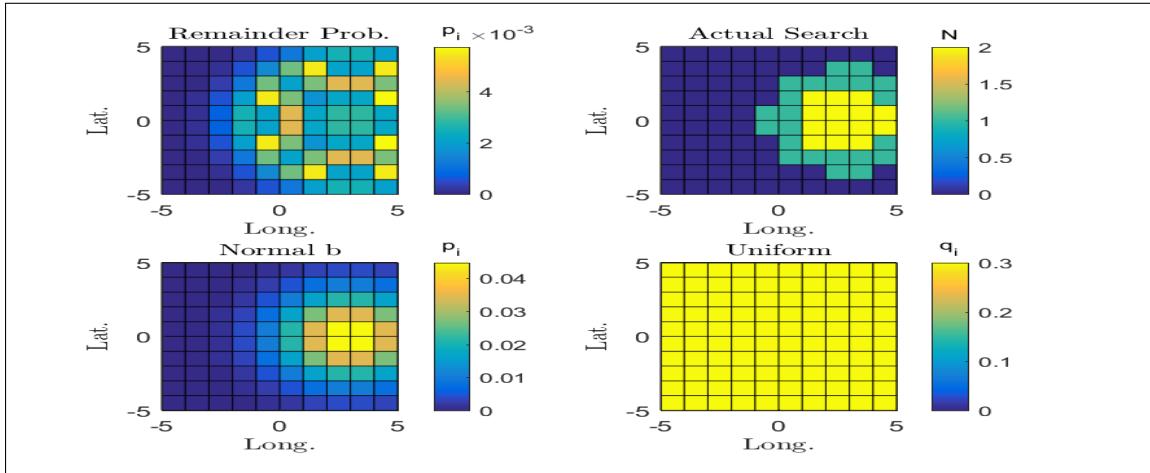


Figure A.13. Remaining probability of detection and boxes search based on side Gaussian  $p_i$  and uniform  $q_i$

### Side Gaussian Location and Longitudinal Stratified Search Density Results

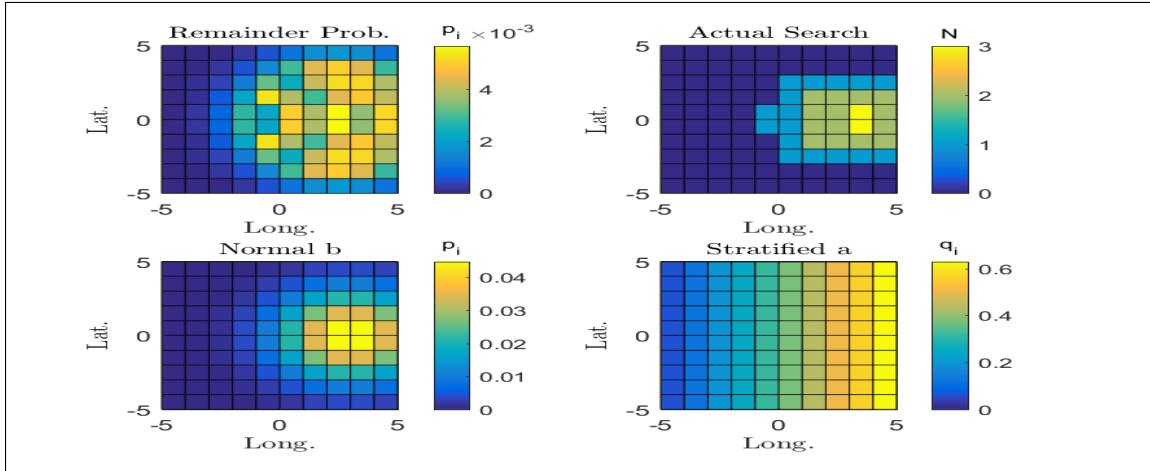


Figure A.14. Remaining probability of detection and boxes search based on side Gaussian  $p_i$  and linear stratified  $q_i$

### Side Gaussian Location and Corner Stratified Search Density Results

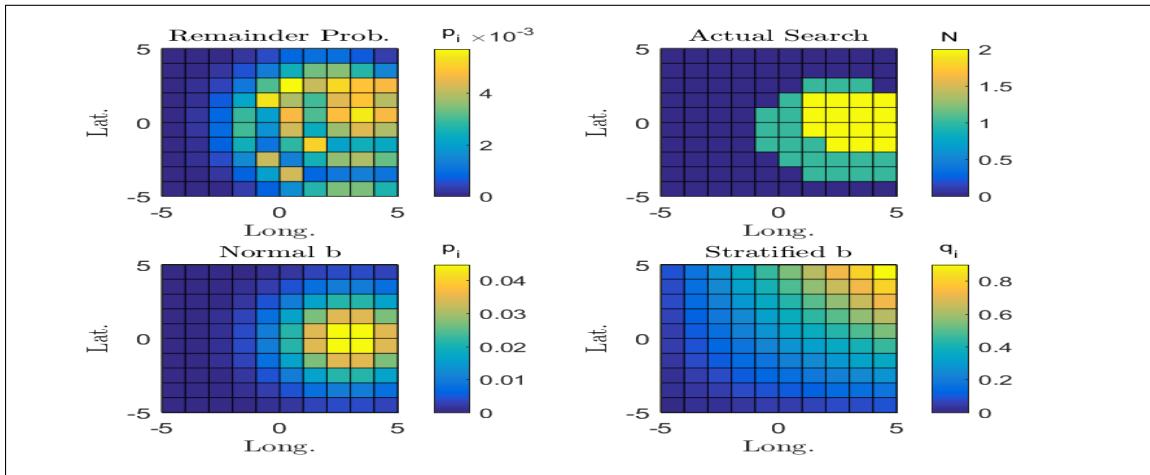


Figure A.15. Remaining probability of detection and boxes search based on side Gaussian  $p_i$  and corner stratified  $q_i$

### Side Gaussian Location Cumulative Probability and Fit Lines

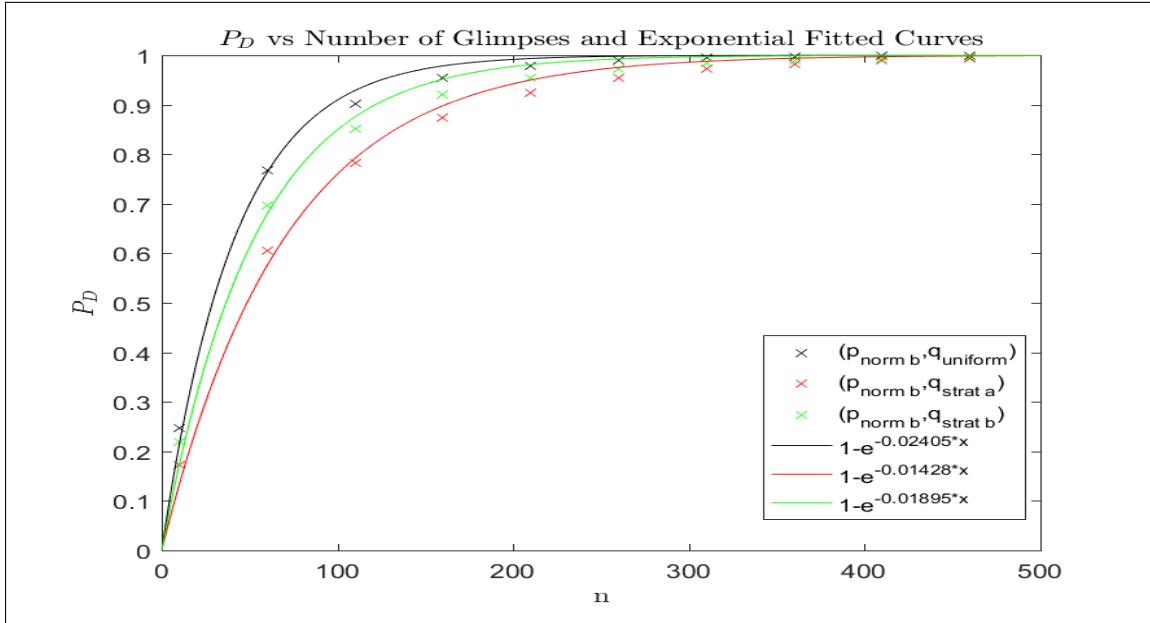


Figure A.16. Cumulative probabilities of detection based on the number of glimpses. Included are the fit best fit lines

## A.2 Bayesian

### Uniform Location and Uniform Search Density Results

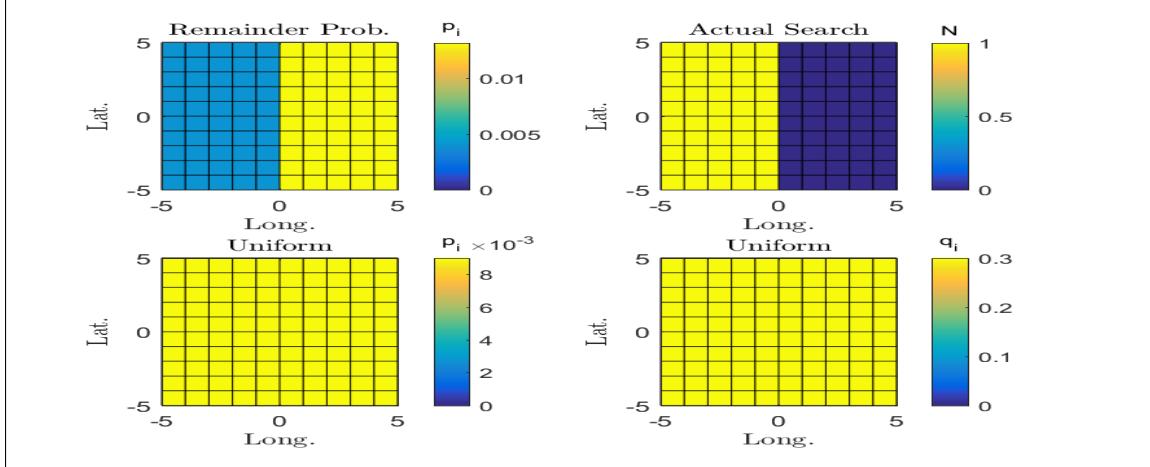


Figure A.17. Remaining probability of detection and boxes search based on uniform  $p_i$  and uniform  $q_i$

### Uniform Location and Longitudinal Stratified Search Density Results

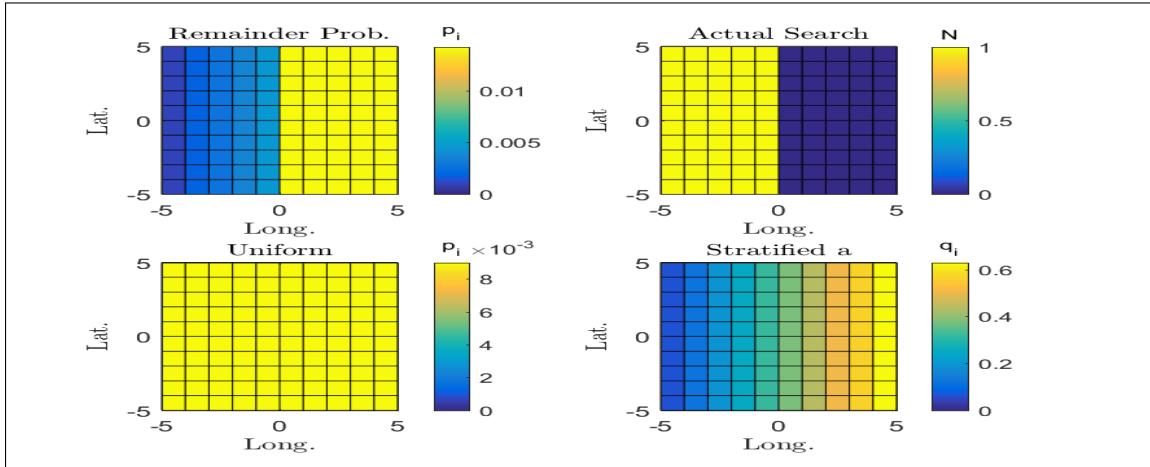


Figure A.18. Remaining probability of detection and boxes search based on uniform  $p_i$  and linear stratified  $q_i$

### Uniform Location and Corner Stratified Search Density Results

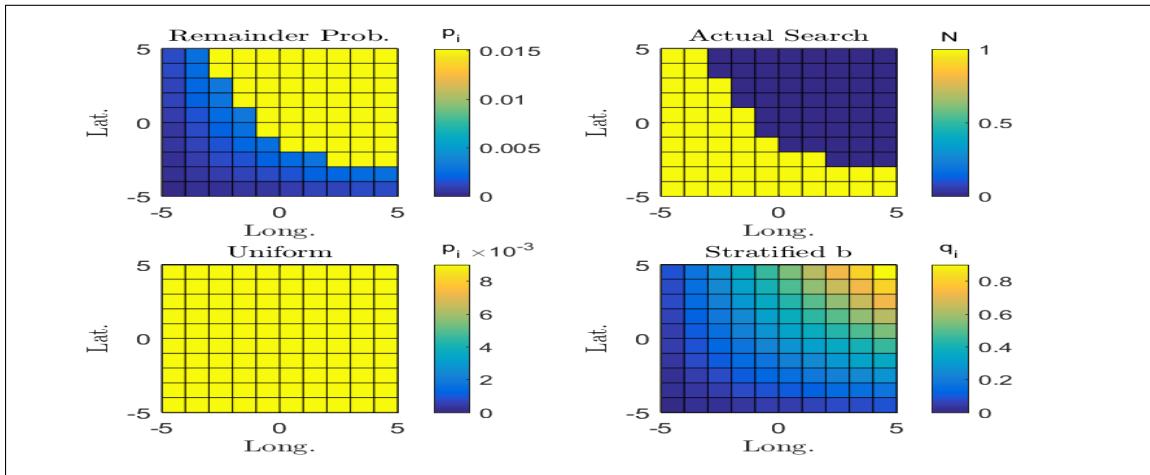


Figure A.19. Remaining probability of detection and boxes search based on uniform  $p_i$  and corner stratified  $q_i$

### Uniform Location Cumulative Probability and Fit Lines

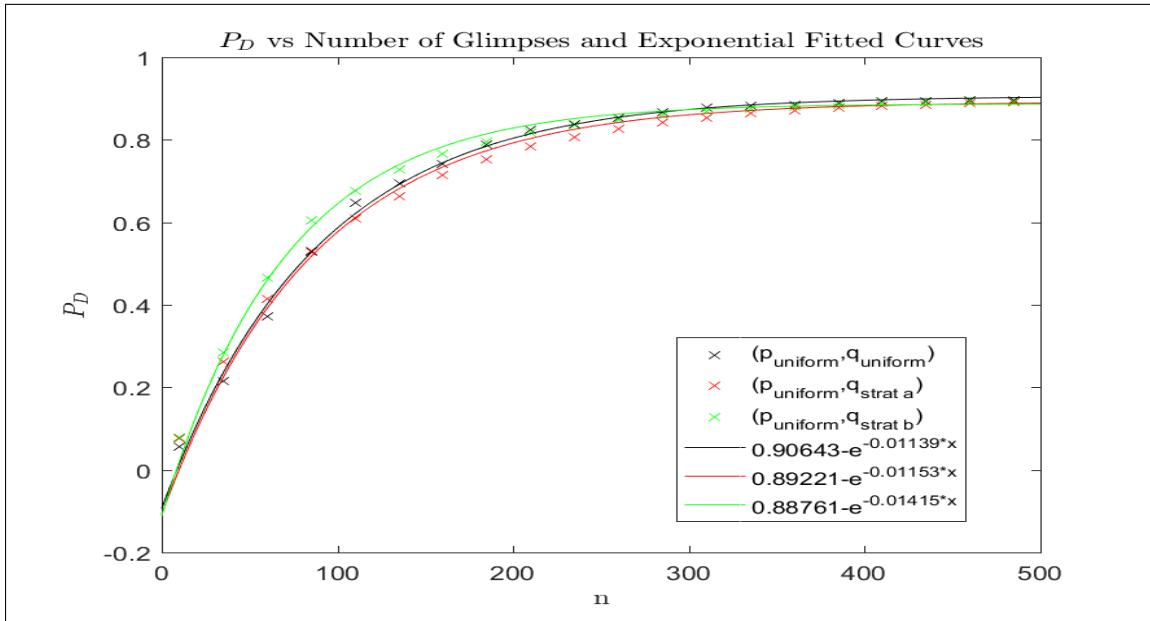


Figure A.20. Cumulative probabilities of detection based on the number of glimpses. Included are the fit best fit lines

### Corner Stratified Location and Uniform Search Density Results

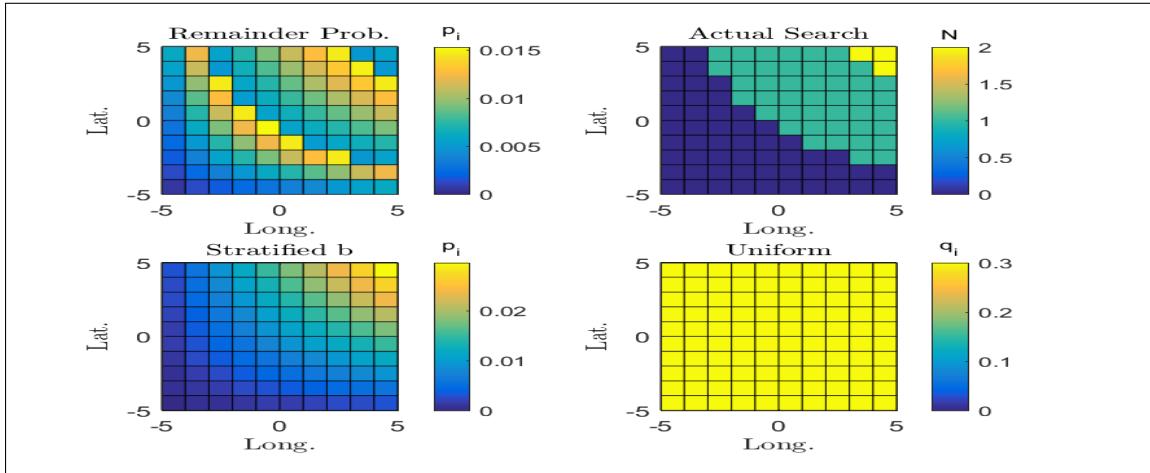


Figure A.21. Remaining probability of detection and boxes search based on corner stratified  $p_i$  and uniform  $q_i$

### Corner Stratified Location and Longitudinal Stratified Search Density Results

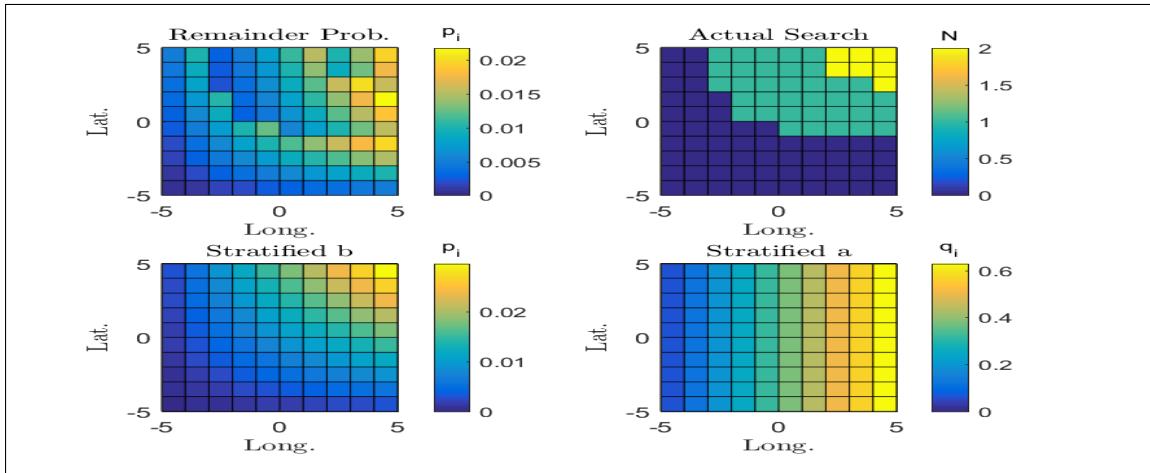


Figure A.22. Remaining probability of detection and boxes search based on corner stratified  $p_i$  and linear stratified  $q_i$

### Corner Stratified Location and Corner Stratified Search Density Results

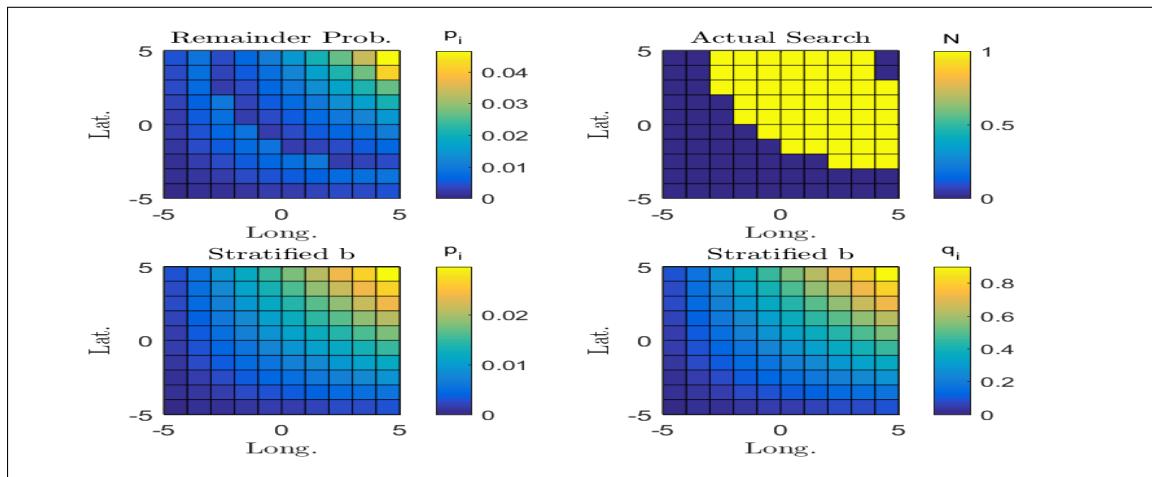


Figure A.23. Remaining probability of detection and boxes search based on corner stratified  $p_i$  and corner stratified  $q_i$

### Corner Stratified Location Cumulative Probability and Fit Lines

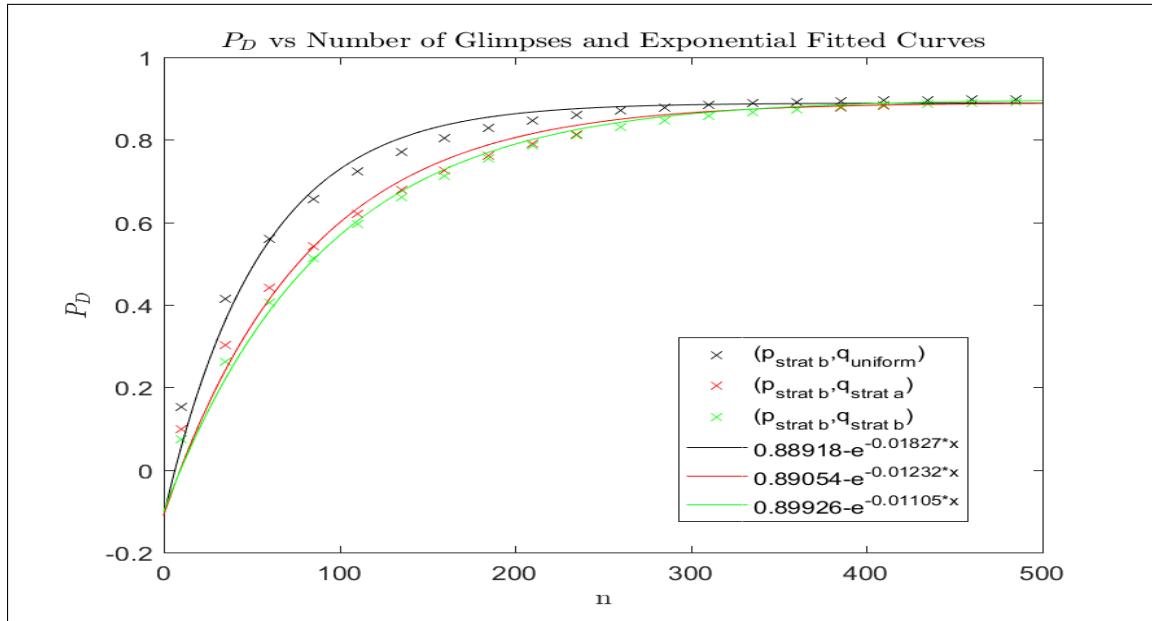


Figure A.24. Cumulative probabilities of detection based on the number of glimpses. Included are the fit best fit lines

### Gaussian Location and Uniform Search Density Results

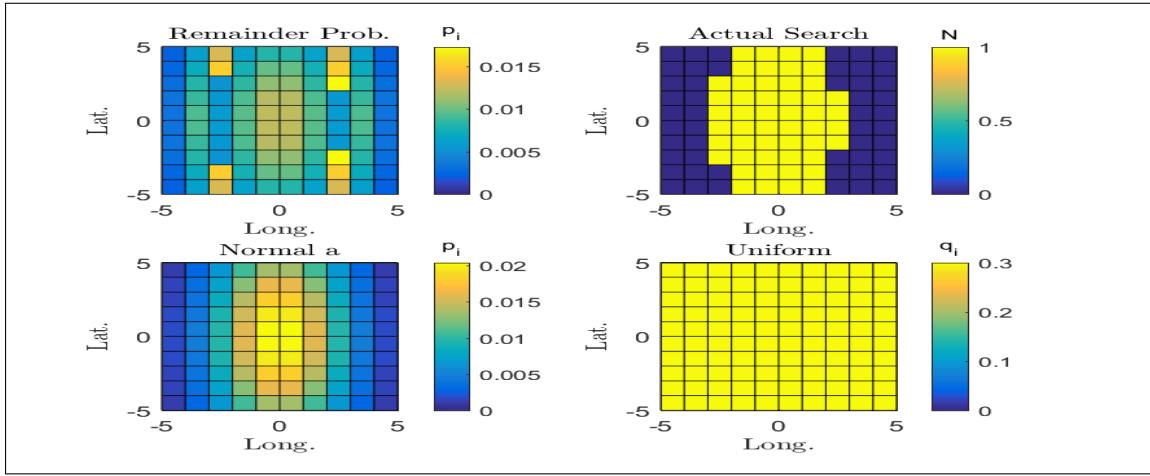


Figure A.25. Remaining probability of detection and boxes search based on Gaussian  $p_i$  and uniform  $q_i$

### Gaussian Location and Longitudinal Stratified Search Density Results

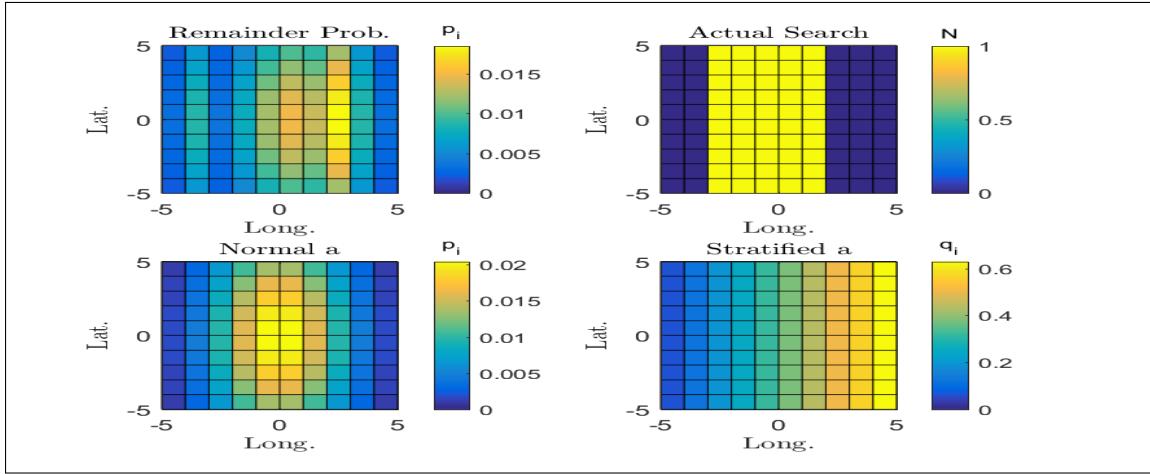


Figure A.26. Remaining probability of detection and boxes search based on Gaussian  $p_i$  and linear stratified  $q_i$

### Gaussian Location and Corner Stratified Search Density Results

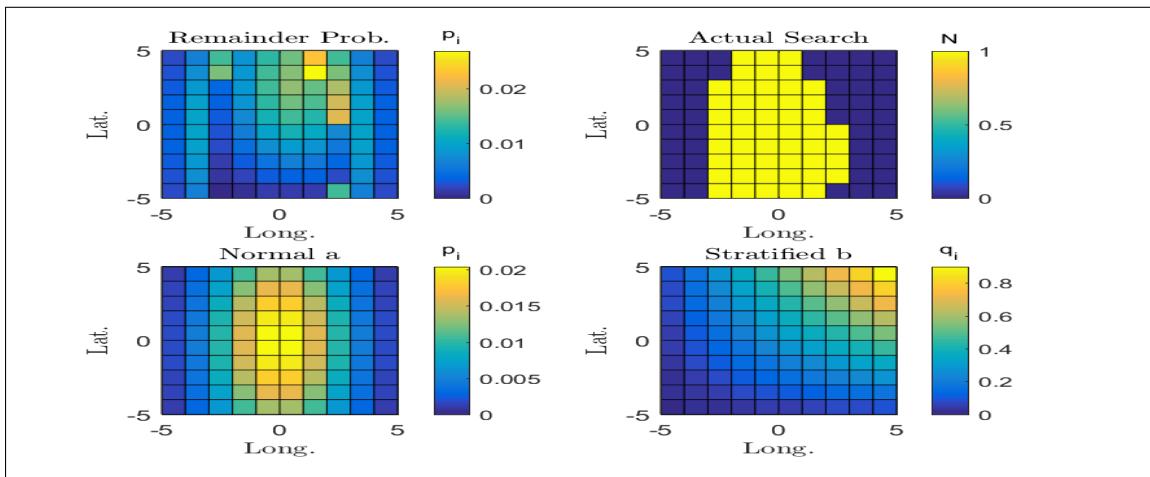


Figure A.27. Remaining probability of detection and boxes search based on Gaussian  $p_i$  and corner stratified  $q_i$

### Gaussian Location Cumulative Probability and Fit Lines

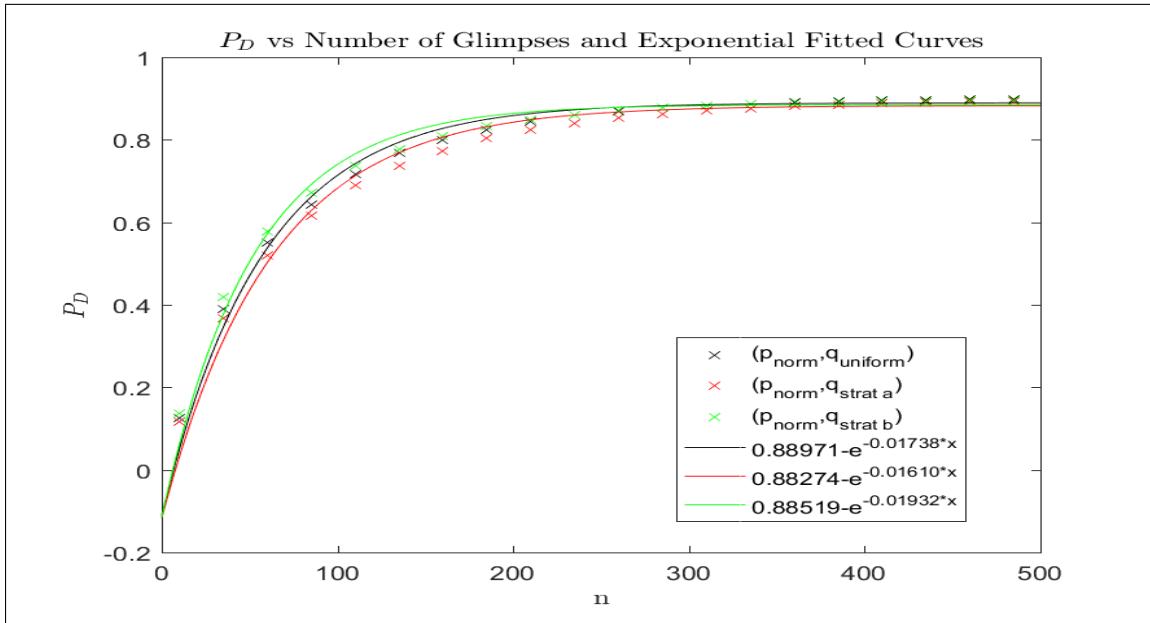


Figure A.28. Cumulative probabilities of detection based on the number of glimpses. Included are the fit best fit lines

### Side Gaussian Location and Uniform Search Density Results

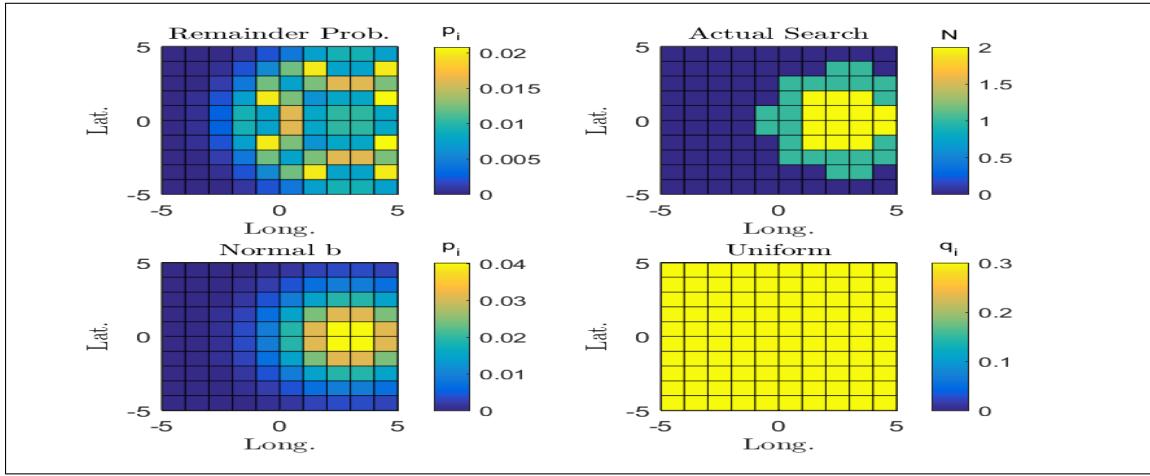


Figure A.29. Remaining probability of detection and boxes search based on side Gaussian  $p_i$  and uniform  $q_i$

### Side Gaussian Location and Longitudinal Stratified Search Density Results

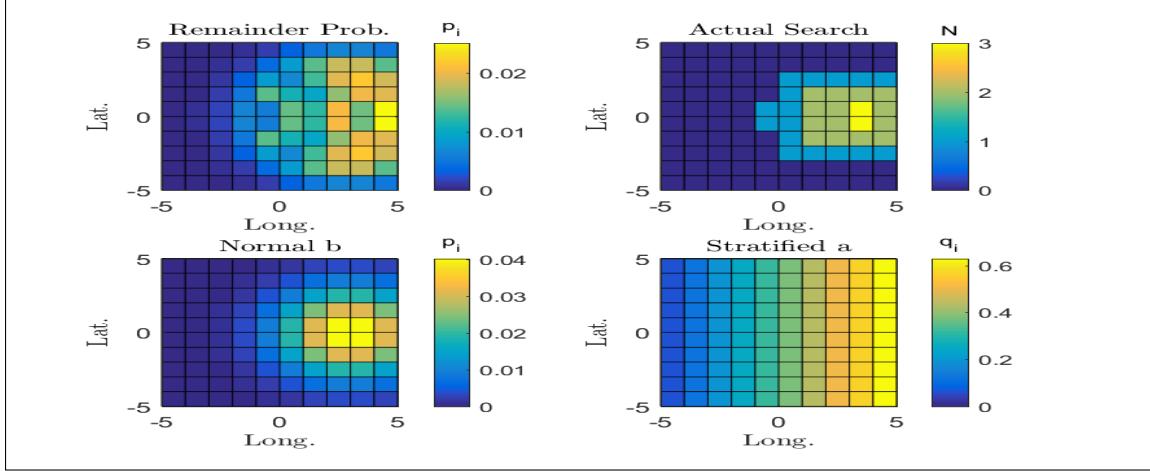


Figure A.30. Remaining probability of detection and boxes search based on side Gaussian  $p_i$  and linear stratified  $q_i$

### Side Gaussian Location and Corner Stratified Search Density Results

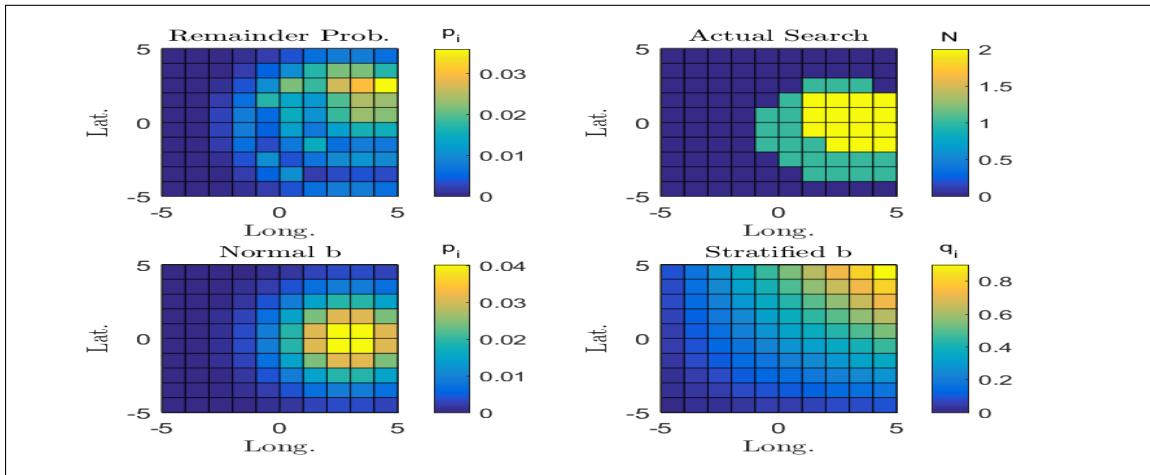


Figure A.31. Remaining probability of detection and boxes search based on side Gaussian  $p_i$  and corner stratified  $q_i$

### Side Gaussian Location Cumulative Probability and Fit Lines

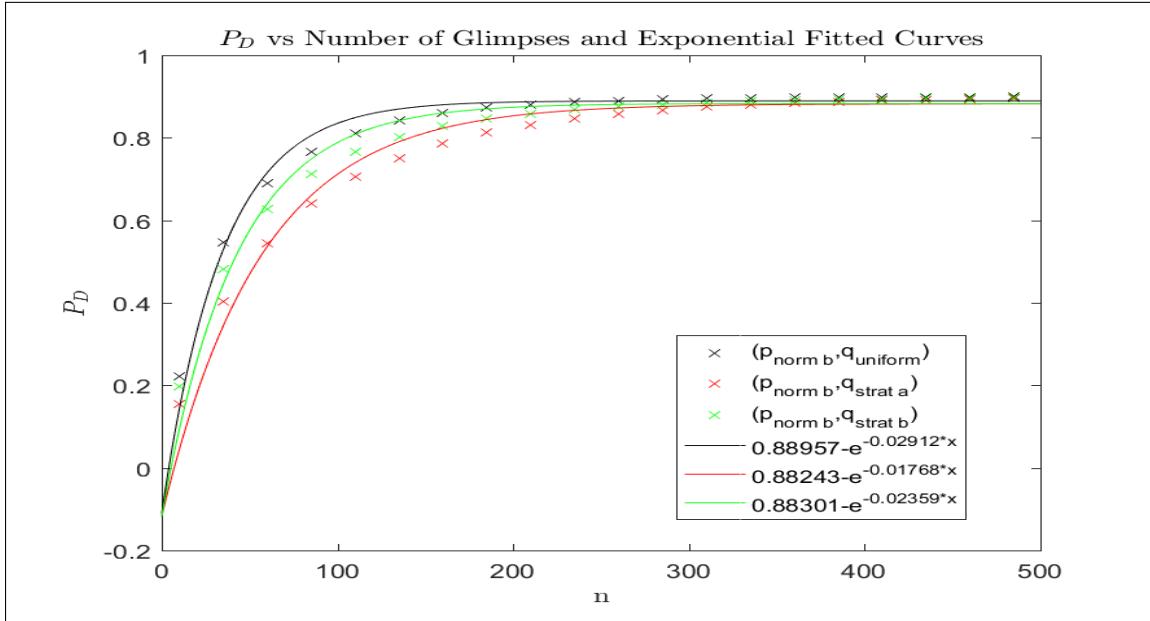
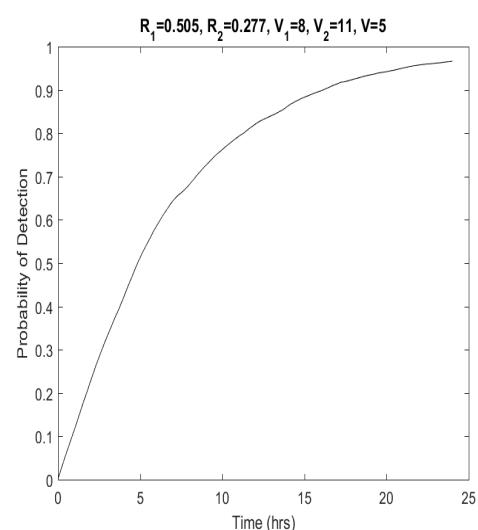
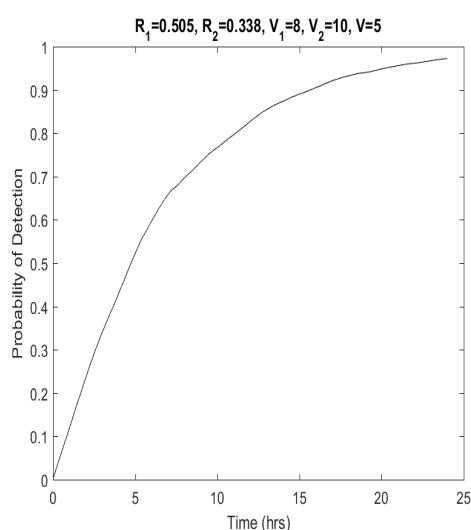
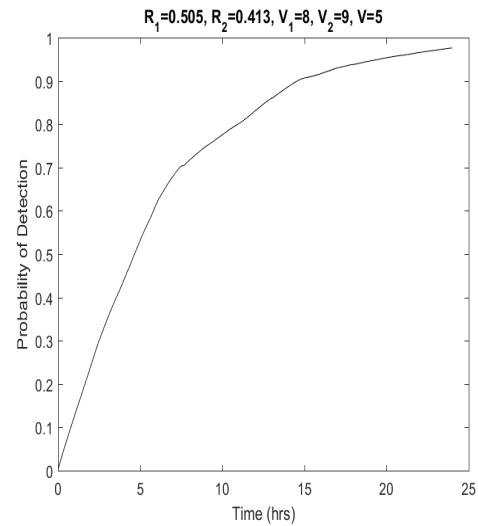
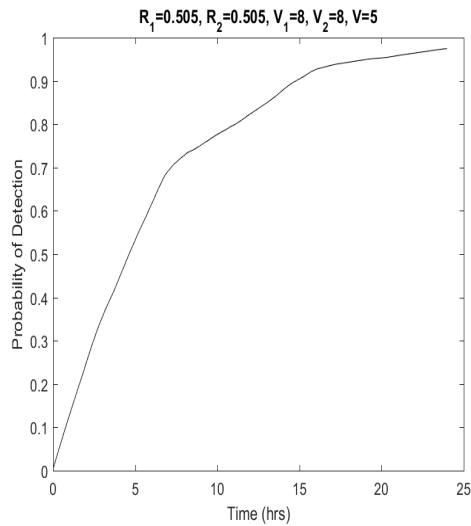


Figure A.32. Cumulative probabilities of detection based on the number of glimpses. Included are the fit best fit lines

## A.3 Continuous

### A.3.1 Inline Ladder Search



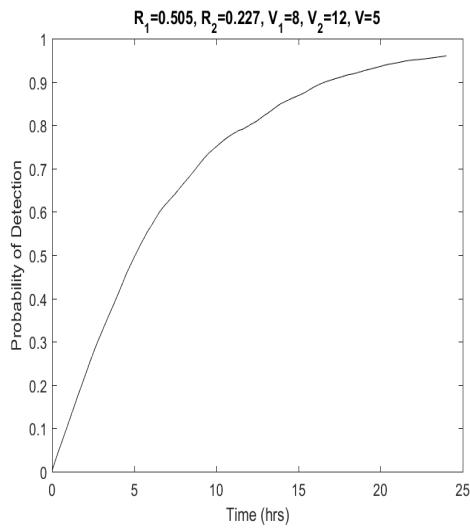


Figure A.37.  $P_D$  vs Time

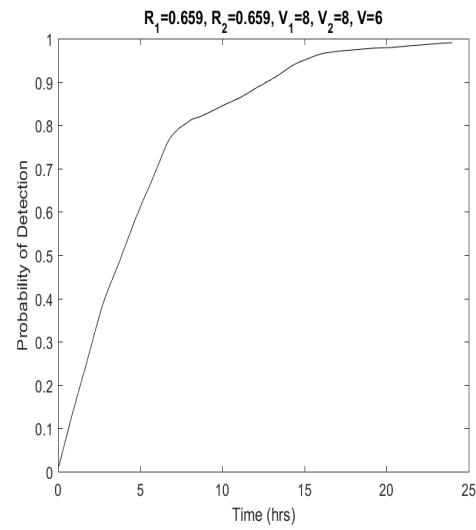


Figure A.38.  $P_D$  vs Time

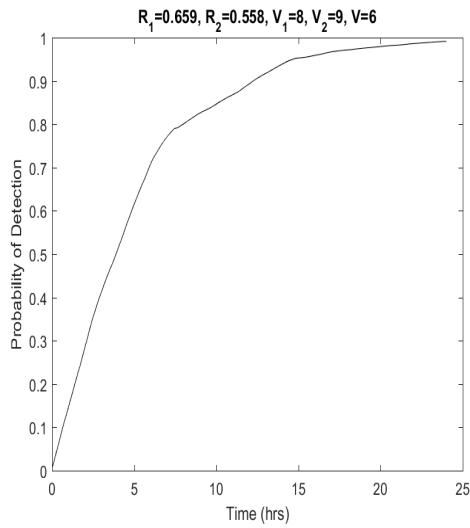


Figure A.39.  $P_D$  vs Time

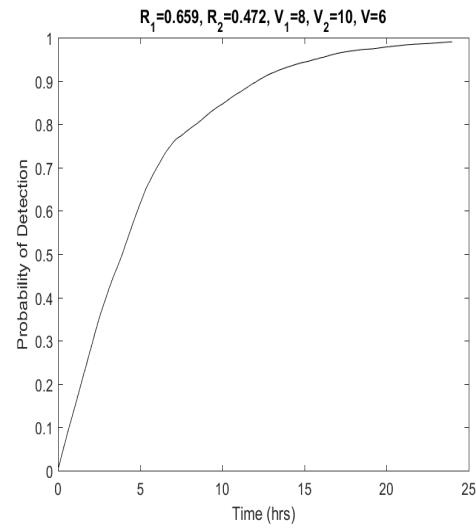


Figure A.40.  $P_D$  vs Time

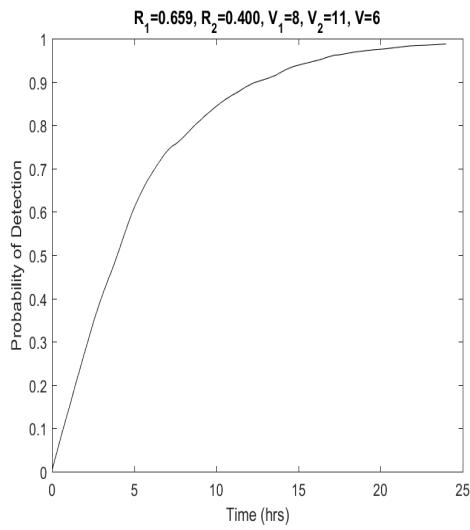


Figure A.41.  $P_D$  vs Time

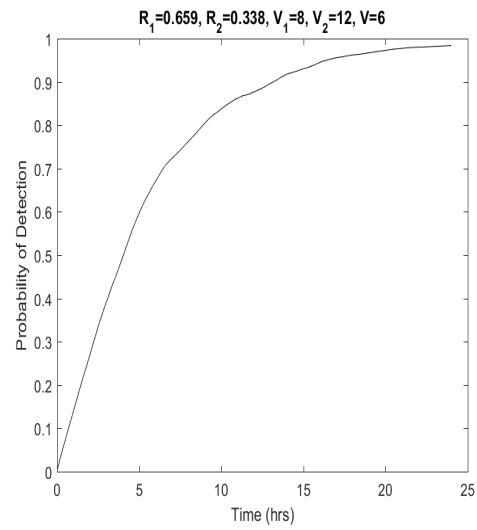


Figure A.42.  $P_D$  vs Time

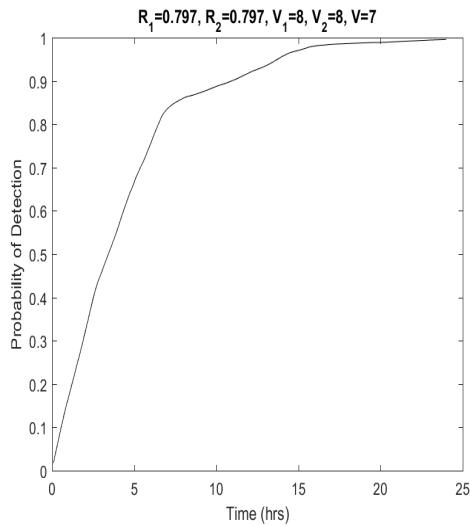


Figure A.43.  $P_D$  vs Time

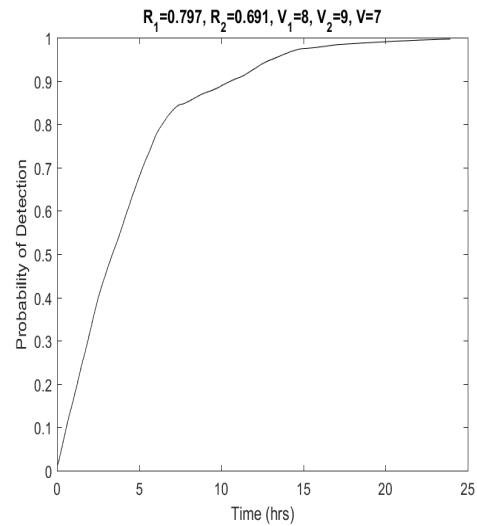


Figure A.44.  $P_D$  vs Time

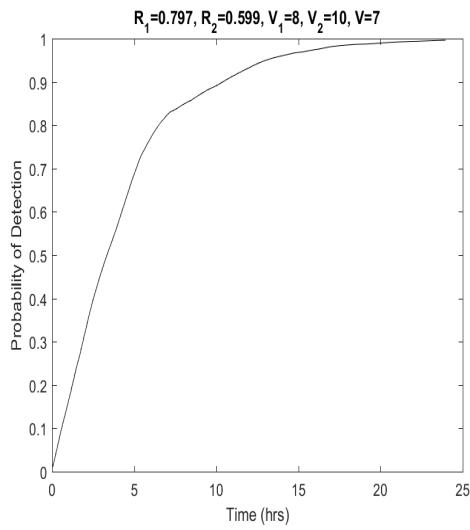


Figure A.45.  $P_D$  vs Time

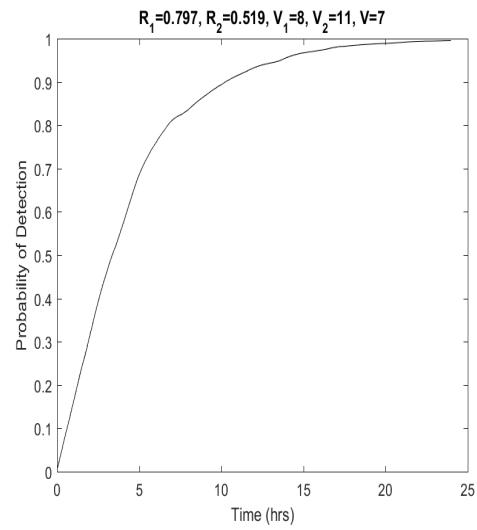


Figure A.46.  $P_D$  vs Time

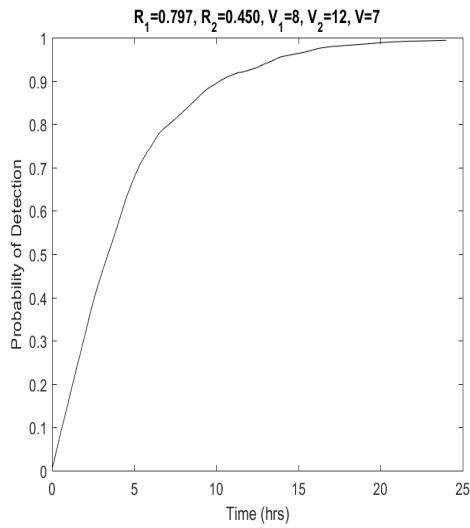


Figure A.47.  $P_D$  vs Time

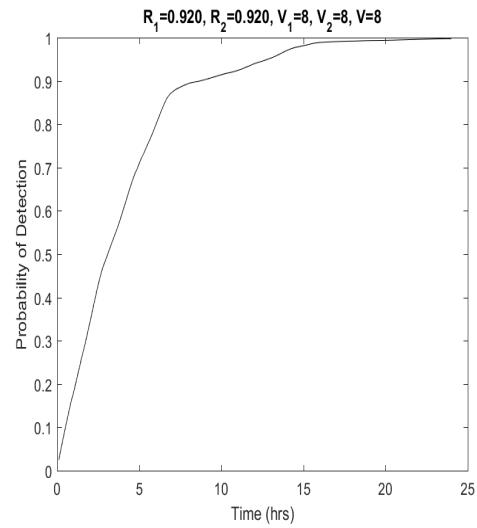


Figure A.48.  $P_D$  vs Time

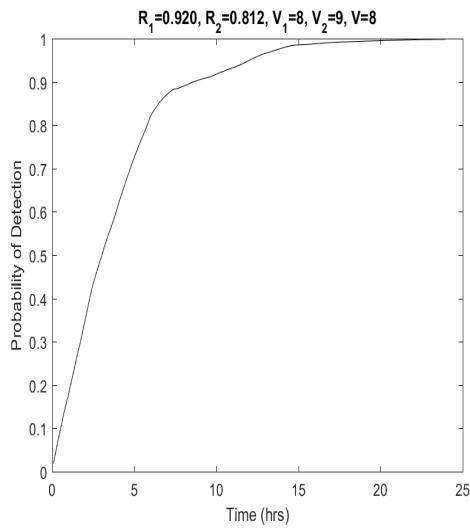


Figure A.49.  $P_D$  vs Time

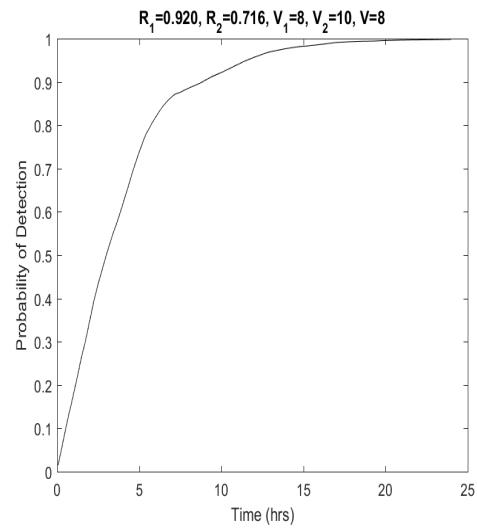


Figure A.50.  $P_D$  vs Time

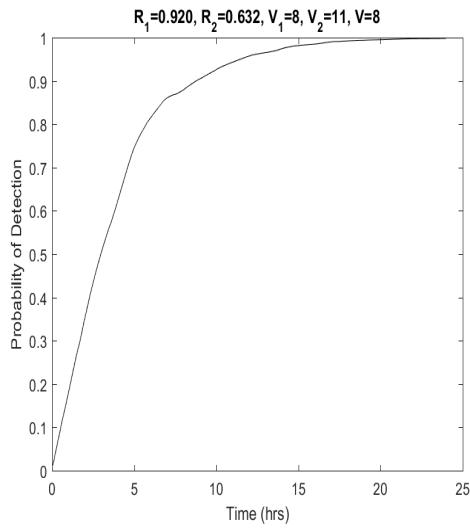


Figure A.51.  $P_D$  vs Time

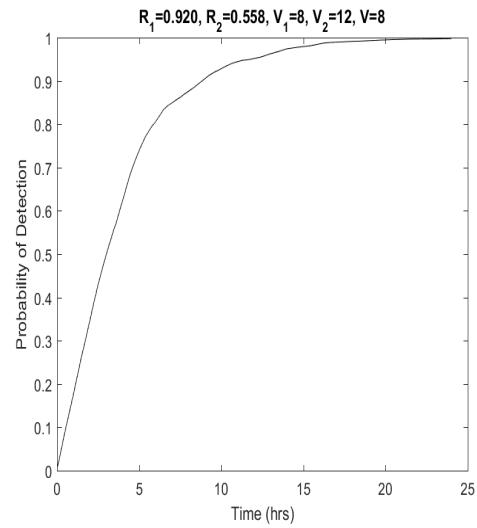


Figure A.52.  $P_D$  vs Time

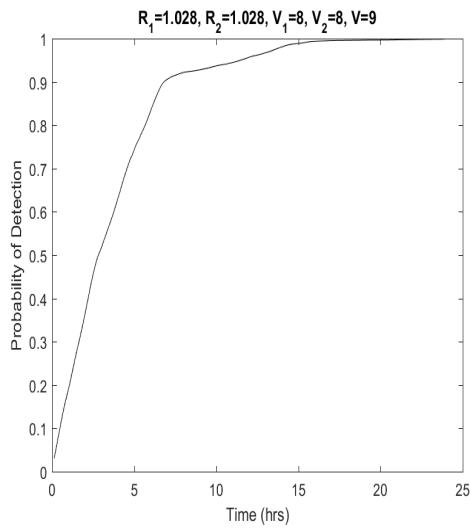


Figure A.53.  $P_D$  vs Time

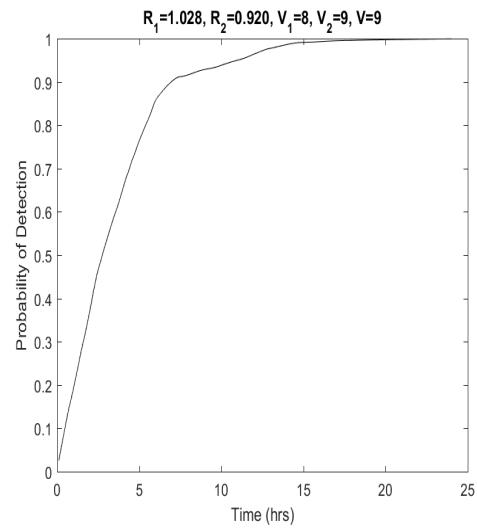


Figure A.54.  $P_D$  vs Time

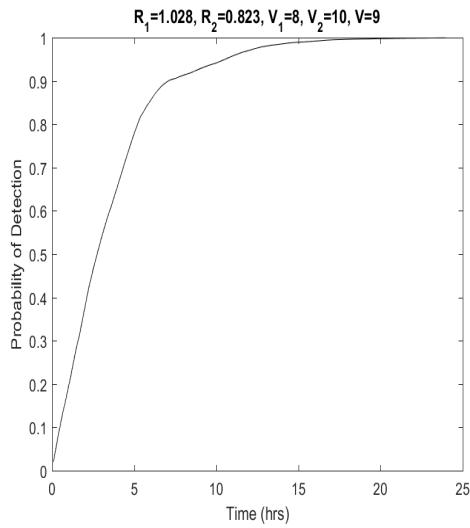


Figure A.55.  $P_D$  vs Time

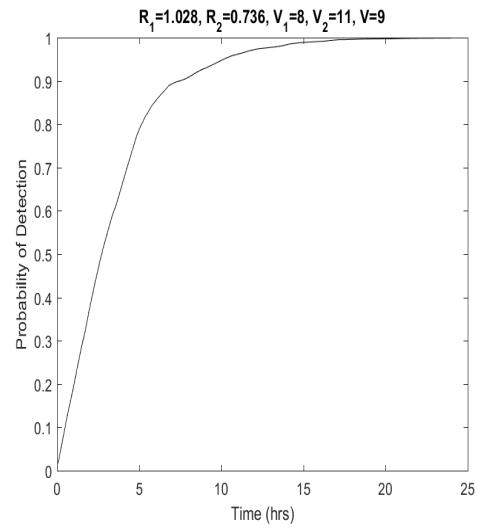


Figure A.56.  $P_D$  vs Time

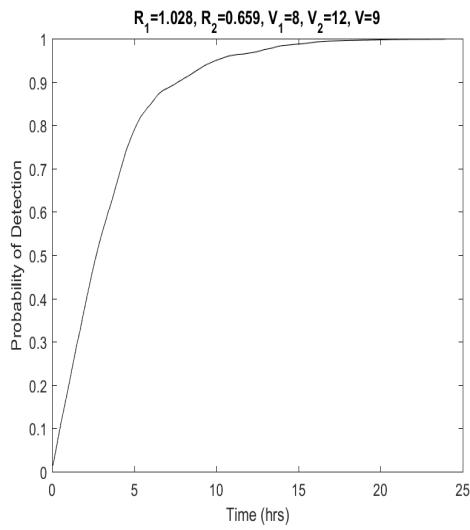


Figure A.57.  $P_D$  vs Time

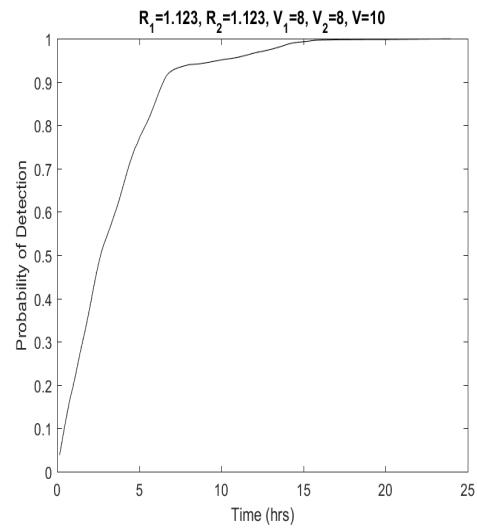


Figure A.58.  $P_D$  vs Time

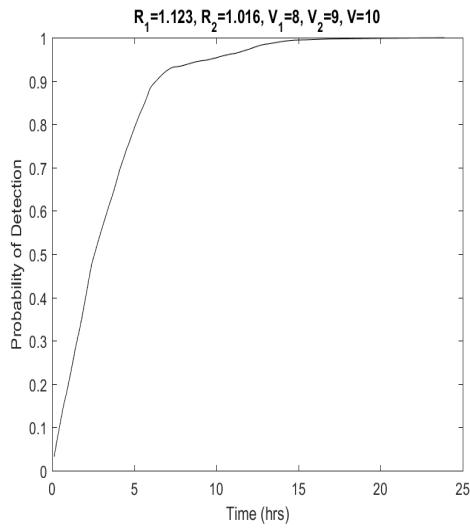


Figure A.59.  $P_D$  vs Time

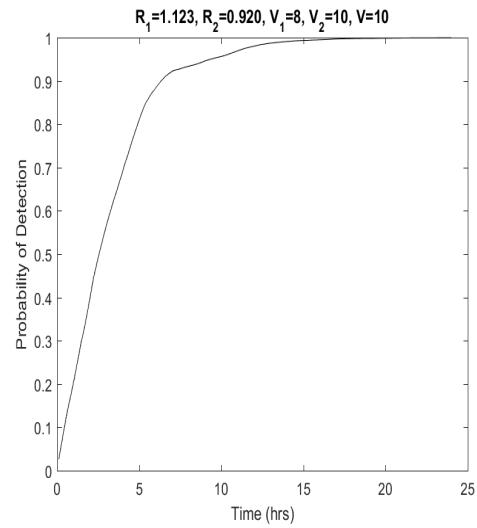


Figure A.60.  $P_D$  vs Time

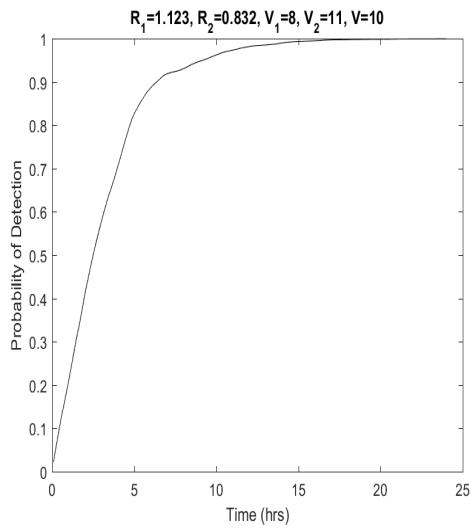


Figure A.61.  $P_D$  vs Time

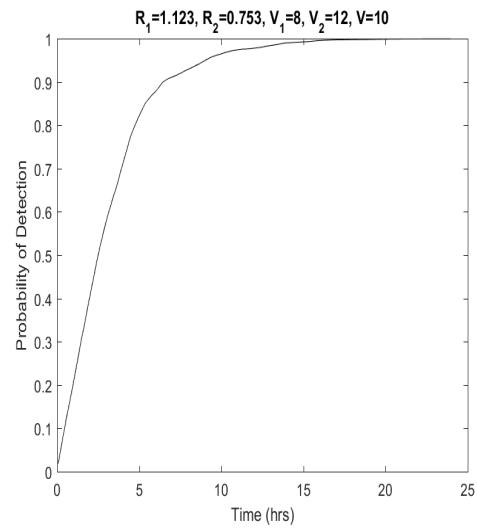


Figure A.62.  $P_D$  vs Time

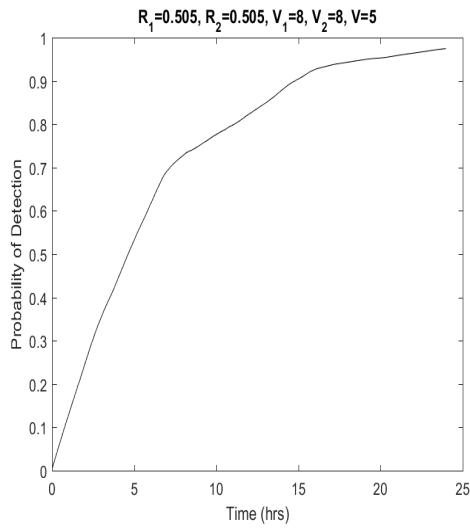


Figure A.63.  $P_D$  vs Time

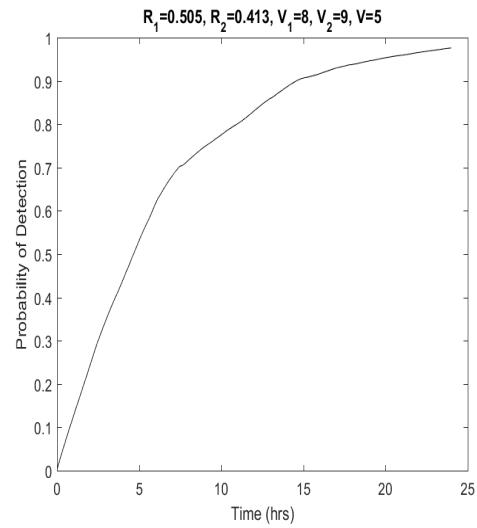


Figure A.64.  $P_D$  vs Time

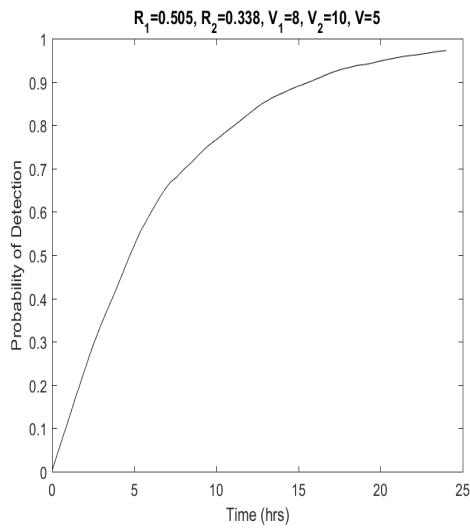


Figure A.65.  $P_D$  vs Time

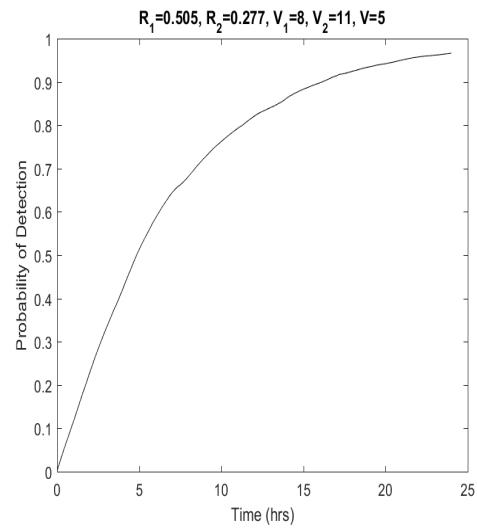


Figure A.66.  $P_D$  vs Time

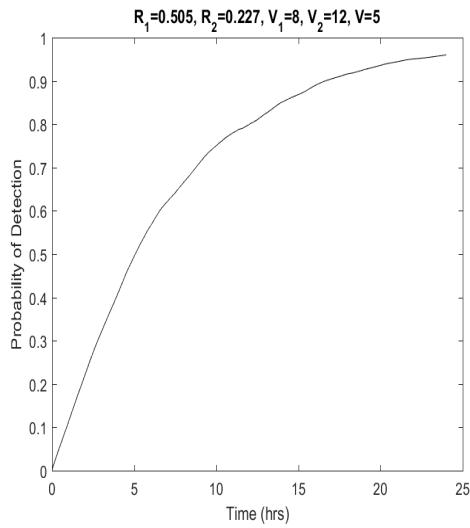


Figure A.67.  $P_D$  vs Time

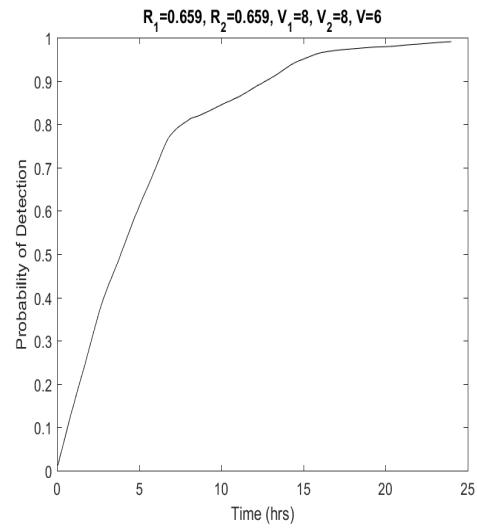


Figure A.68.  $P_D$  vs Time

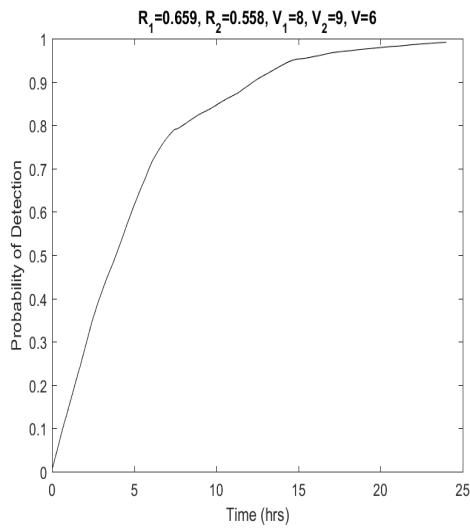


Figure A.69.  $P_D$  vs Time

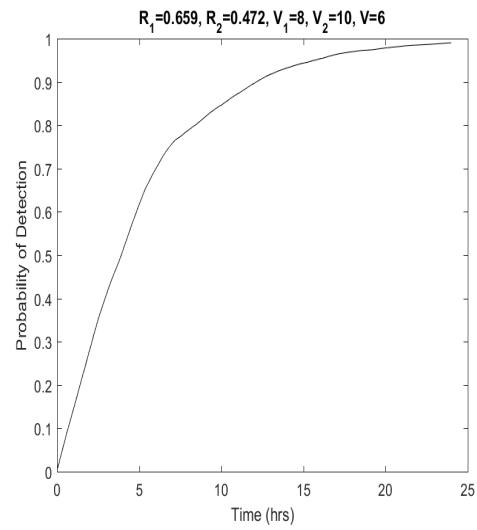


Figure A.70.  $P_D$  vs Time

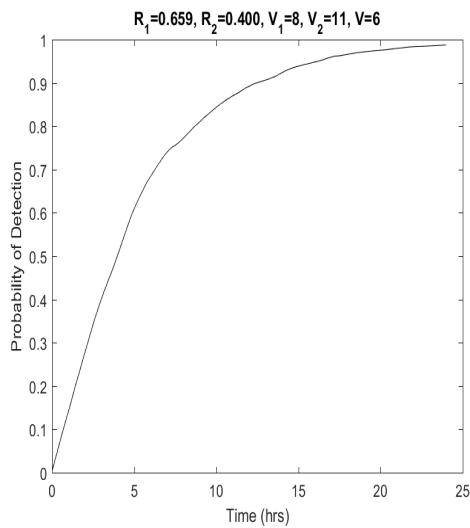


Figure A.71.  $P_D$  vs Time

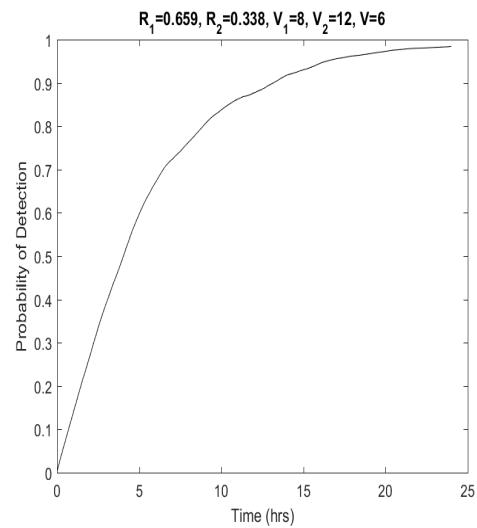


Figure A.72.  $P_D$  vs Time

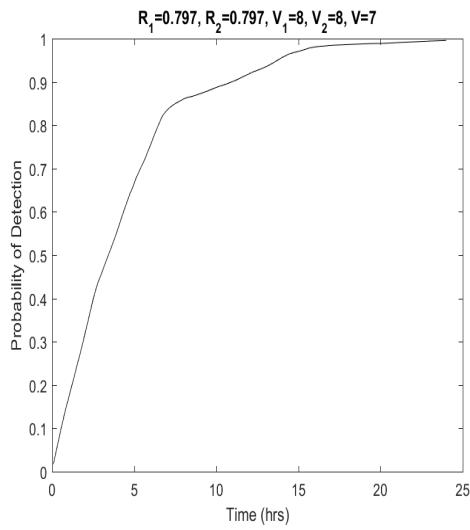


Figure A.73.  $P_D$  vs Time

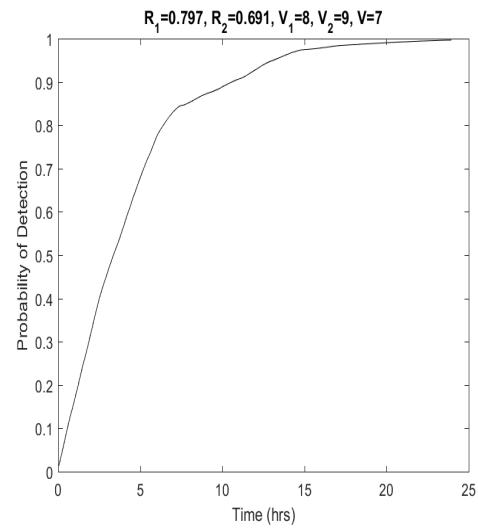


Figure A.74.  $P_D$  vs Time

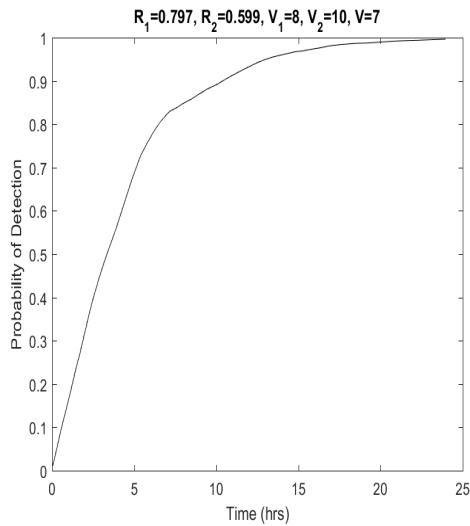


Figure A.75.  $P_D$  vs Time

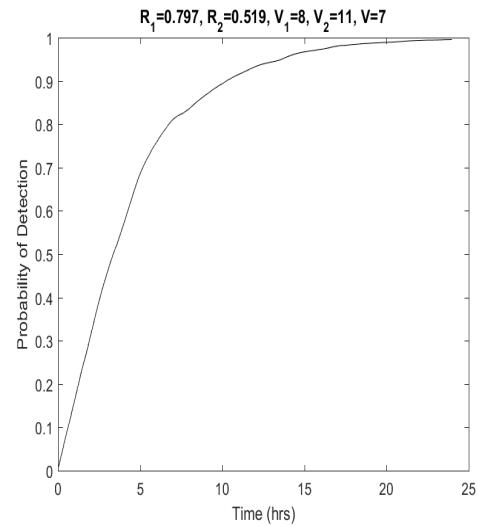


Figure A.76.  $P_D$  vs Time

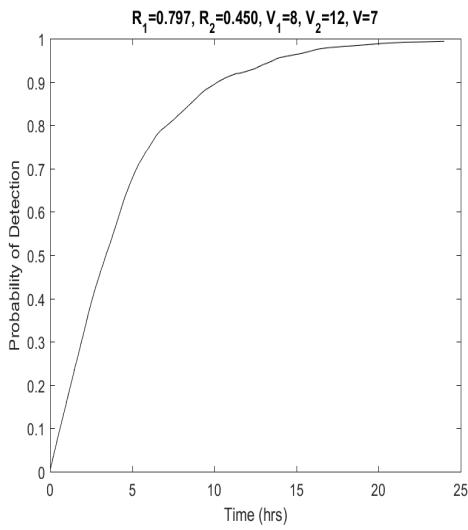


Figure A.77.  $P_D$  vs Time

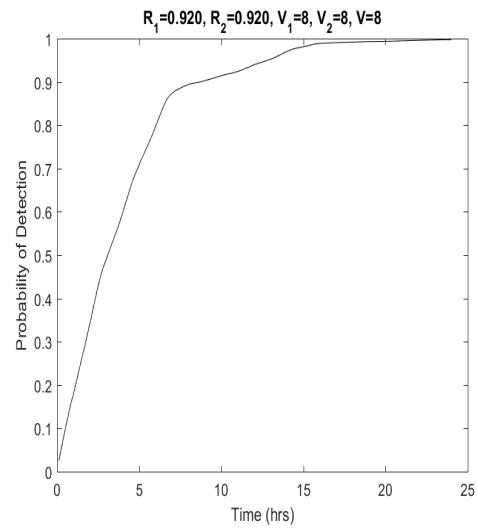


Figure A.78.  $P_D$  vs Time

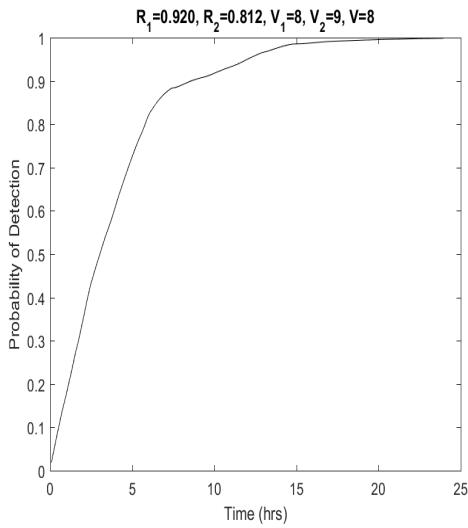


Figure A.79.  $P_D$  vs Time

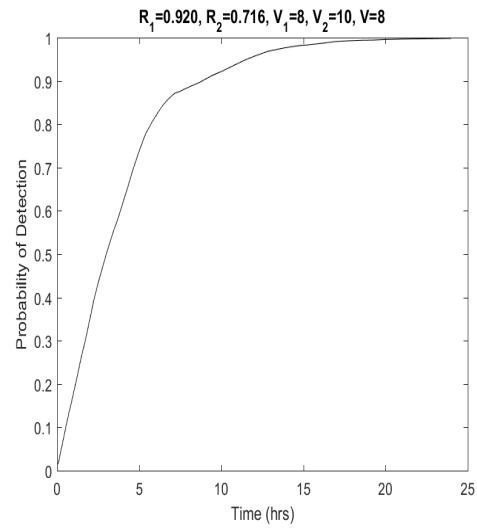


Figure A.80.  $P_D$  vs Time

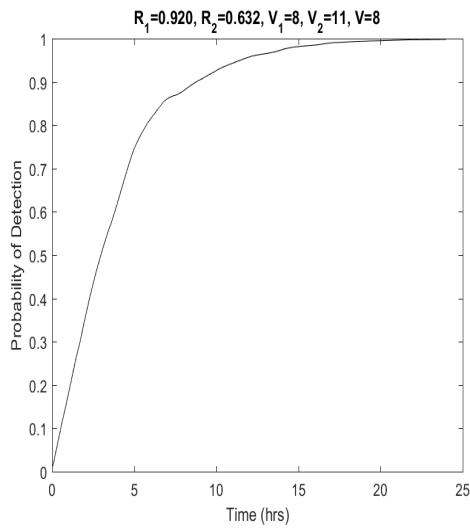


Figure A.81.  $P_D$  vs Time

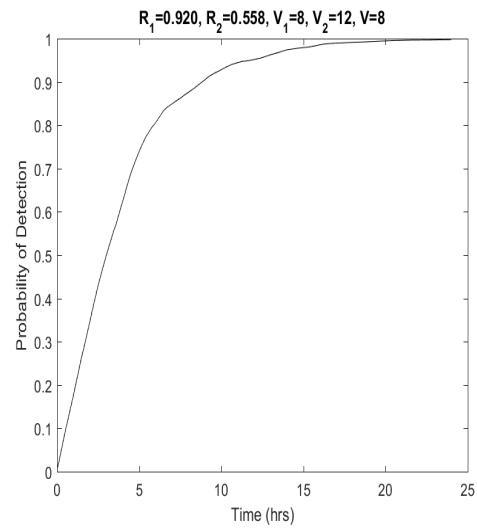


Figure A.82.  $P_D$  vs Time

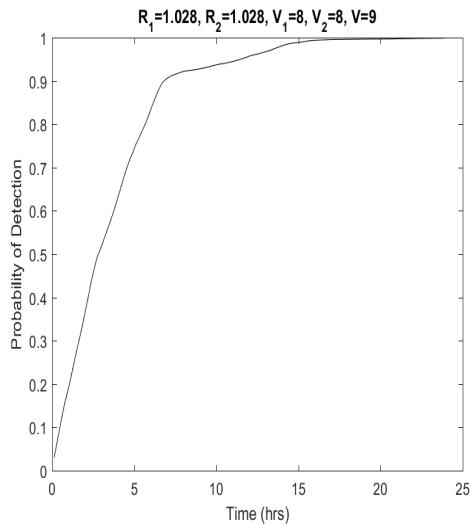


Figure A.83.  $P_D$  vs Time

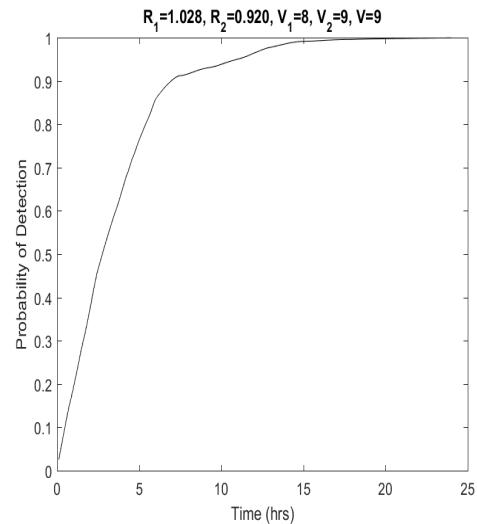


Figure A.84.  $P_D$  vs Time

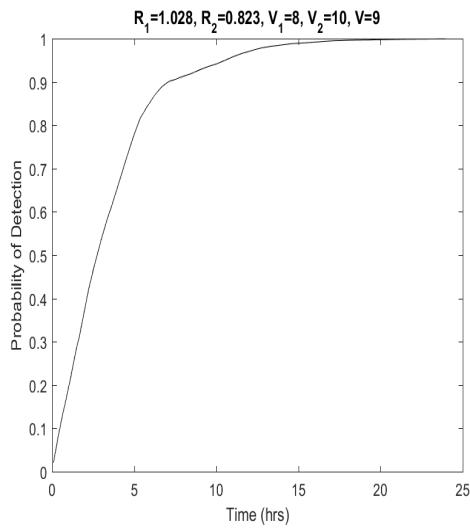


Figure A.85.  $P_D$  vs Time

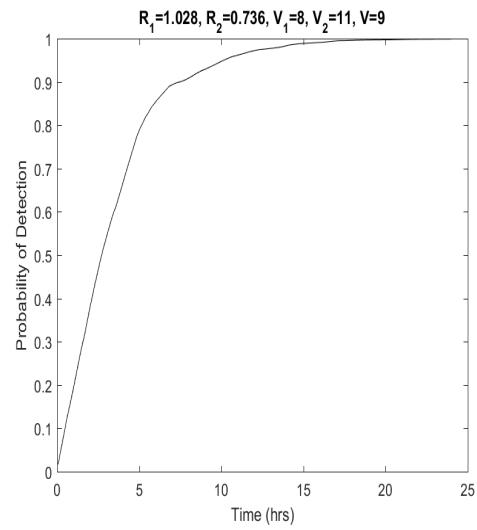


Figure A.86.  $P_D$  vs Time

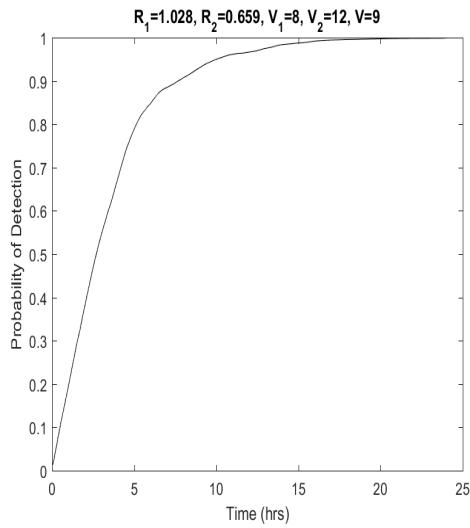


Figure A.87.  $P_D$  vs Time

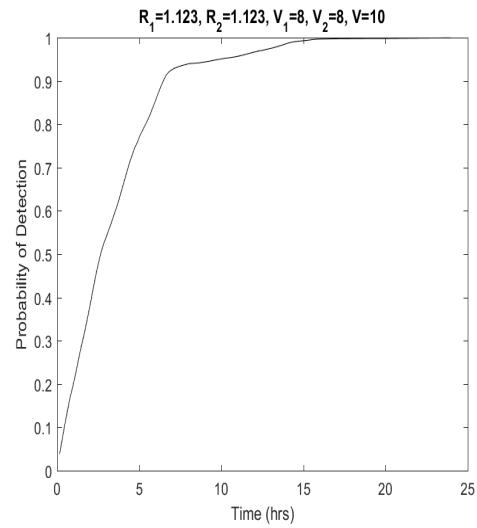


Figure A.88.  $P_D$  vs Time

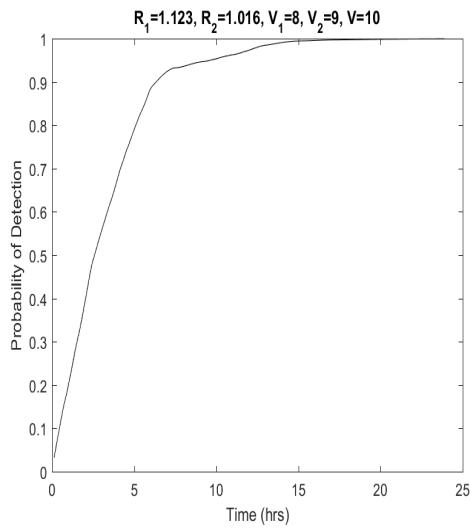


Figure A.89.  $P_D$  vs Time

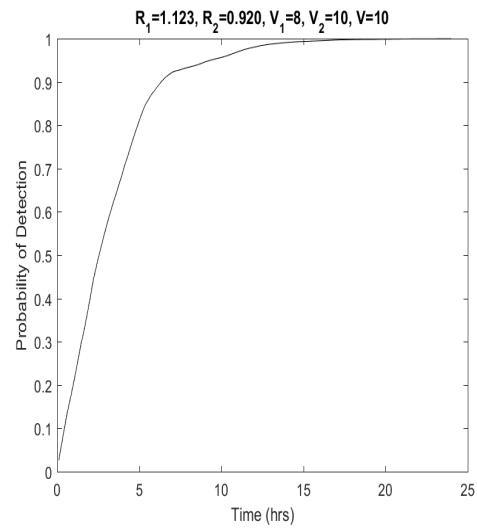


Figure A.90.  $P_D$  vs Time

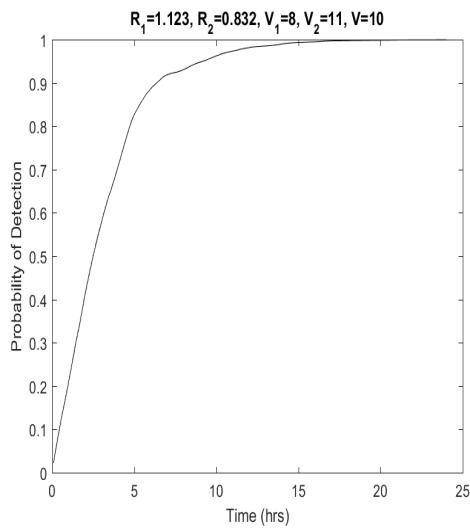


Figure A.91.  $P_D$  vs Time

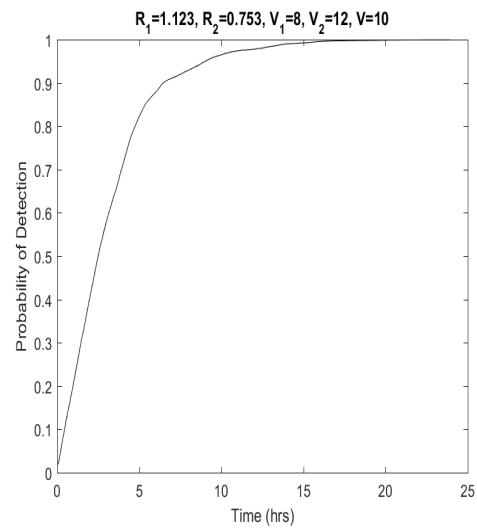


Figure A.92.  $P_D$  vs Time

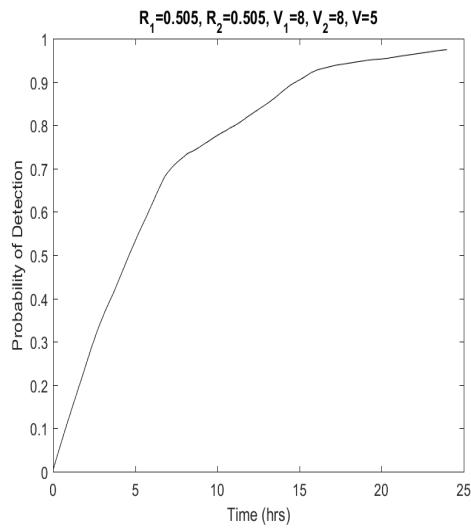


Figure A.93.  $P_D$  vs Time

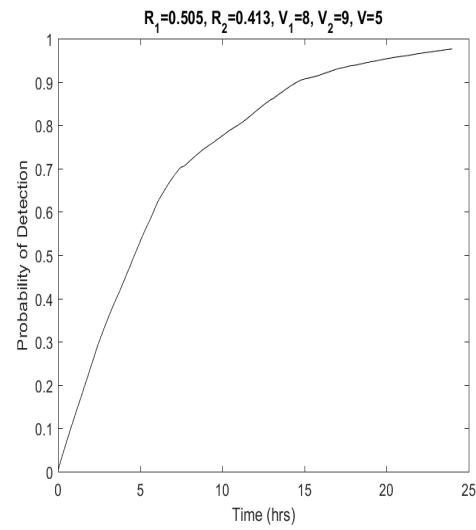


Figure A.94.  $P_D$  vs Time

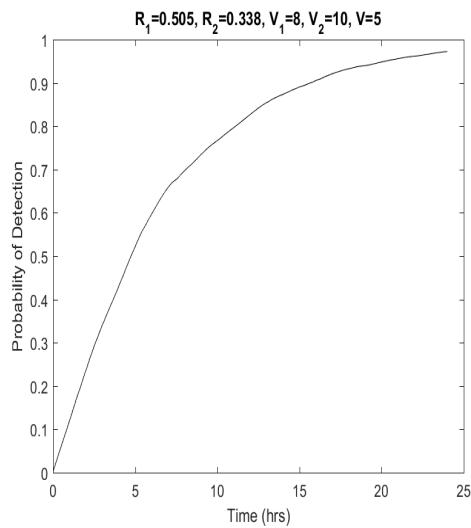


Figure A.95.  $P_D$  vs Time

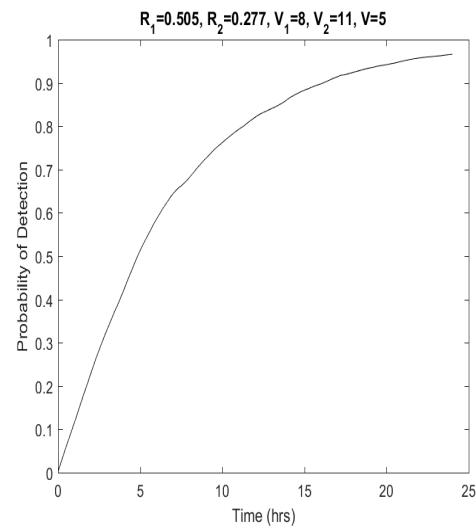


Figure A.96.  $P_D$  vs Time

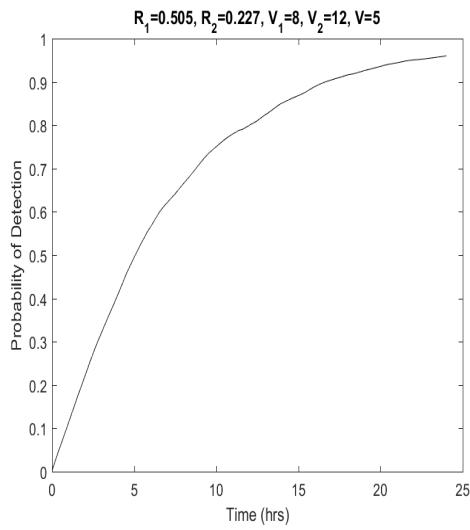


Figure A.97.  $P_D$  vs Time

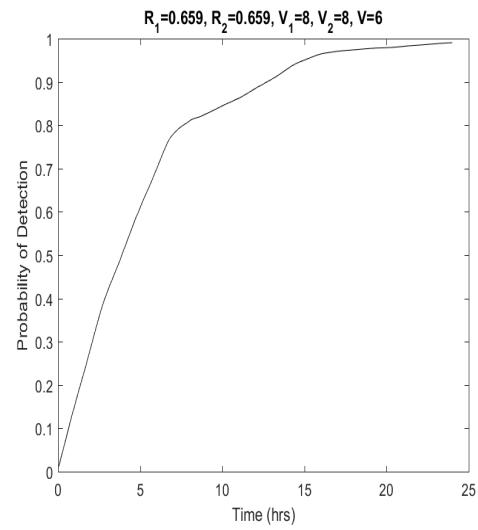


Figure A.98.  $P_D$  vs Time

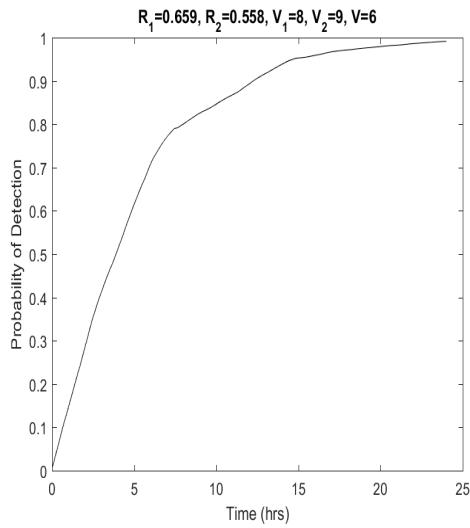


Figure A.99.  $P_D$  vs Time

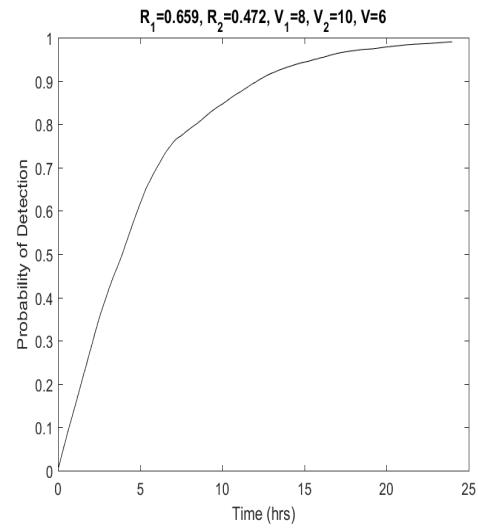


Figure A.100.  $P_D$  vs Time

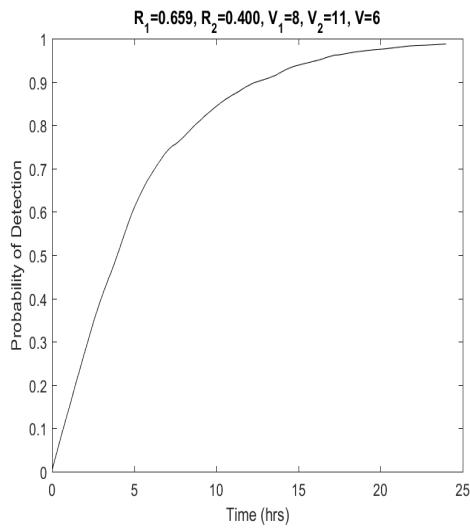


Figure A.101.  $P_D$  vs Time

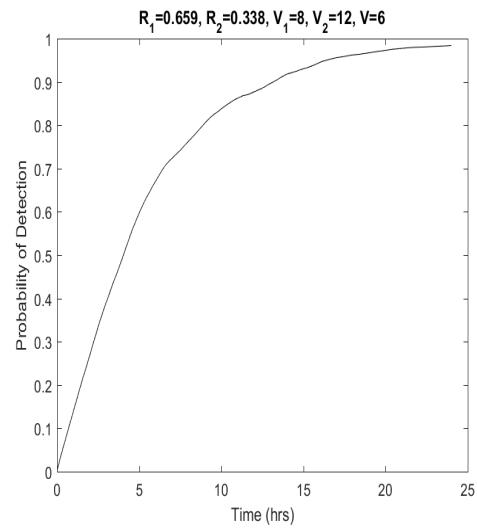


Figure A.102.  $P_D$  vs Time

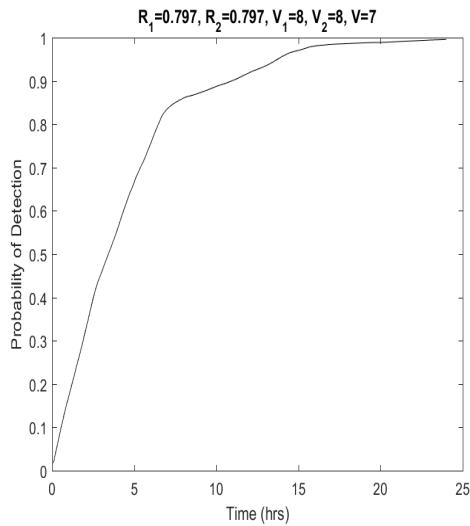


Figure A.103.  $P_D$  vs Time

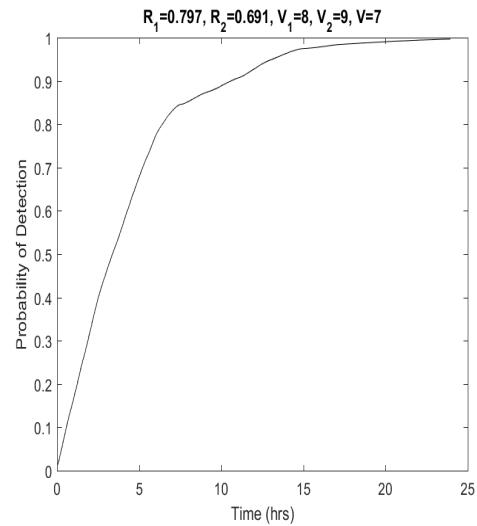


Figure A.104.  $P_D$  vs Time

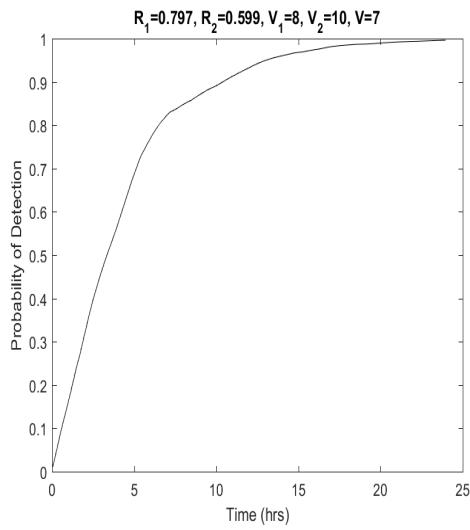


Figure A.105.  $P_D$  vs Time

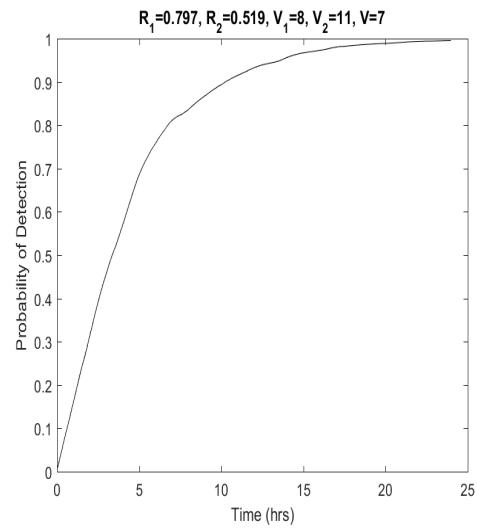


Figure A.106.  $P_D$  vs Time

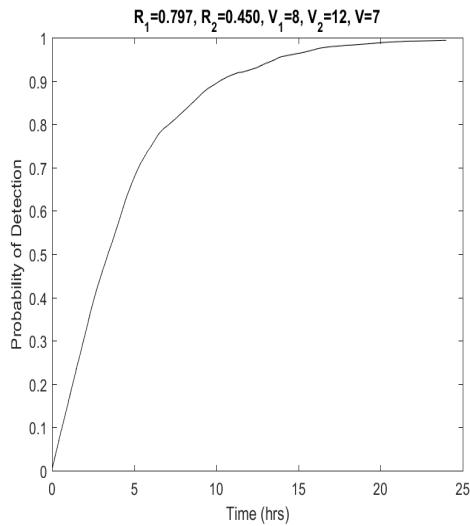


Figure A.107.  $P_D$  vs Time

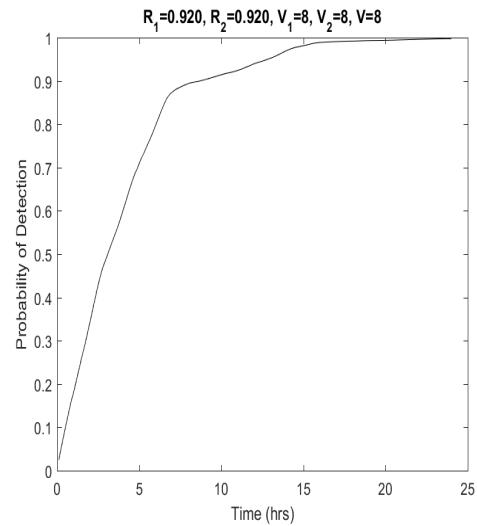


Figure A.108.  $P_D$  vs Time

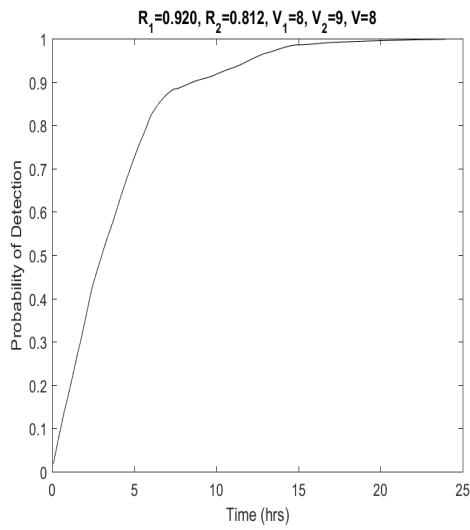


Figure A.109.  $P_D$  vs Time

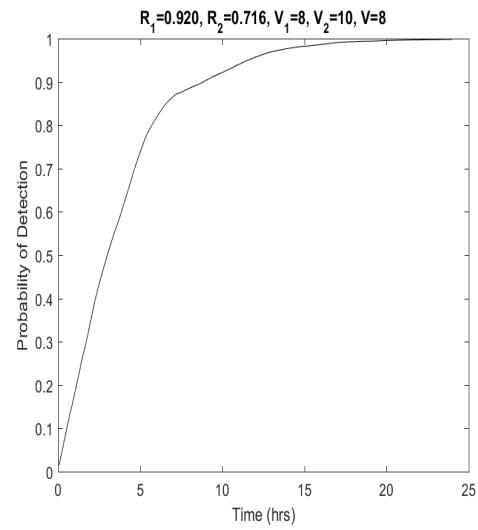


Figure A.110.  $P_D$  vs Time

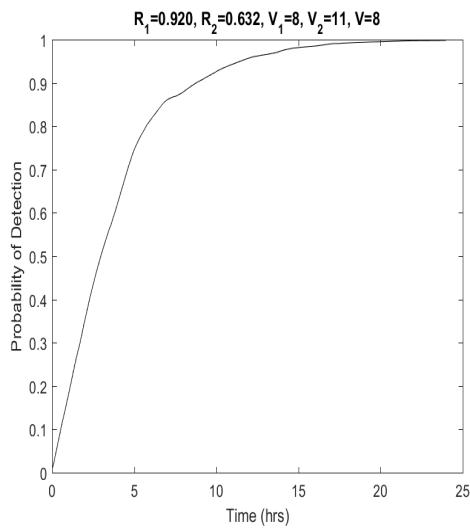


Figure A.111.  $P_D$  vs Time

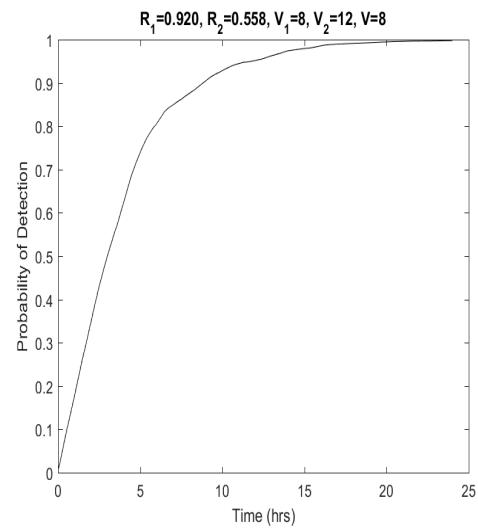


Figure A.112.  $P_D$  vs Time

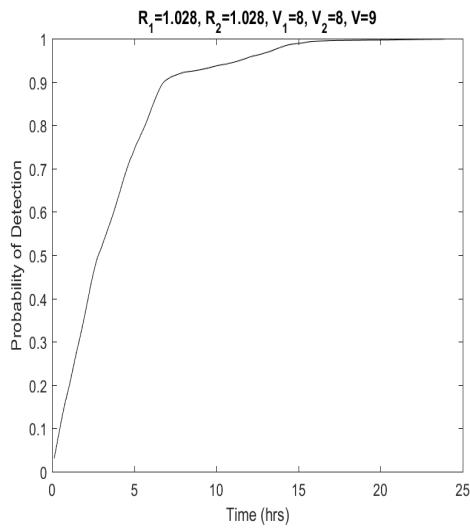


Figure A.113.  $P_D$  vs Time

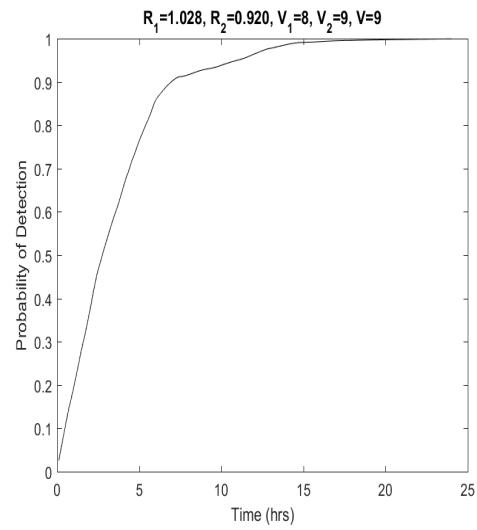


Figure A.114.  $P_D$  vs Time

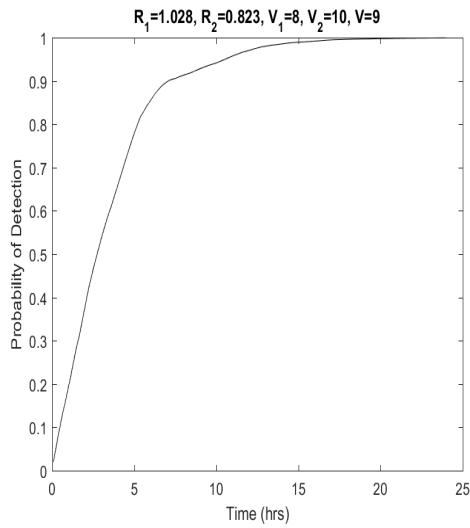


Figure A.115.  $P_D$  vs Time

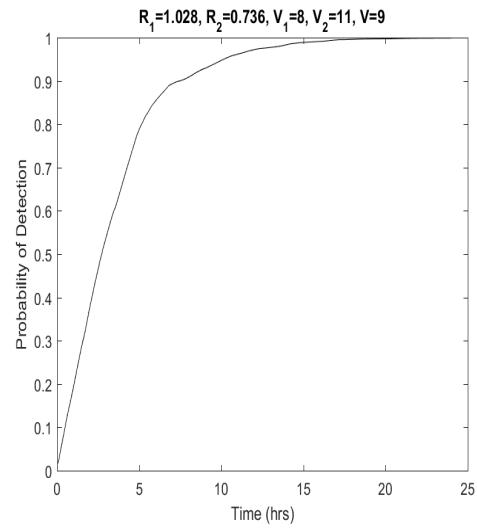


Figure A.116.  $P_D$  vs Time

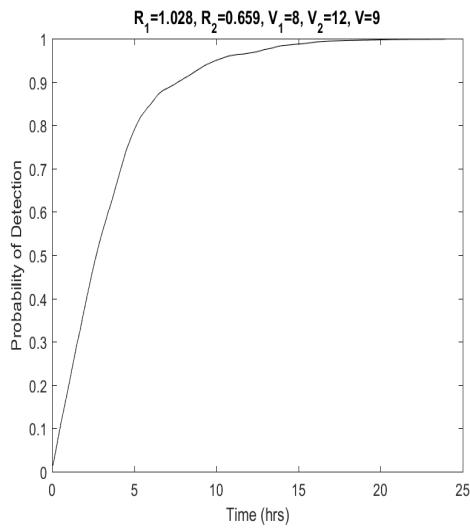


Figure A.117.  $P_D$  vs Time

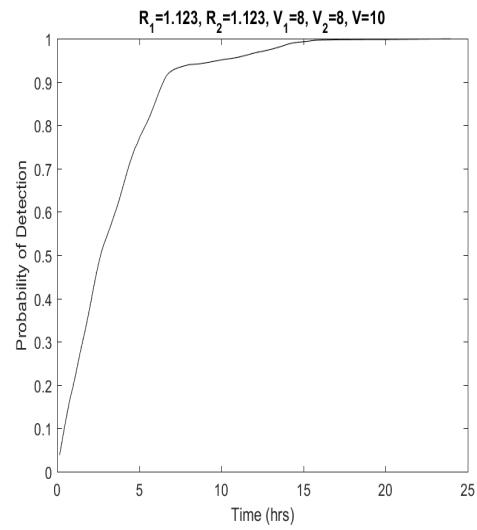


Figure A.118.  $P_D$  vs Time

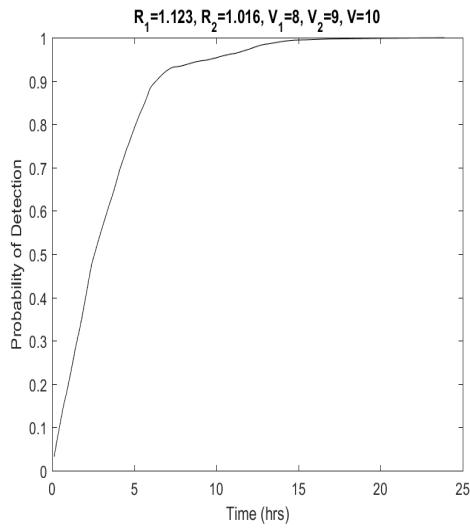


Figure A.119.  $P_D$  vs Time

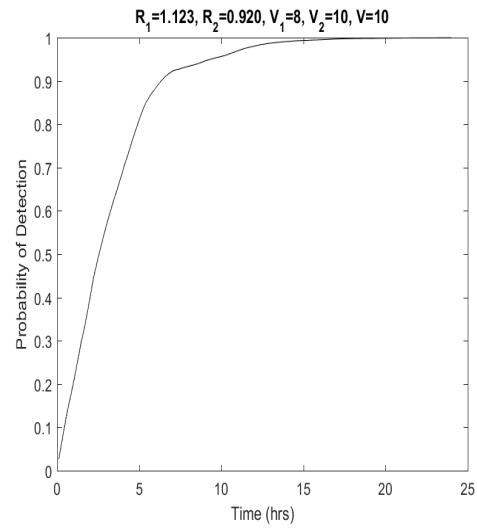


Figure A.120.  $P_D$  vs Time

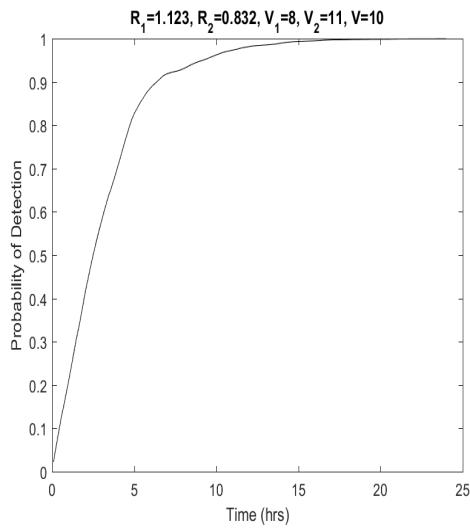


Figure A.121.  $P_D$  vs Time

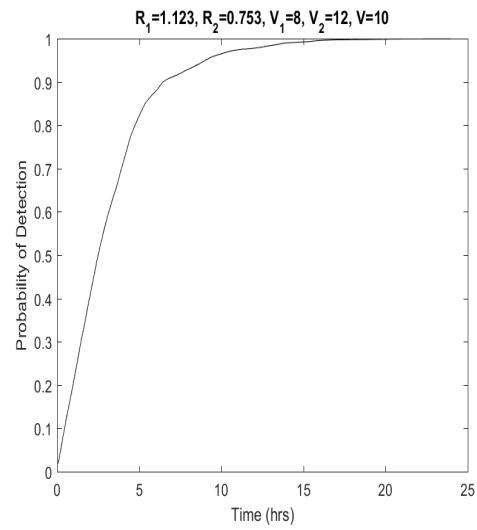


Figure A.122.  $P_D$  vs Time

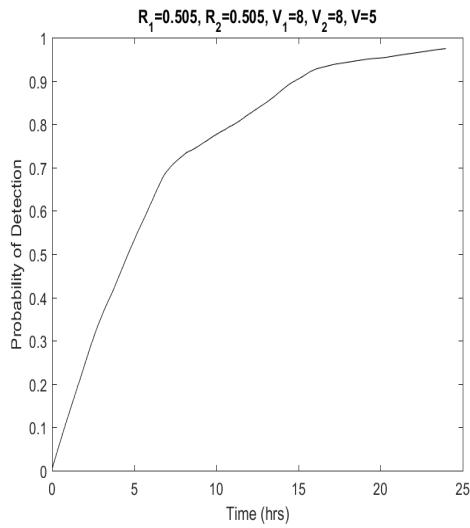


Figure A.123.  $P_D$  vs Time

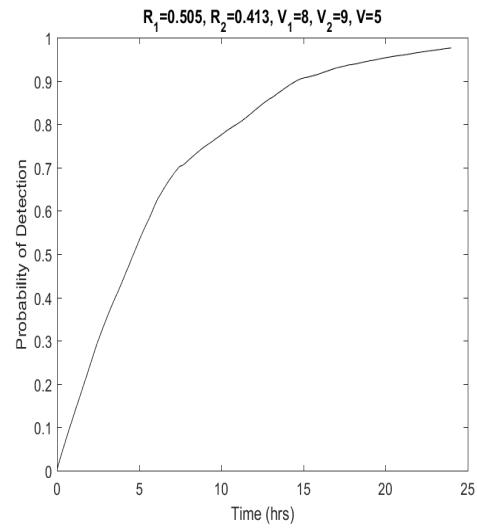


Figure A.124.  $P_D$  vs Time

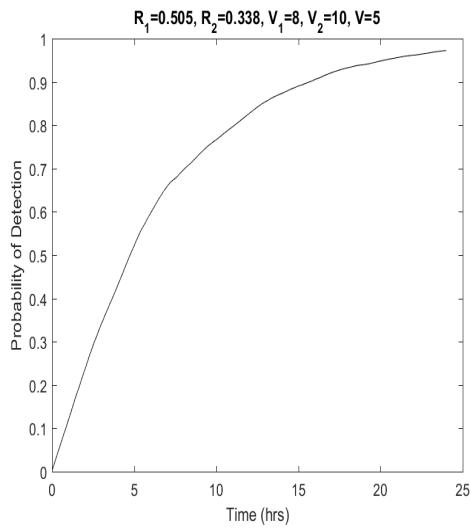


Figure A.125.  $P_D$  vs Time

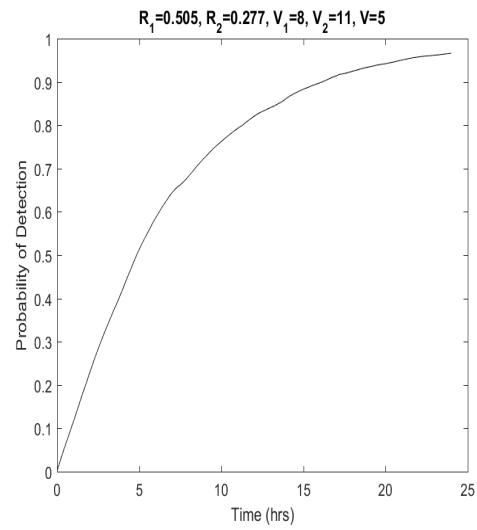


Figure A.126.  $P_D$  vs Time

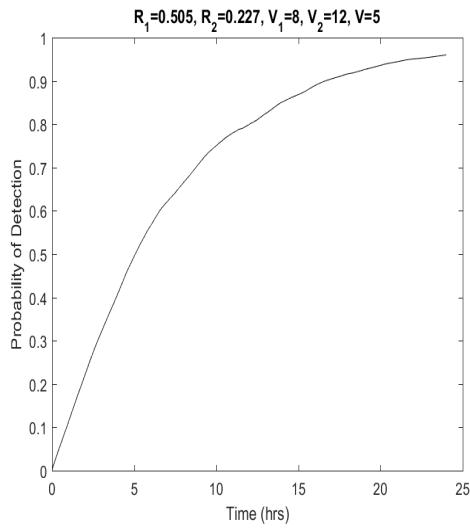


Figure A.127.  $P_D$  vs Time

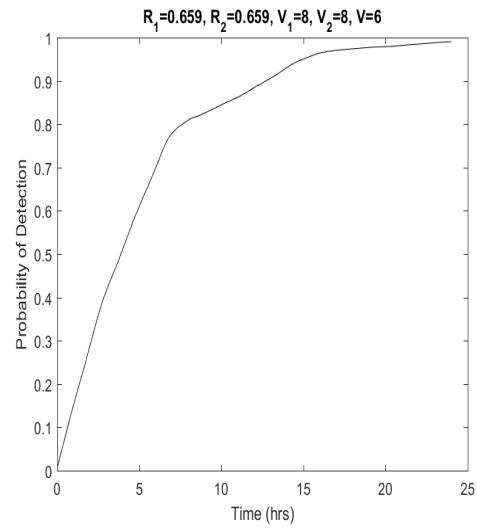


Figure A.128.  $P_D$  vs Time

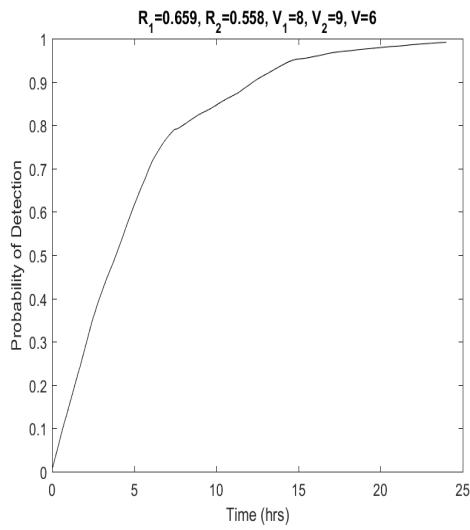


Figure A.129.  $P_D$  vs Time

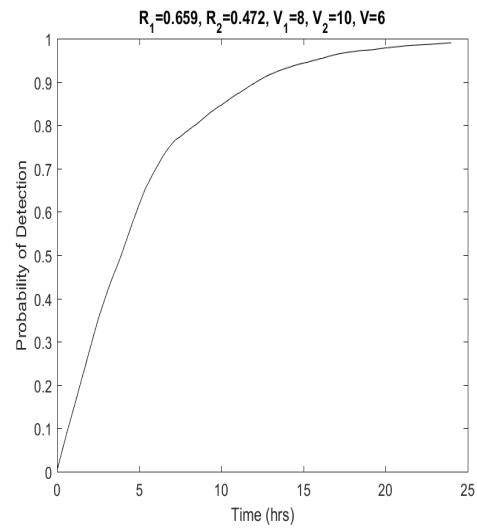


Figure A.130.  $P_D$  vs Time

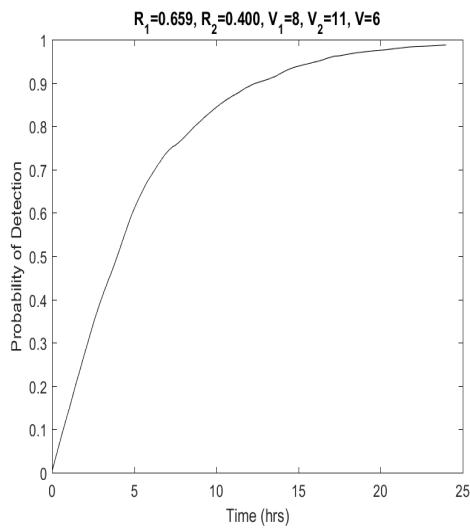


Figure A.131.  $P_D$  vs Time

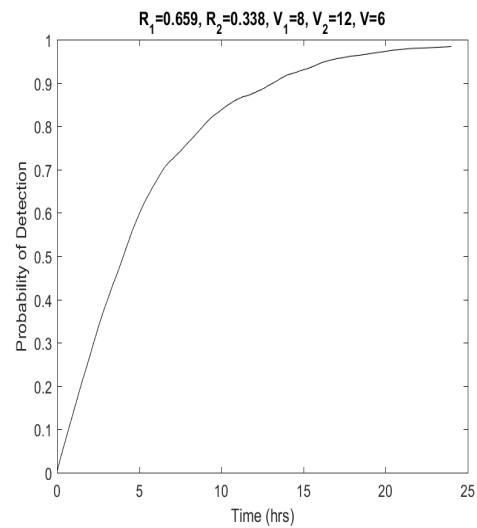


Figure A.132.  $P_D$  vs Time

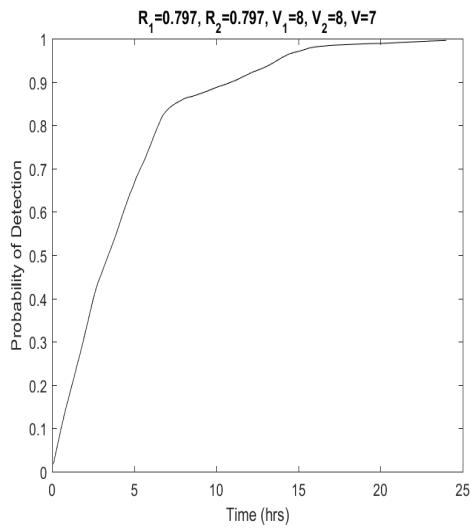


Figure A.133.  $P_D$  vs Time

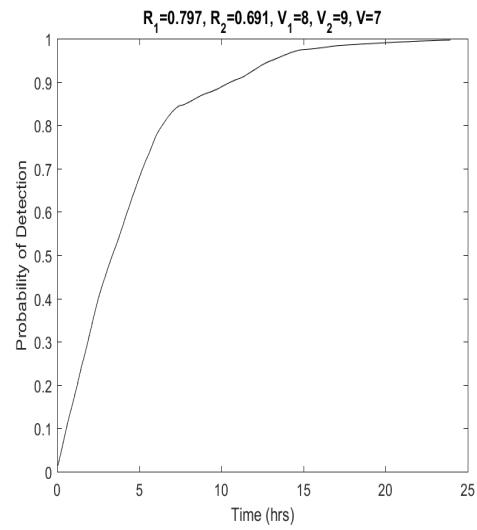


Figure A.134.  $P_D$  vs Time

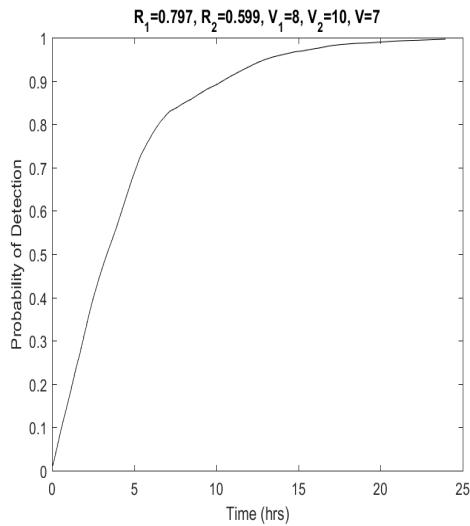


Figure A.135.  $P_D$  vs Time

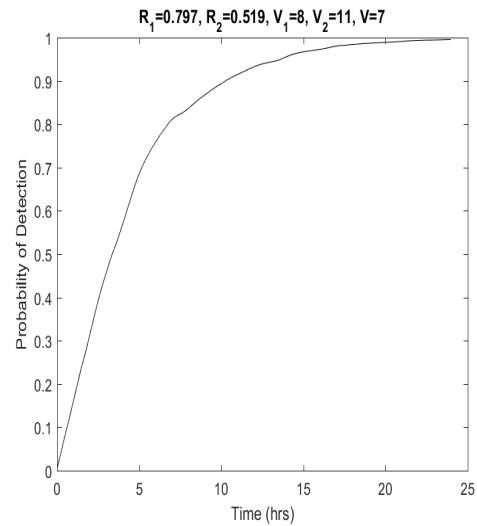


Figure A.136.  $P_D$  vs Time

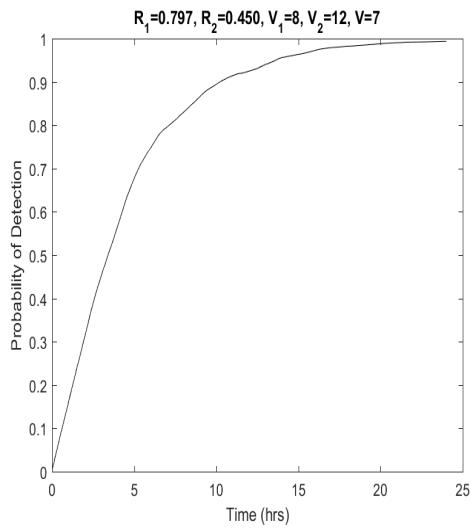


Figure A.137.  $P_D$  vs Time

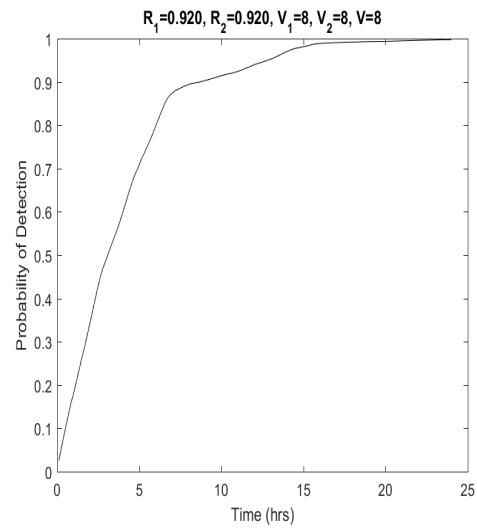


Figure A.138.  $P_D$  vs Time

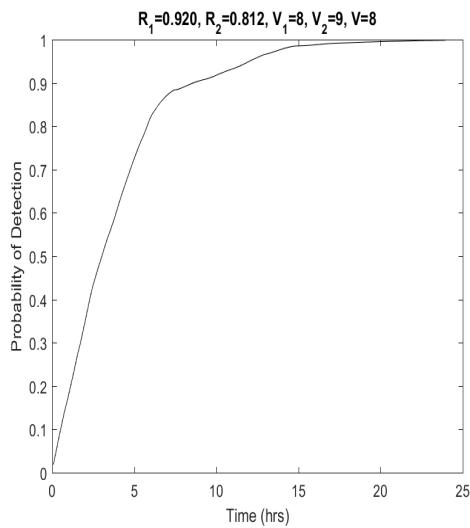


Figure A.139.  $P_D$  vs Time

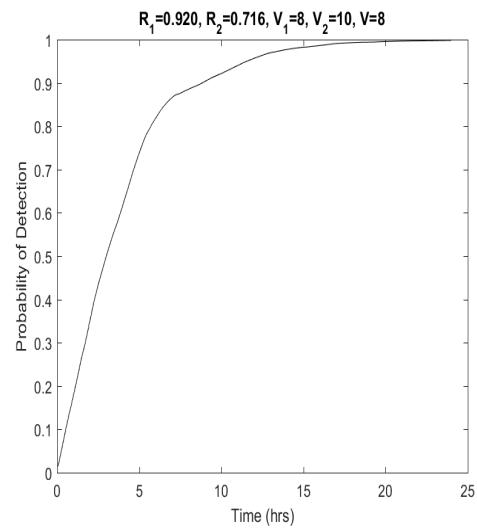


Figure A.140.  $P_D$  vs Time

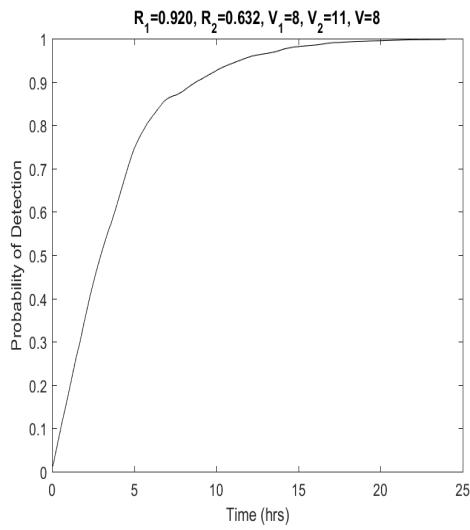


Figure A.141.  $P_D$  vs Time

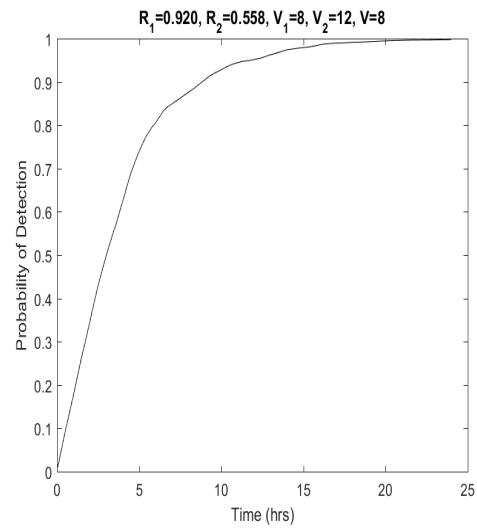


Figure A.142.  $P_D$  vs Time

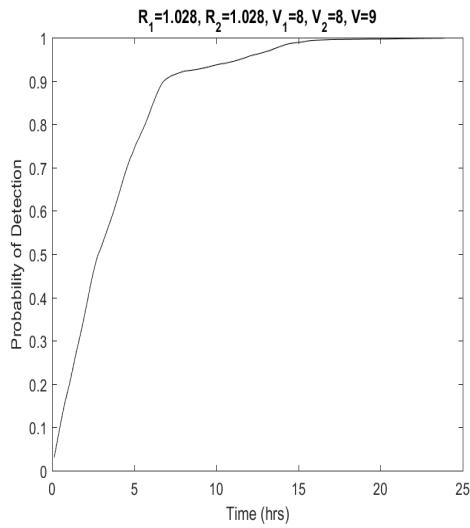


Figure A.143.  $P_D$  vs Time

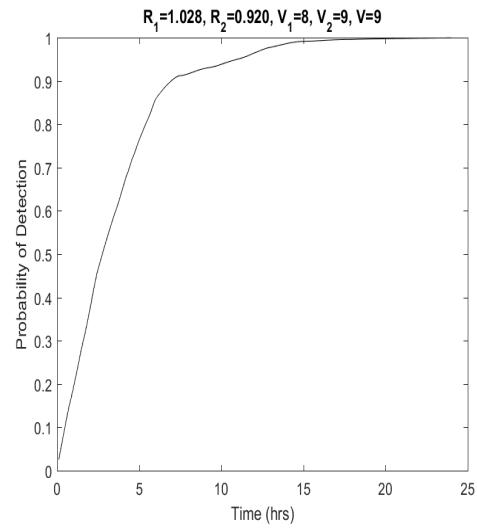


Figure A.144.  $P_D$  vs Time

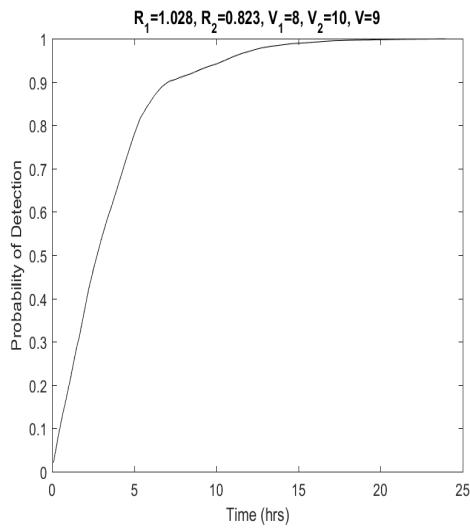


Figure A.145.  $P_D$  vs Time

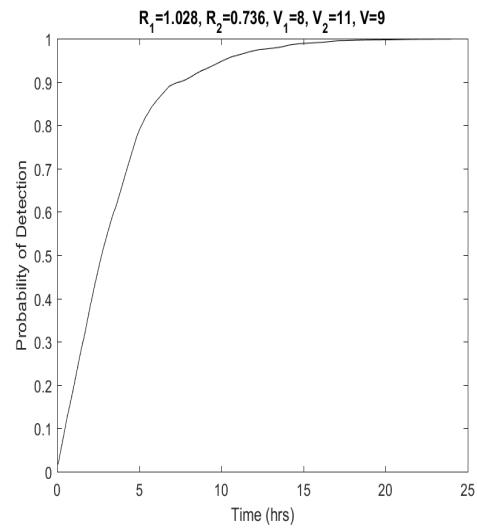


Figure A.146.  $P_D$  vs Time

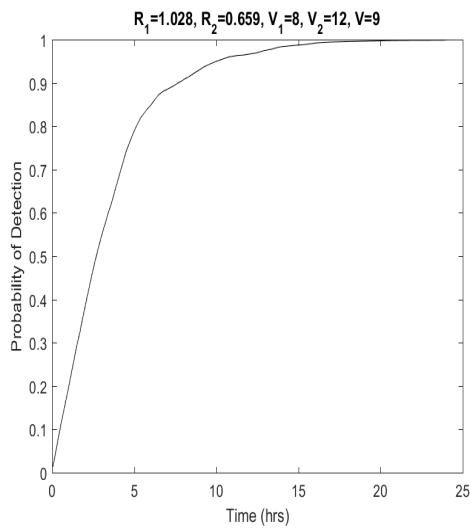


Figure A.147.  $P_D$  vs Time

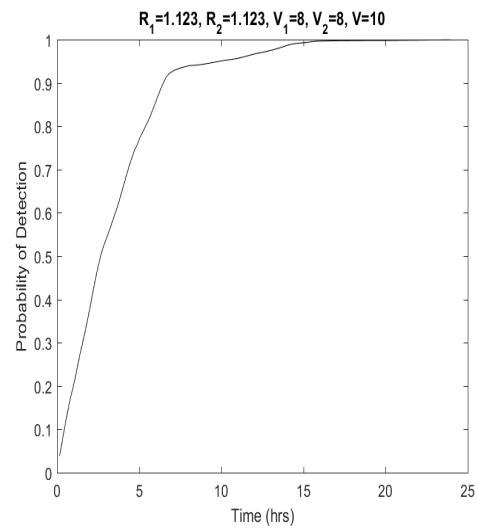


Figure A.148.  $P_D$  vs Time

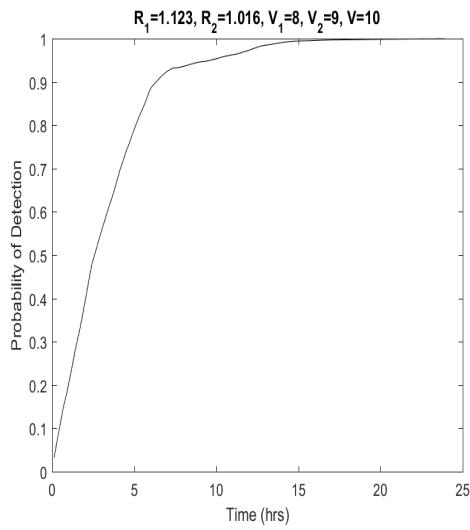


Figure A.149.  $P_D$  vs Time

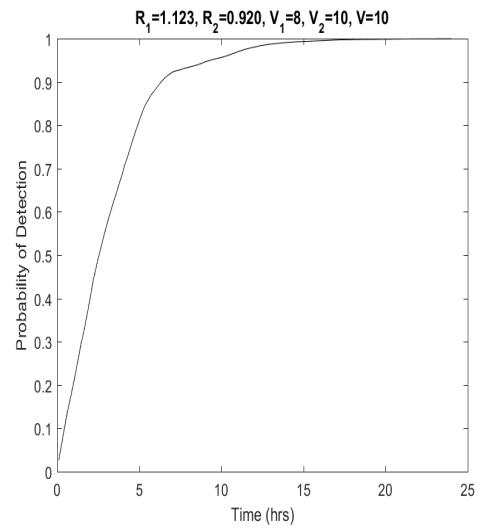


Figure A.150.  $P_D$  vs Time

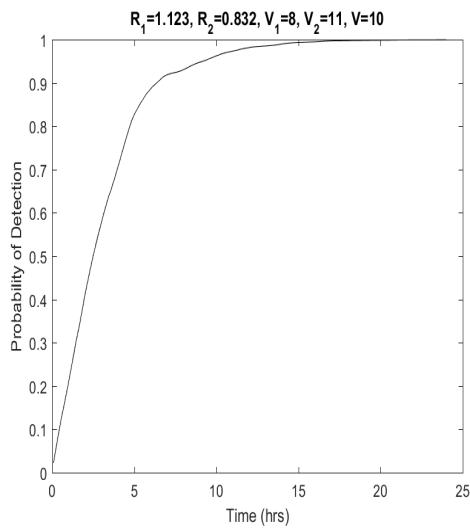


Figure A.151.  $P_D$  vs Time

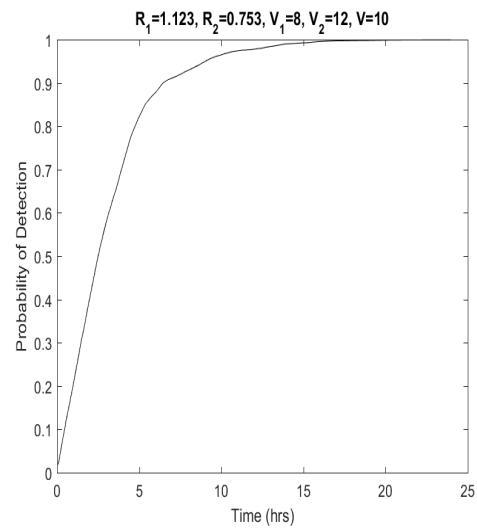


Figure A.152.  $P_D$  vs Time

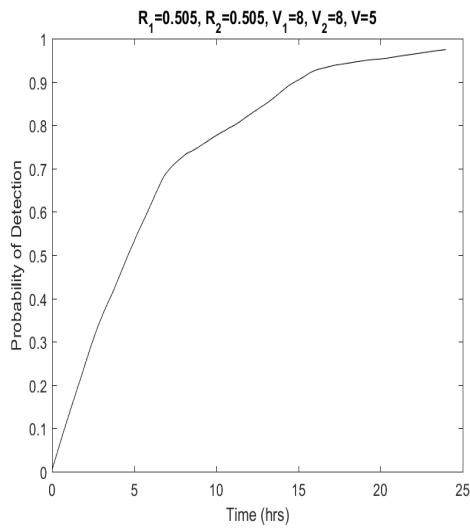


Figure A.153.  $P_D$  vs Time

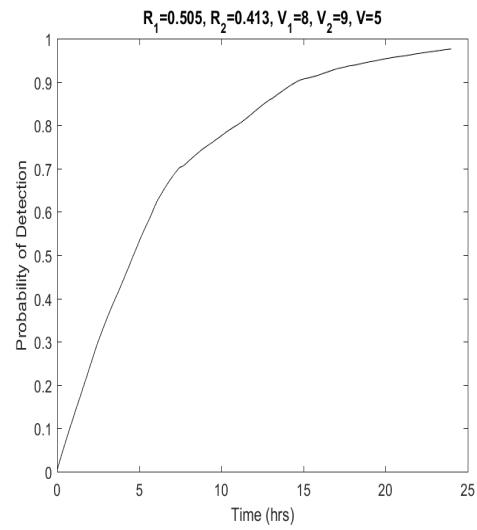


Figure A.154.  $P_D$  vs Time

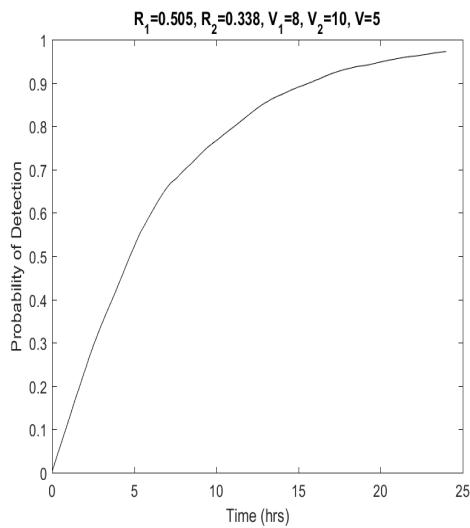


Figure A.155.  $P_D$  vs Time

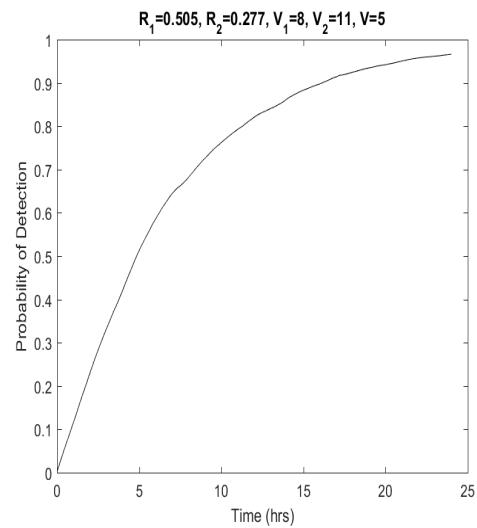


Figure A.156.  $P_D$  vs Time

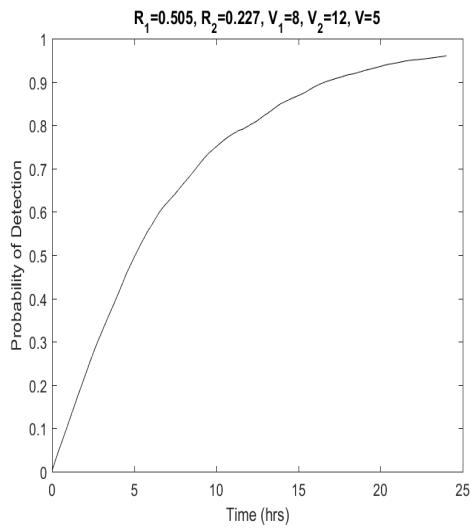


Figure A.157.  $P_D$  vs Time

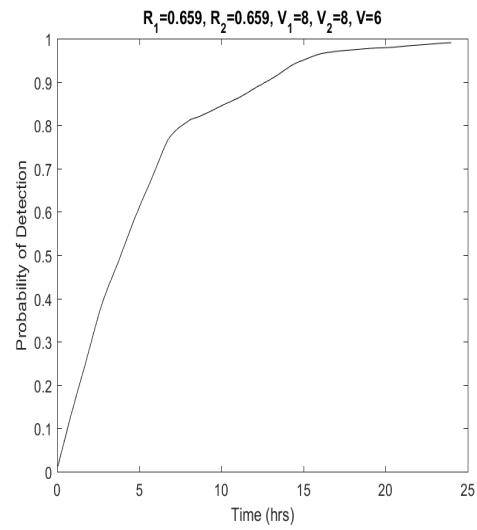


Figure A.158.  $P_D$  vs Time

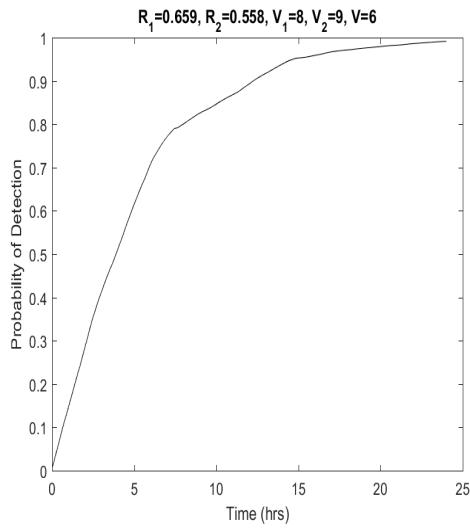


Figure A.159.  $P_D$  vs Time

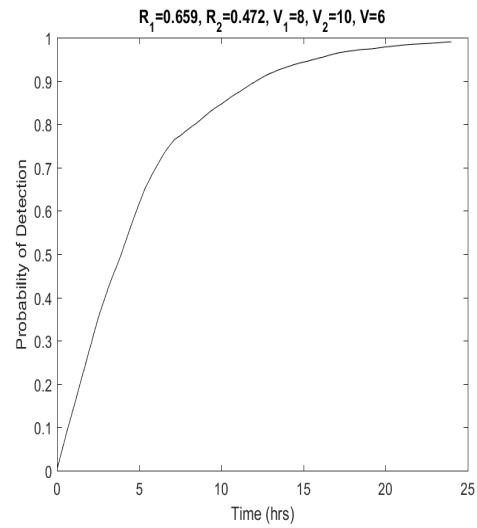


Figure A.160.  $P_D$  vs Time

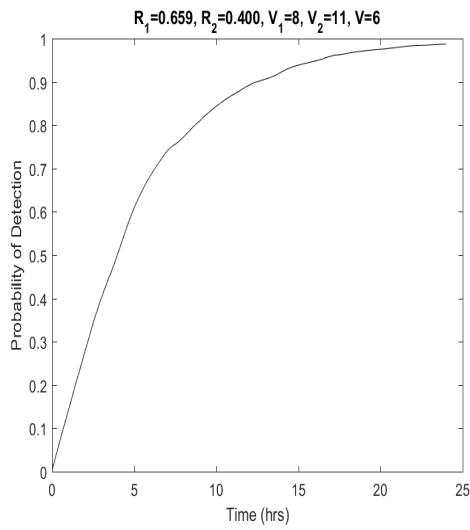


Figure A.161.  $P_D$  vs Time

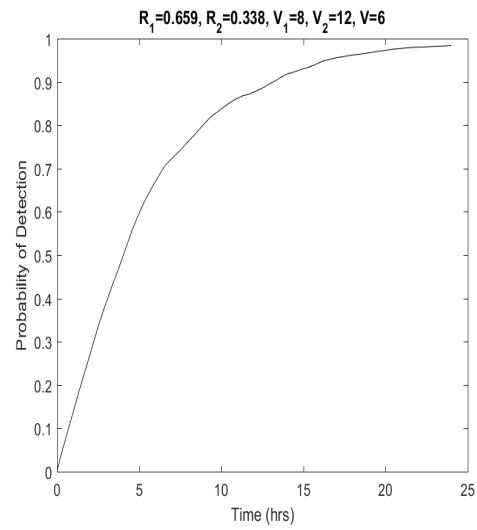


Figure A.162.  $P_D$  vs Time

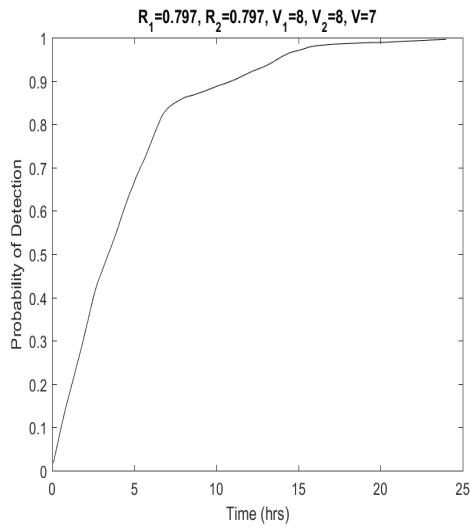


Figure A.163.  $P_D$  vs Time

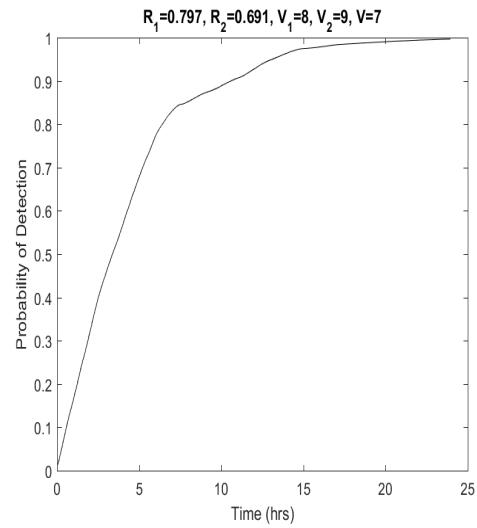


Figure A.164.  $P_D$  vs Time

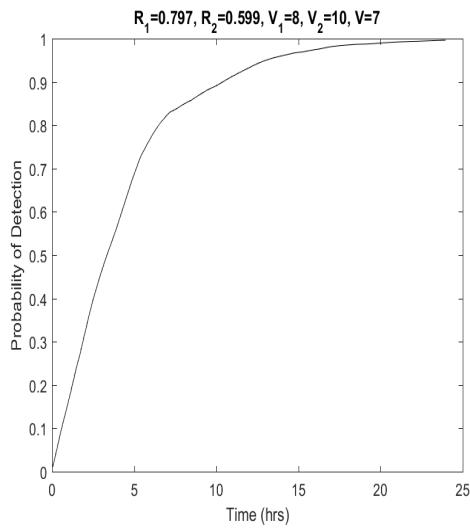


Figure A.165.  $P_D$  vs Time

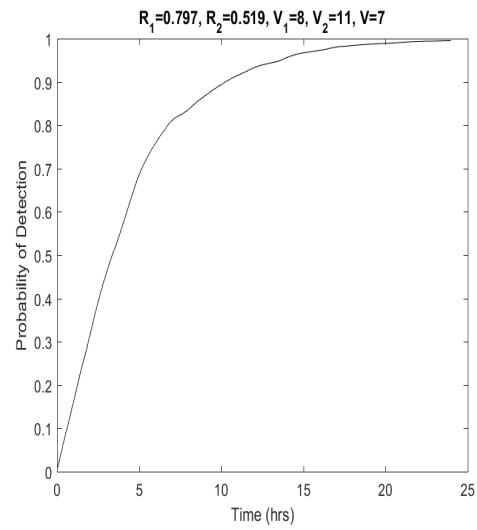


Figure A.166.  $P_D$  vs Time

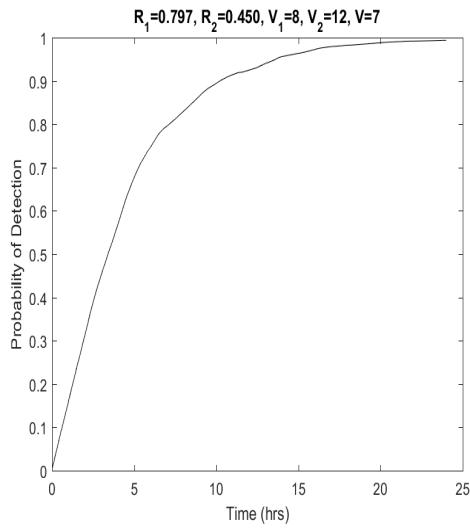


Figure A.167.  $P_D$  vs Time

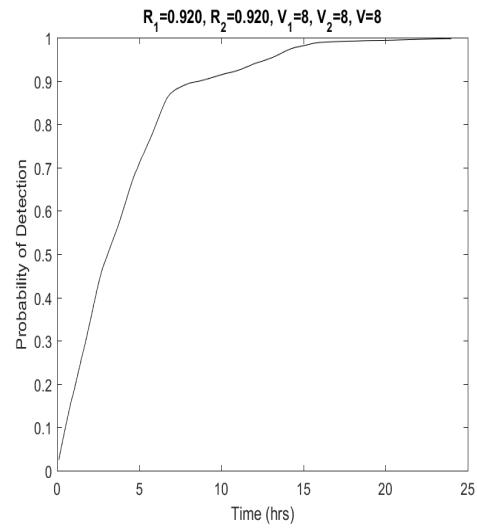


Figure A.168.  $P_D$  vs Time

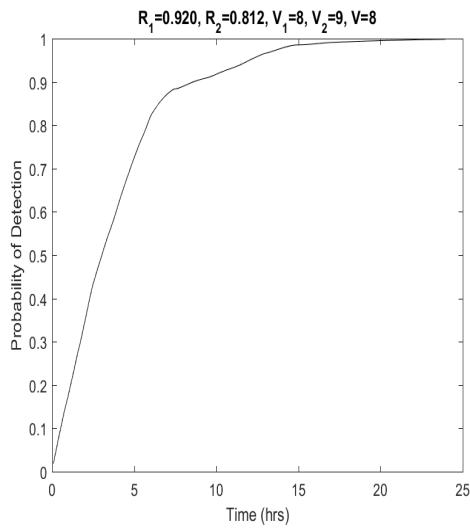


Figure A.169.  $P_D$  vs Time

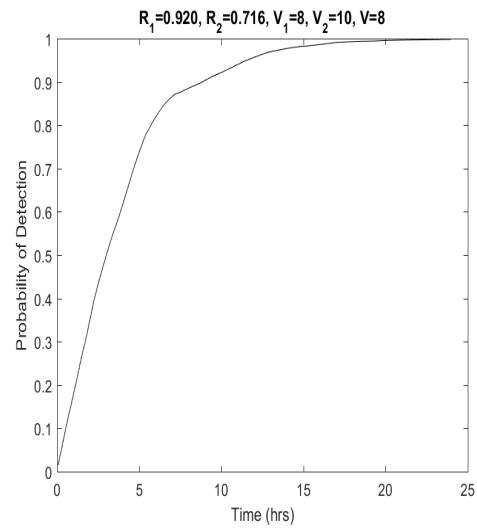


Figure A.170.  $P_D$  vs Time

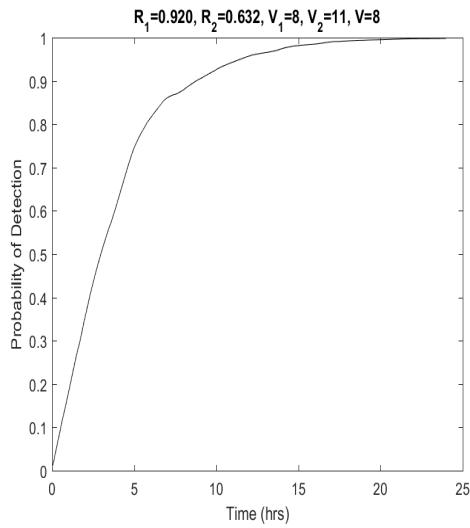


Figure A.171.  $P_D$  vs Time

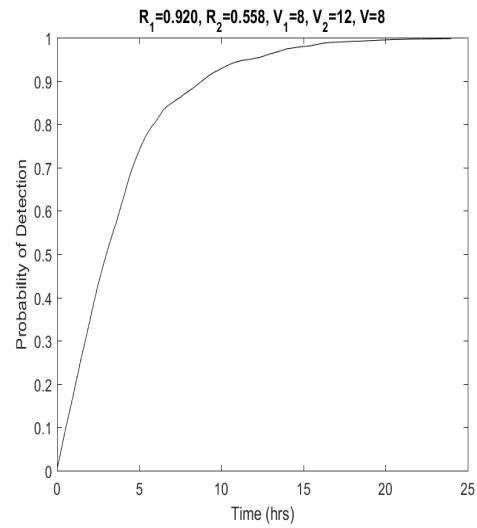


Figure A.172.  $P_D$  vs Time

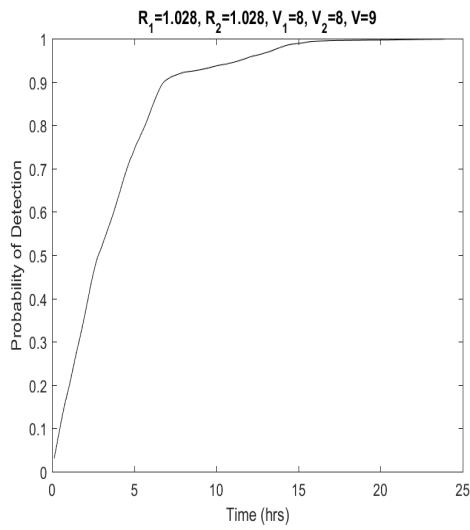


Figure A.173.  $P_D$  vs Time

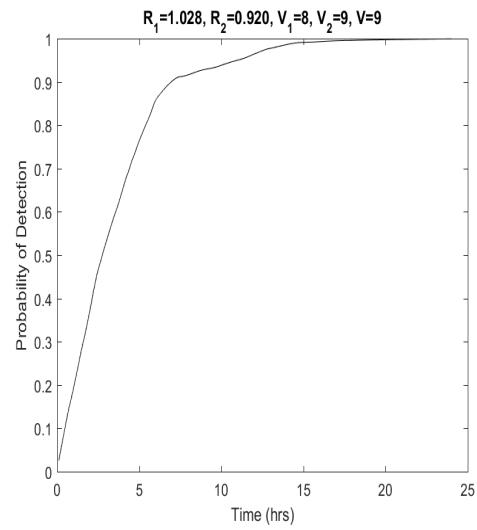


Figure A.174.  $P_D$  vs Time

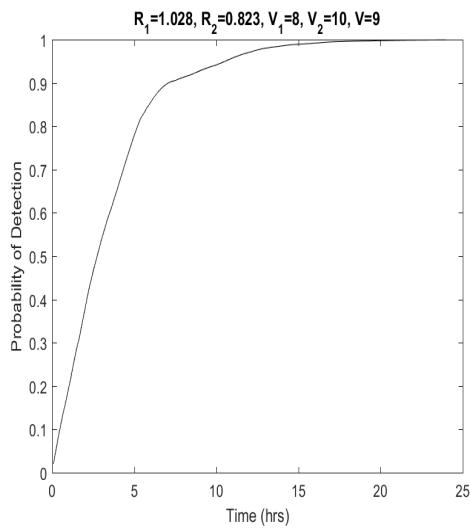


Figure A.175.  $P_D$  vs Time

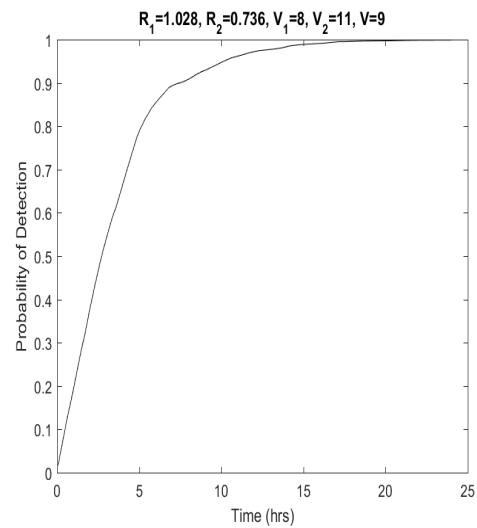


Figure A.176.  $P_D$  vs Time

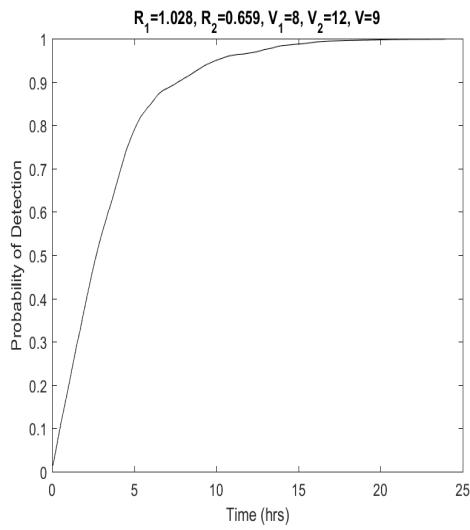


Figure A.177.  $P_D$  vs Time

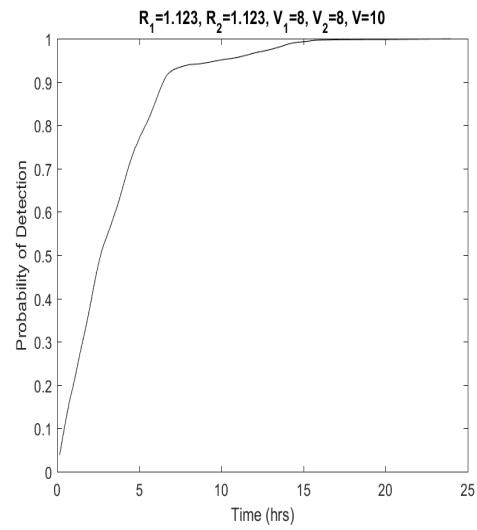


Figure A.178.  $P_D$  vs Time

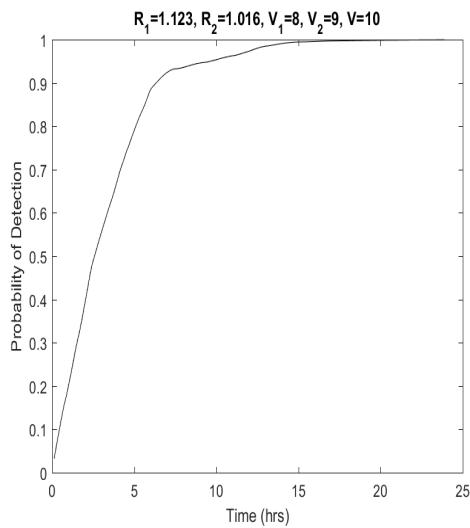


Figure A.179.  $P_D$  vs Time

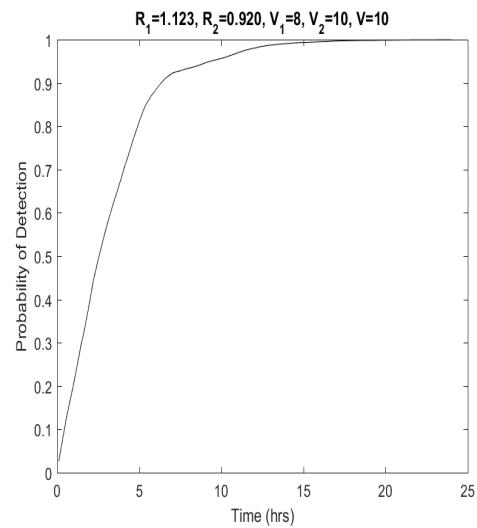


Figure A.180.  $P_D$  vs Time

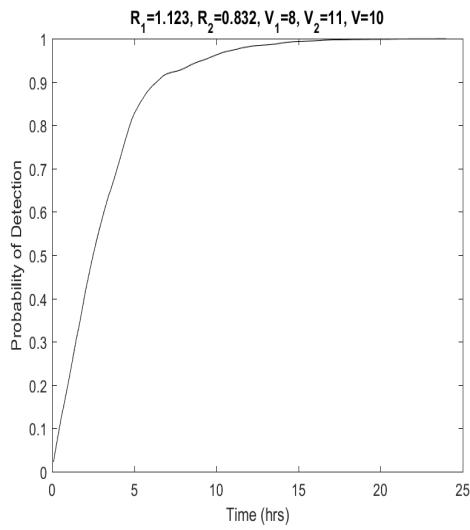


Figure A.181.  $P_D$  vs Time

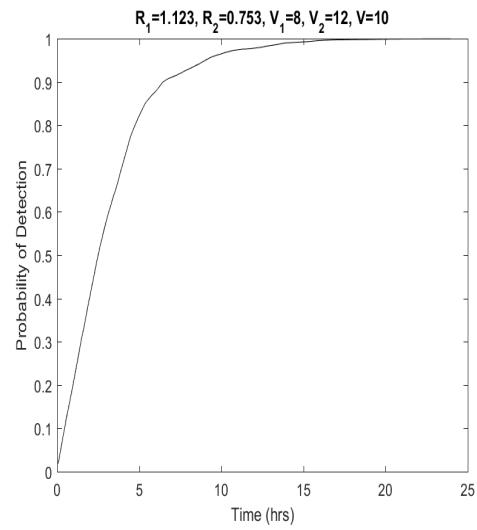


Figure A.182.  $P_D$  vs Time

### A.3.2 Multipath Ladder Search

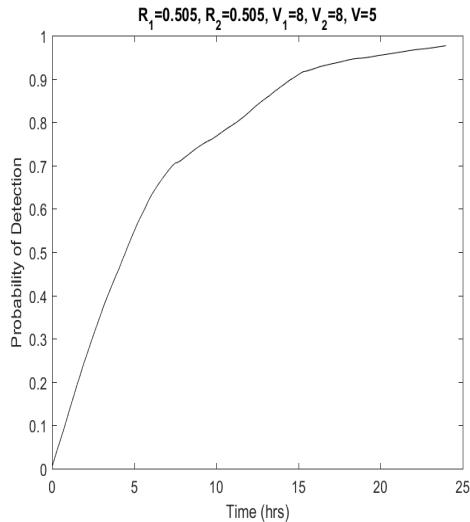


Figure A.183.  $P_D$  vs Time

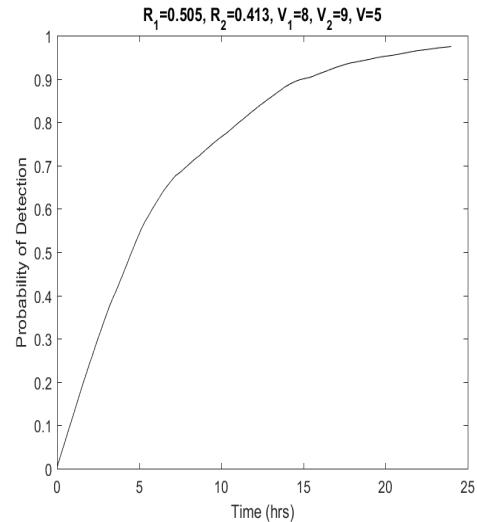


Figure A.184.  $P_D$  vs Time

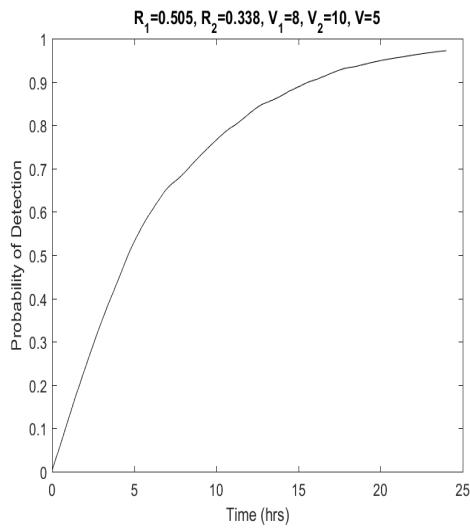


Figure A.185.  $P_D$  vs Time

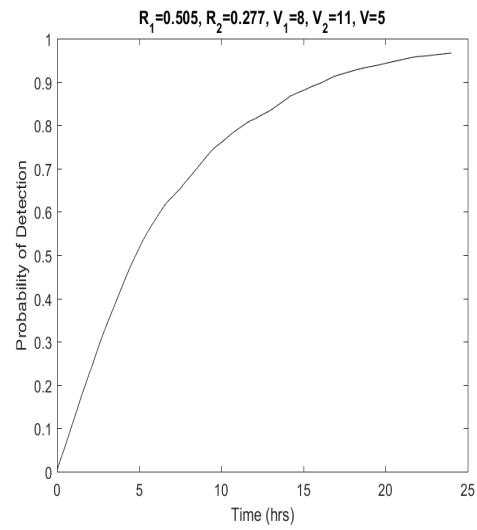


Figure A.186.  $P_D$  vs Time

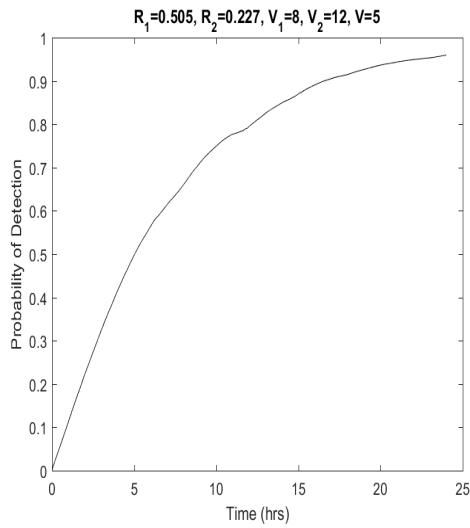


Figure A.187.  $P_D$  vs Time

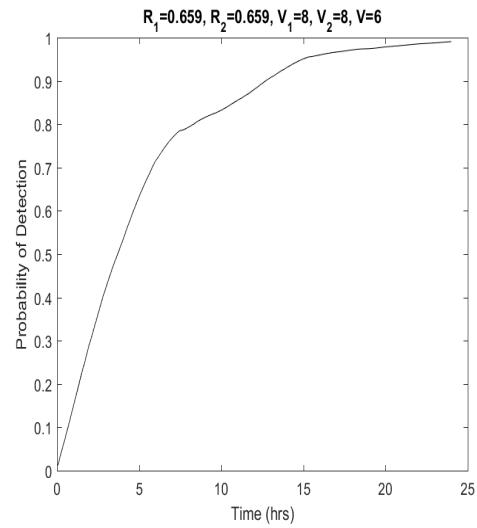


Figure A.188.  $P_D$  vs Time

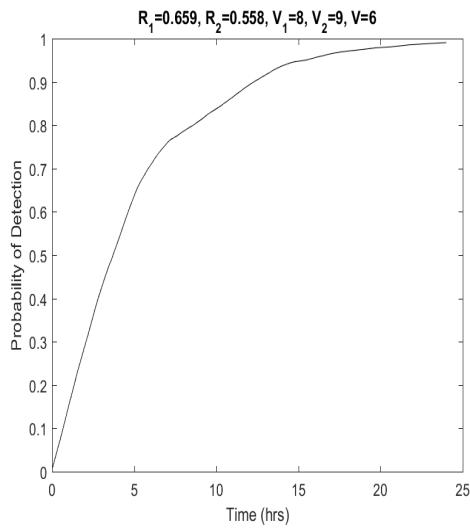


Figure A.189.  $P_D$  vs Time

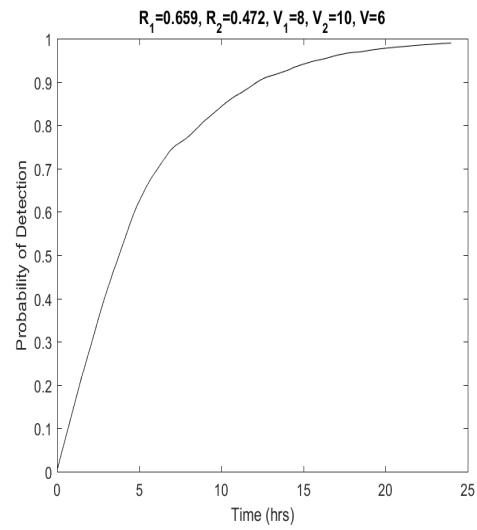


Figure A.190.  $P_D$  vs Time

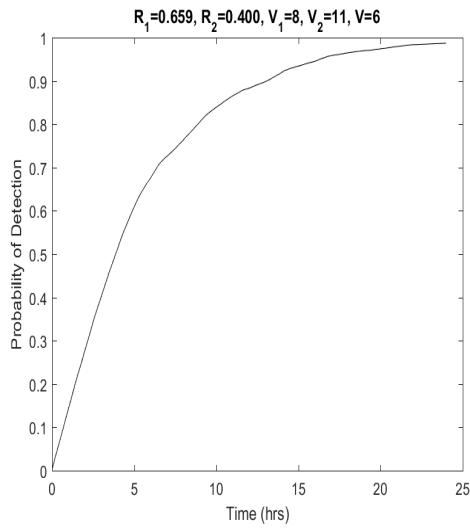


Figure A.191.  $P_D$  vs Time

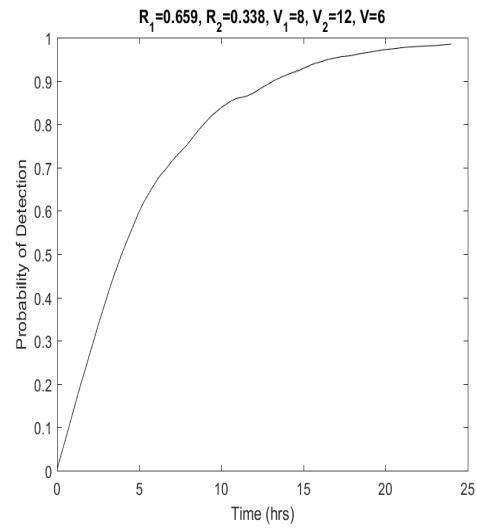


Figure A.192.  $P_D$  vs Time

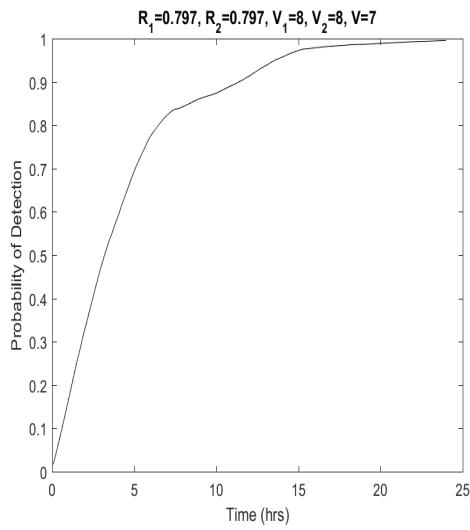


Figure A.193.  $P_D$  vs Time

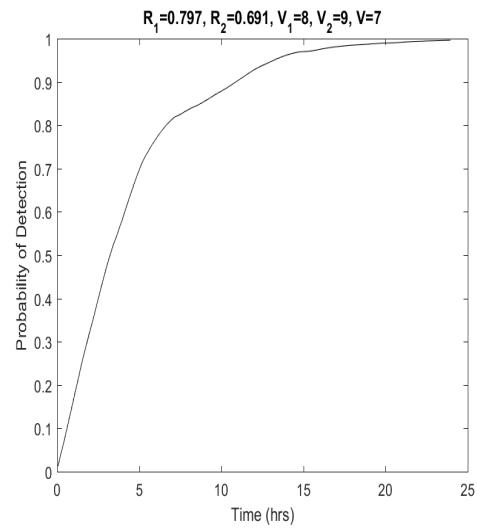


Figure A.194.  $P_D$  vs Time

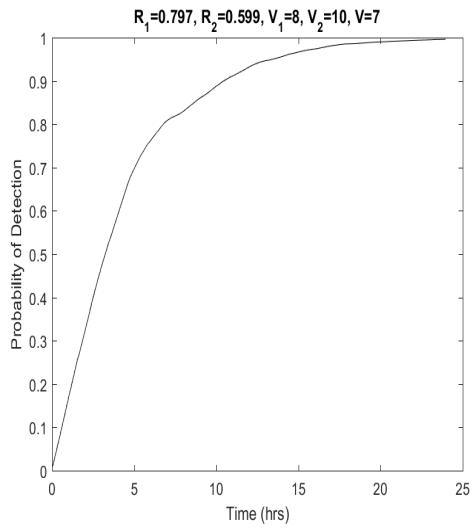


Figure A.195.  $P_D$  vs Time

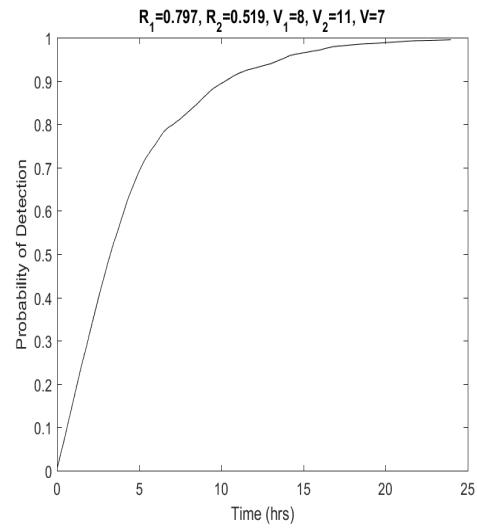


Figure A.196.  $P_D$  vs Time

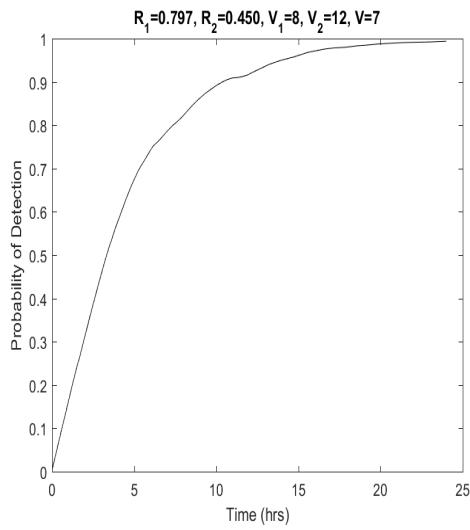


Figure A.197.  $P_D$  vs Time

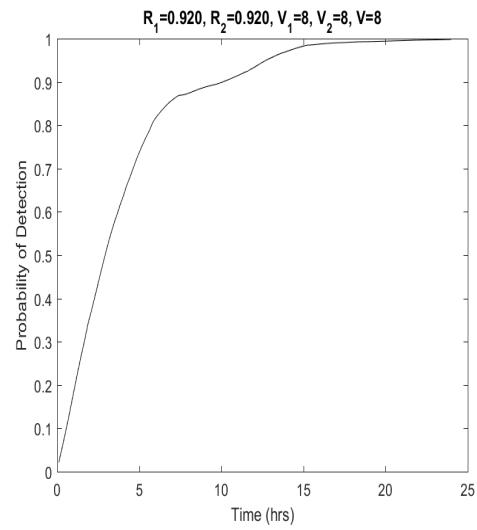


Figure A.198.  $P_D$  vs Time

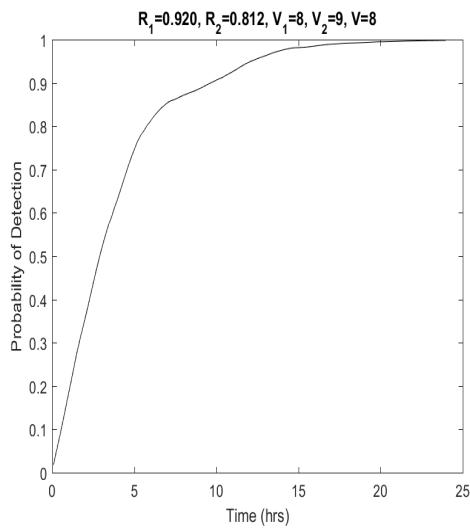


Figure A.199.  $P_D$  vs Time

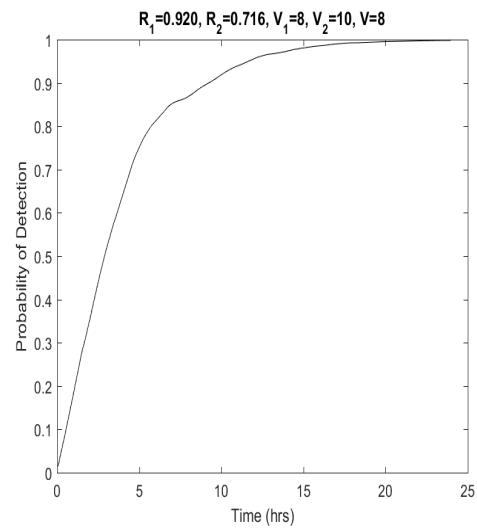


Figure A.200.  $P_D$  vs Time

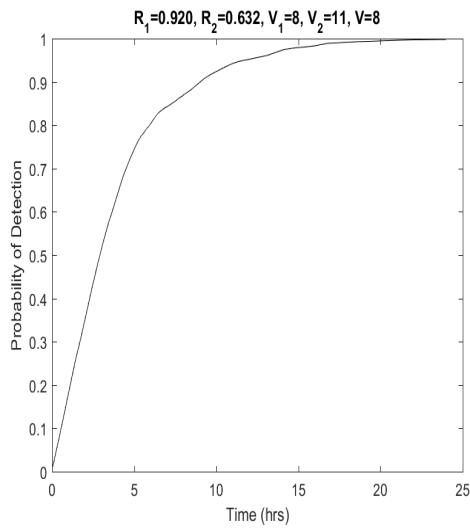


Figure A.201.  $P_D$  vs Time

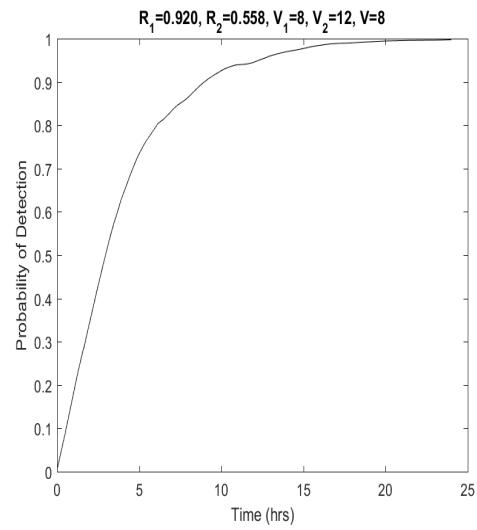


Figure A.202.  $P_D$  vs Time

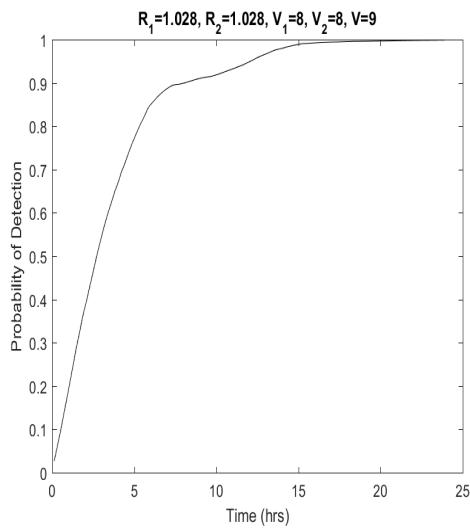


Figure A.203.  $P_D$  vs Time

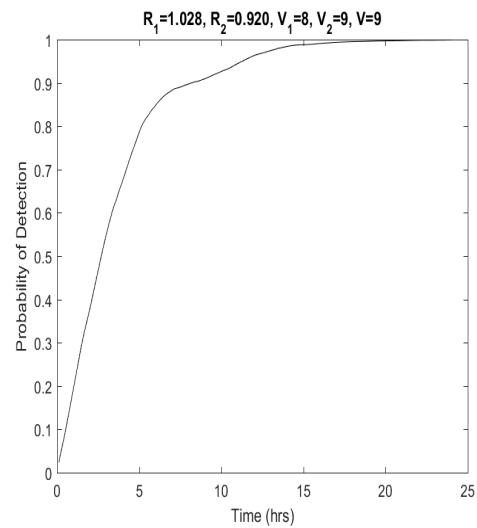


Figure A.204.  $P_D$  vs Time

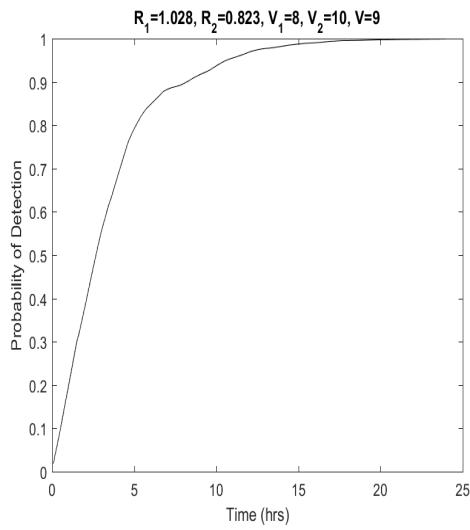


Figure A.205.  $P_D$  vs Time

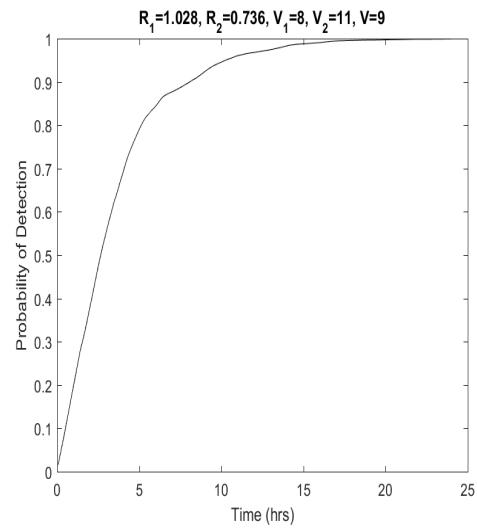


Figure A.206.  $P_D$  vs Time

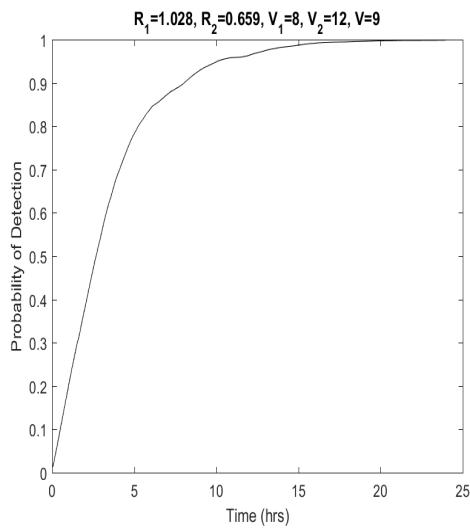


Figure A.207.  $P_D$  vs Time

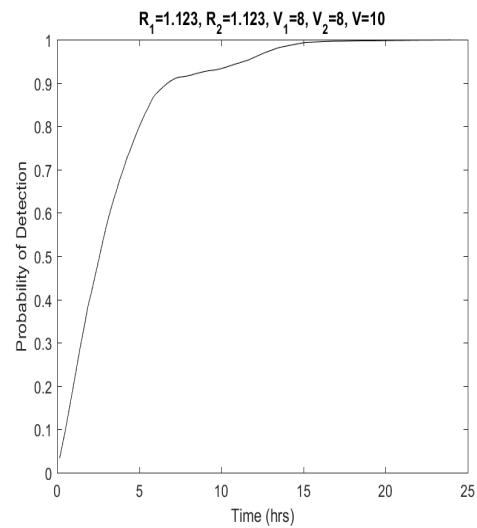


Figure A.208.  $P_D$  vs Time

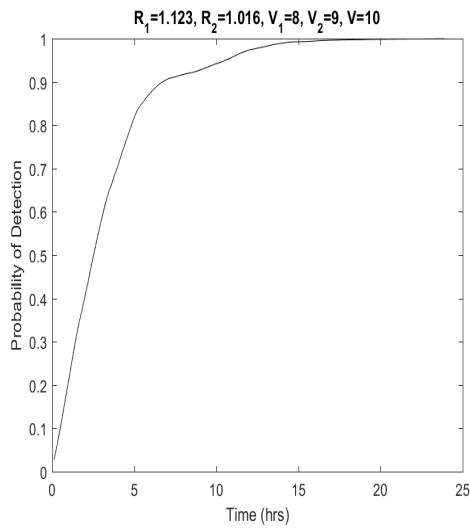


Figure A.209.  $P_D$  vs Time

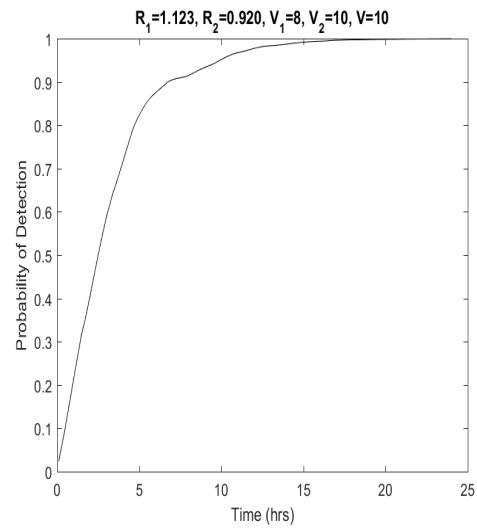


Figure A.210.  $P_D$  vs Time

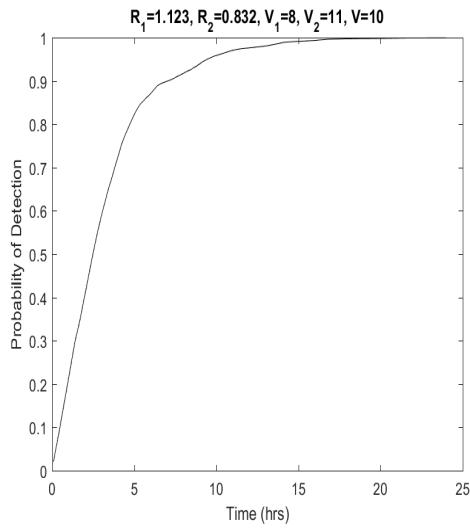


Figure A.211.  $P_D$  vs Time

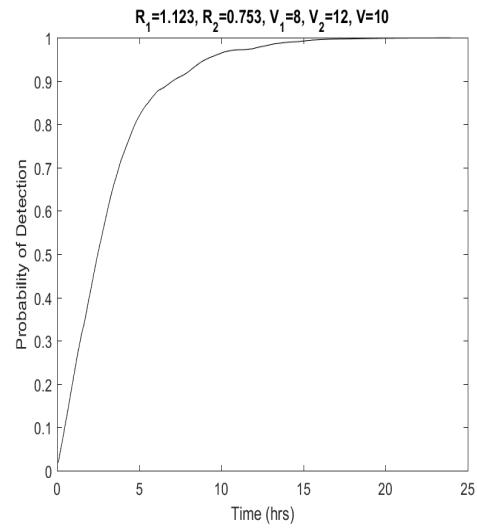


Figure A.212.  $P_D$  vs Time

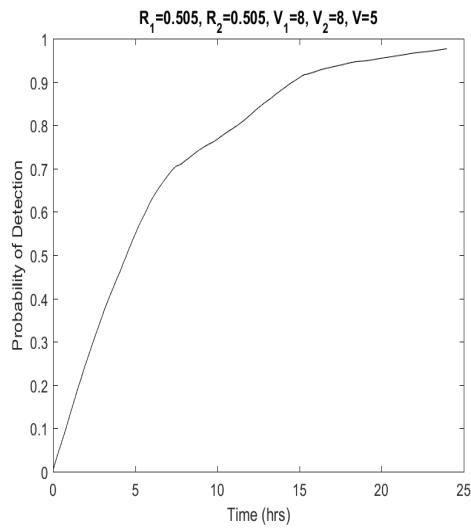


Figure A.213.  $P_D$  vs Time

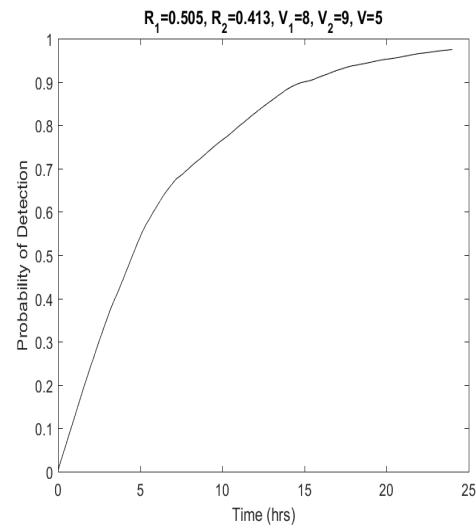


Figure A.214.  $P_D$  vs Time

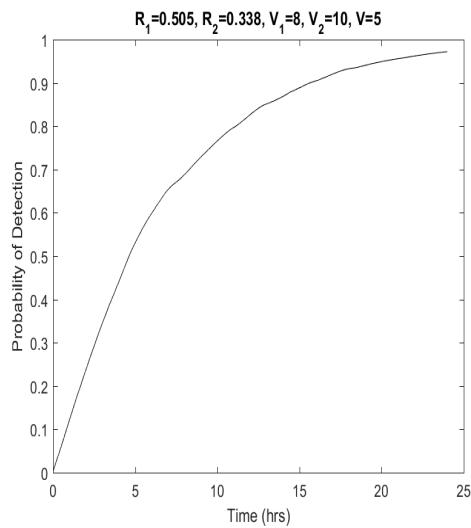


Figure A.215.  $P_D$  vs Time

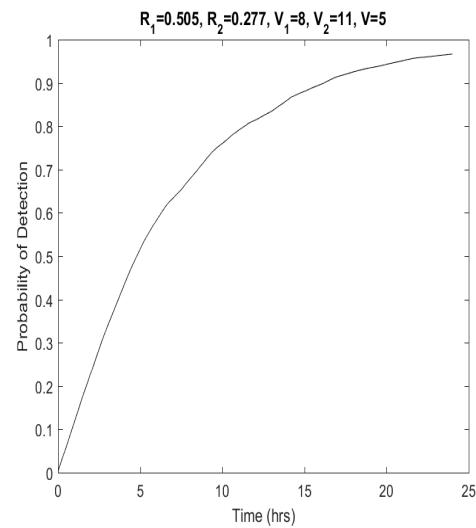


Figure A.216.  $P_D$  vs Time

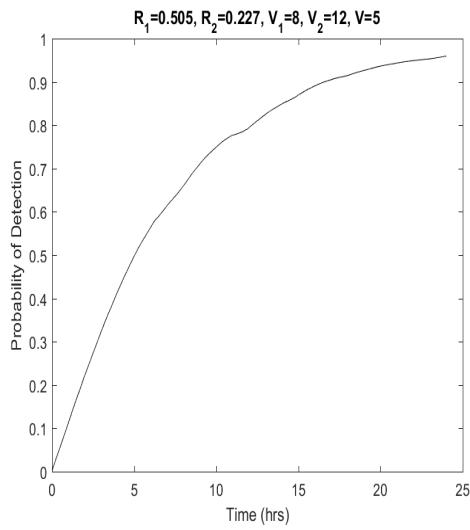


Figure A.217.  $P_D$  vs Time

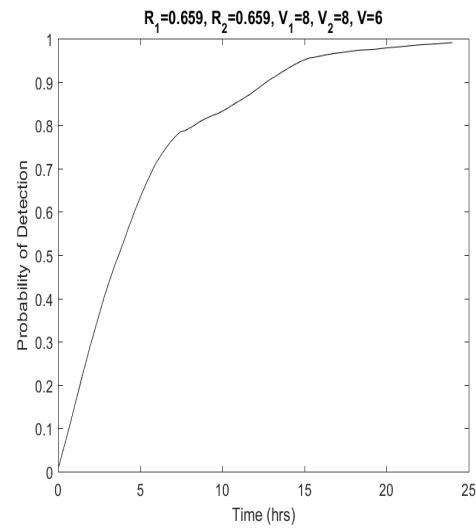


Figure A.218.  $P_D$  vs Time

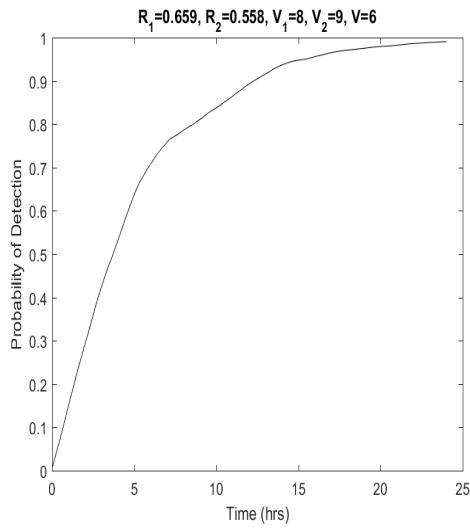


Figure A.219.  $P_D$  vs Time

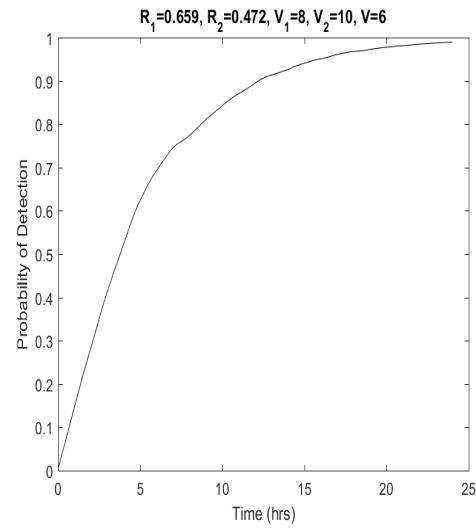


Figure A.220.  $P_D$  vs Time

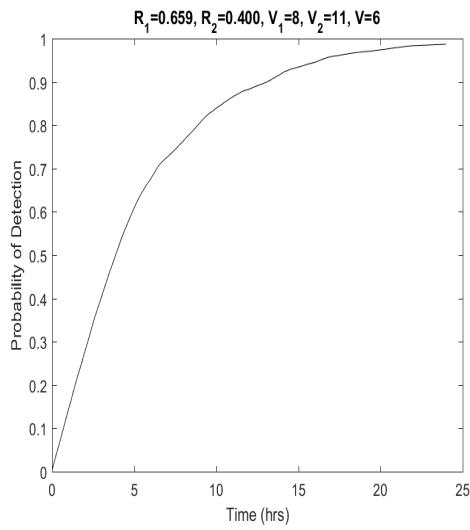


Figure A.221.  $P_D$  vs Time

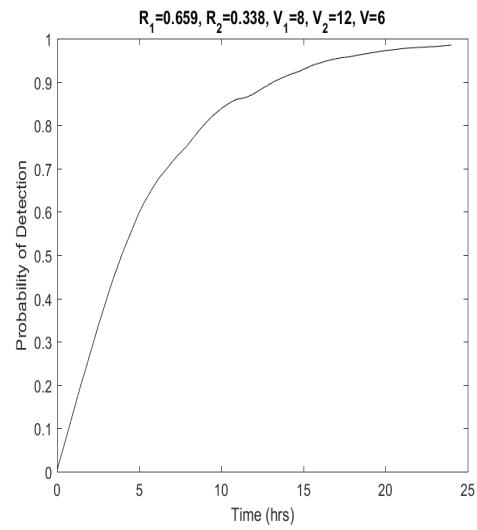


Figure A.222.  $P_D$  vs Time

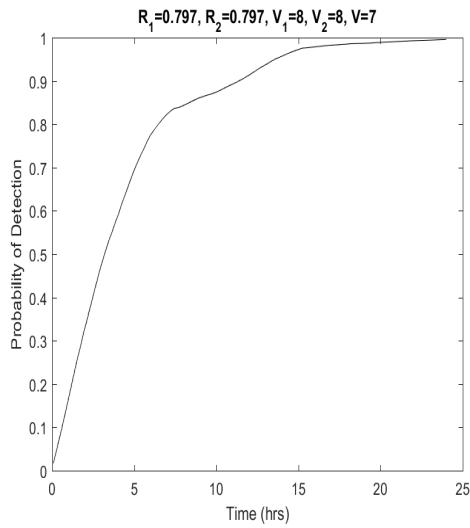


Figure A.223.  $P_D$  vs Time

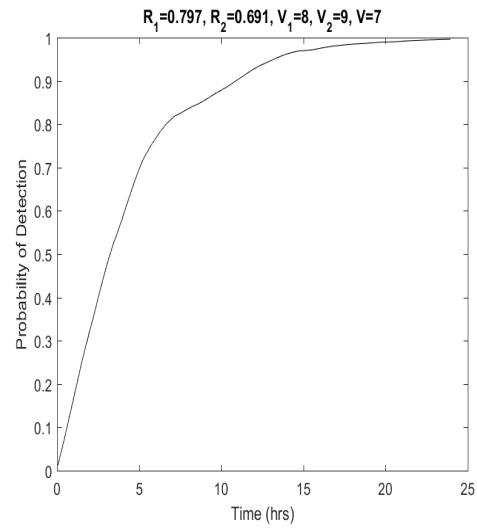


Figure A.224.  $P_D$  vs Time

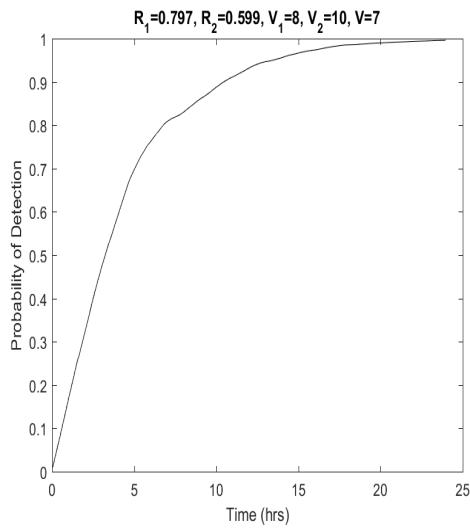


Figure A.225.  $P_D$  vs Time

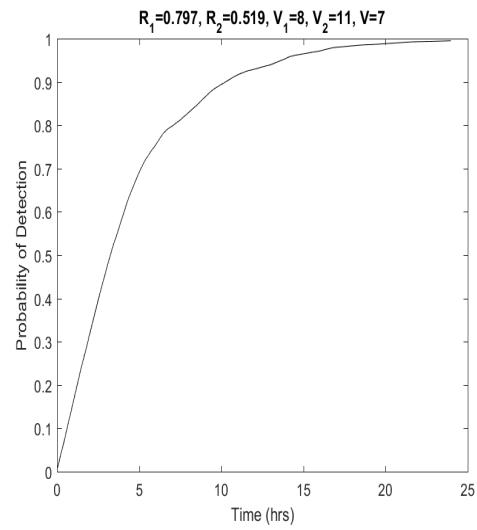


Figure A.226.  $P_D$  vs Time

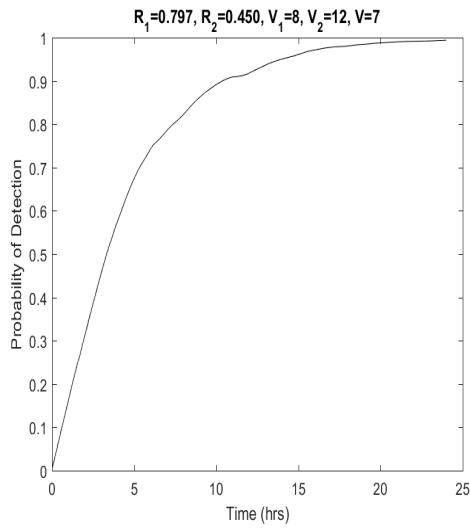


Figure A.227.  $P_D$  vs Time

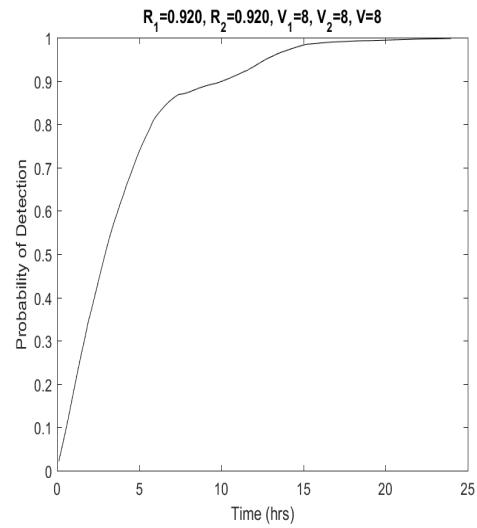


Figure A.228.  $P_D$  vs Time

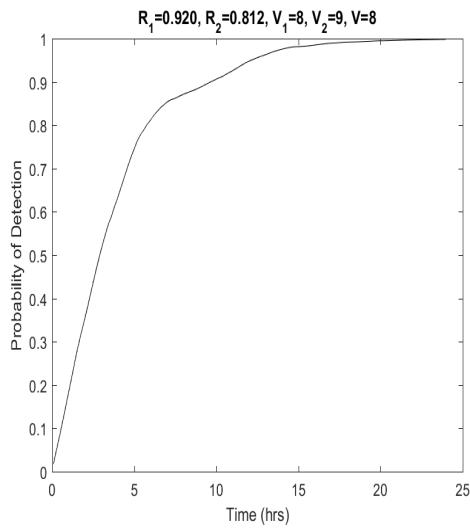


Figure A.229.  $P_D$  vs Time

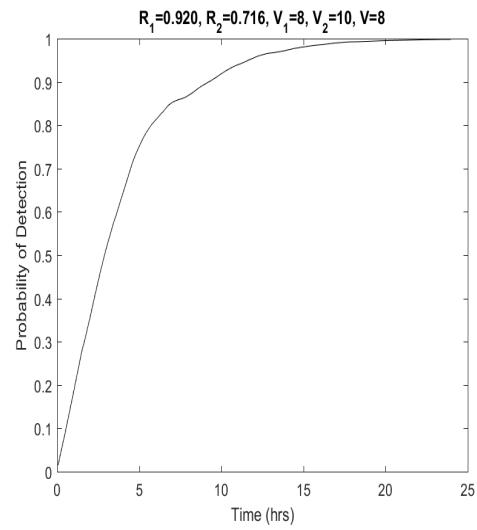


Figure A.230.  $P_D$  vs Time

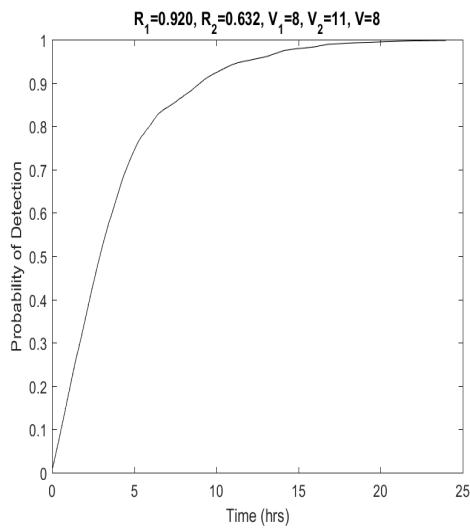


Figure A.231.  $P_D$  vs Time

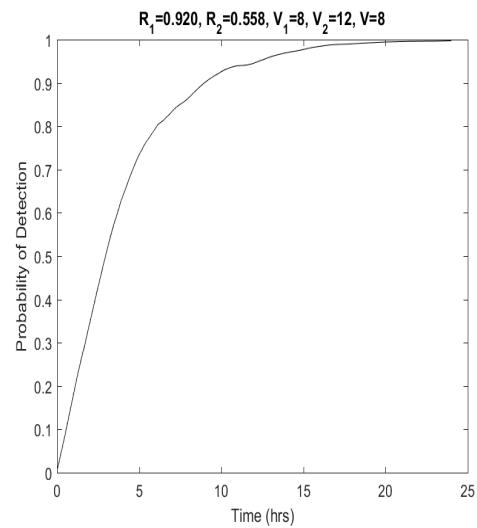


Figure A.232.  $P_D$  vs Time

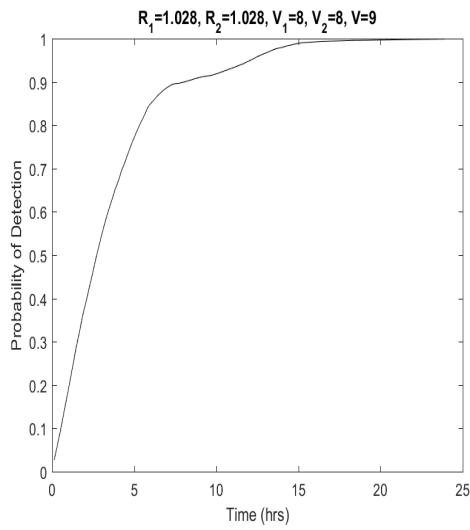


Figure A.233.  $P_D$  vs Time

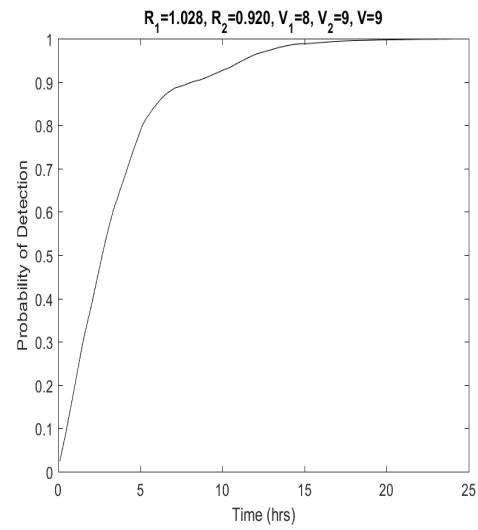


Figure A.234.  $P_D$  vs Time

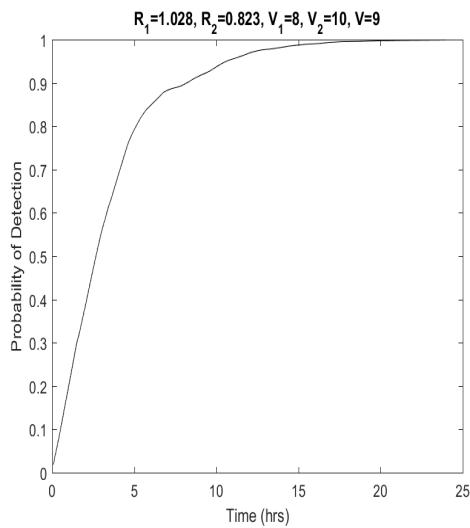


Figure A.235.  $P_D$  vs Time

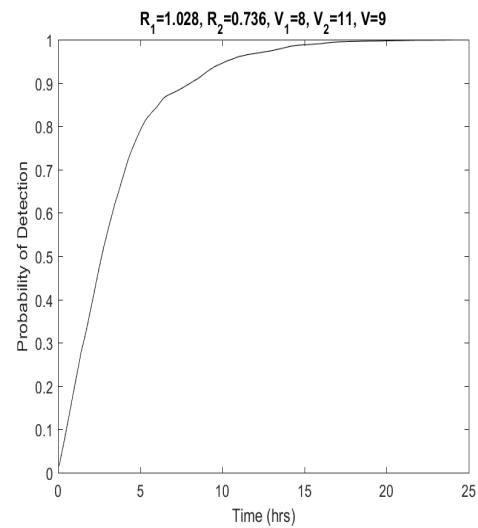


Figure A.236.  $P_D$  vs Time

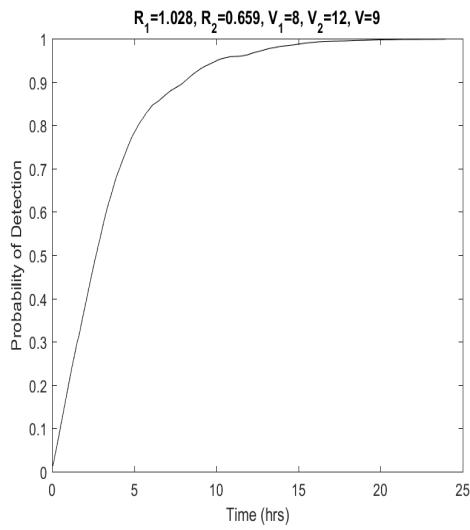


Figure A.237.  $P_D$  vs Time

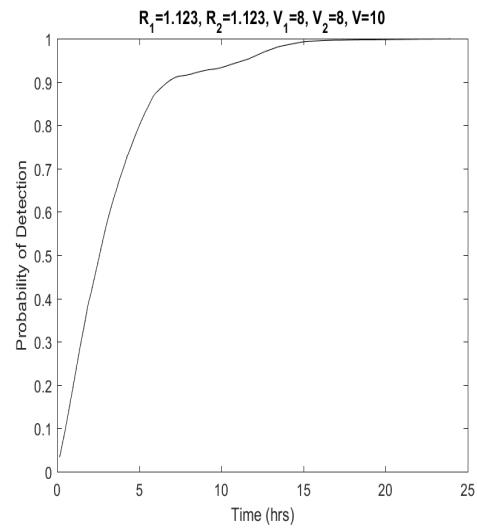


Figure A.238.  $P_D$  vs Time

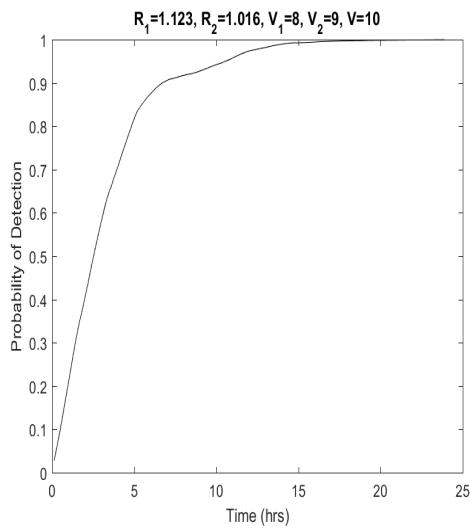


Figure A.239.  $P_D$  vs Time

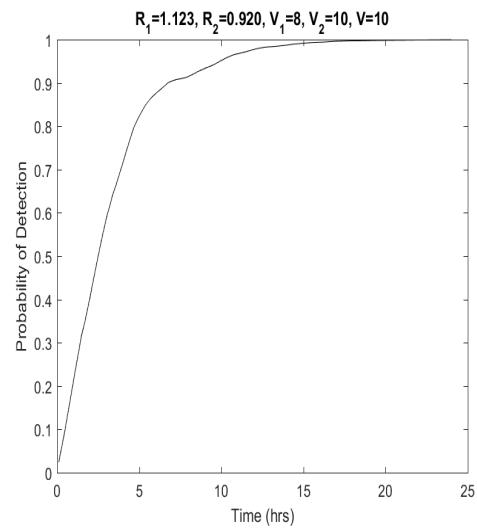


Figure A.240.  $P_D$  vs Time

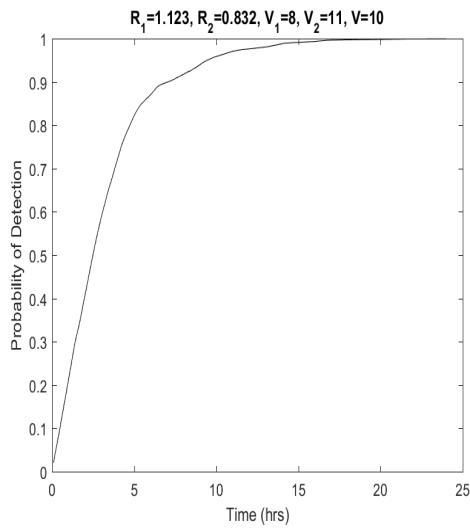


Figure A.241.  $P_D$  vs Time

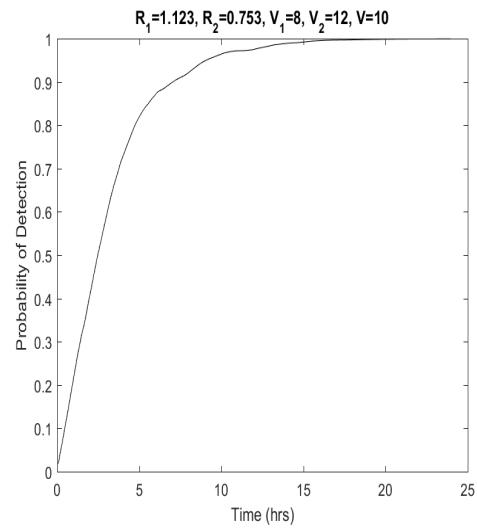


Figure A.242.  $P_D$  vs Time

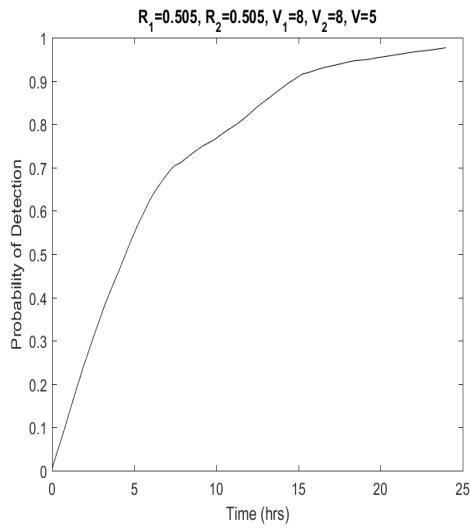


Figure A.243.  $P_D$  vs Time

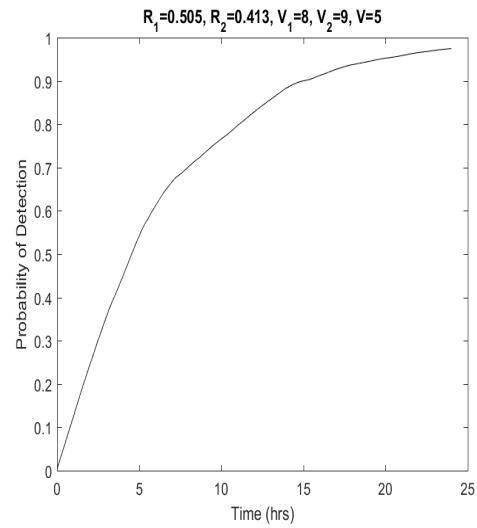


Figure A.244.  $P_D$  vs Time

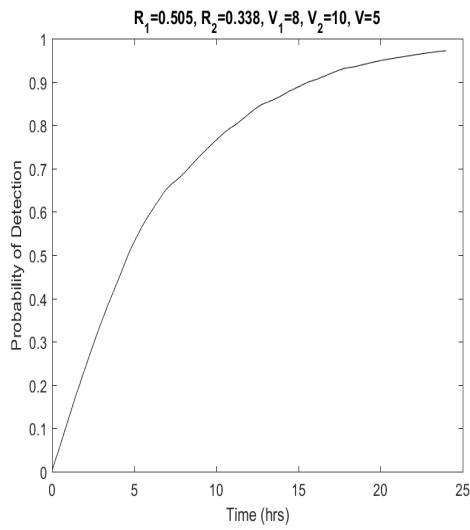


Figure A.245.  $P_D$  vs Time

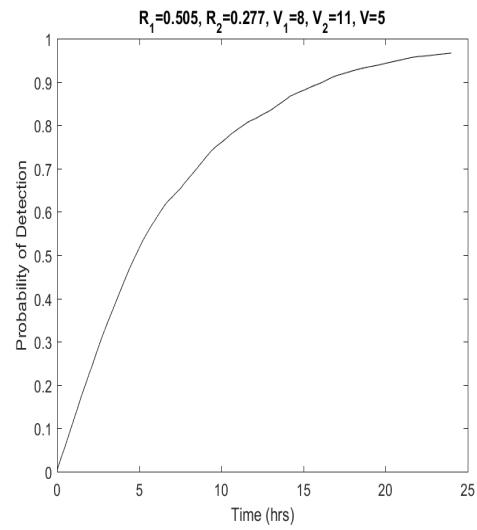


Figure A.246.  $P_D$  vs Time

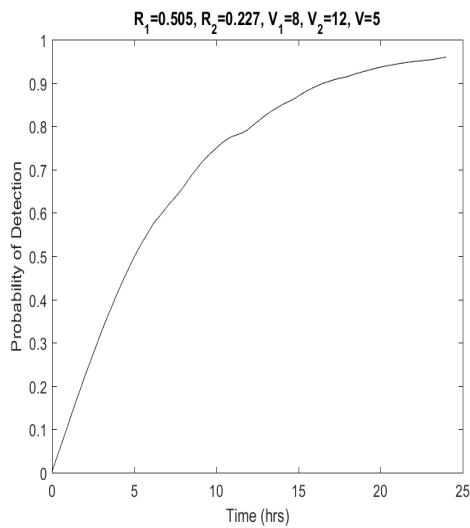


Figure A.247.  $P_D$  vs Time

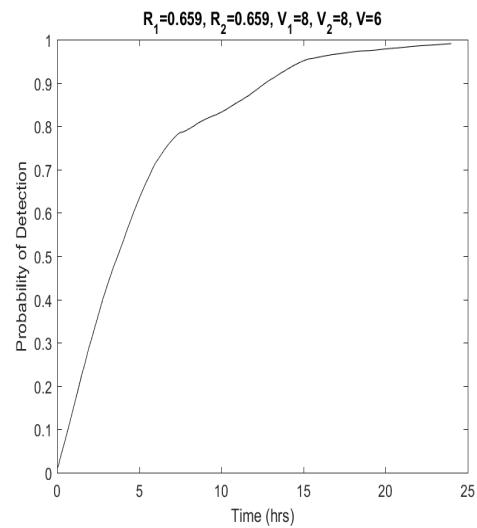


Figure A.248.  $P_D$  vs Time

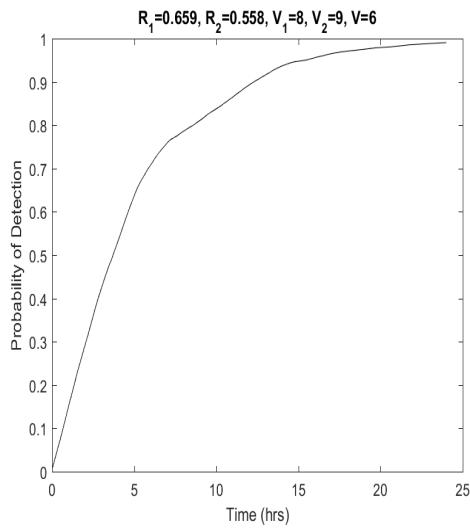


Figure A.249.  $P_D$  vs Time

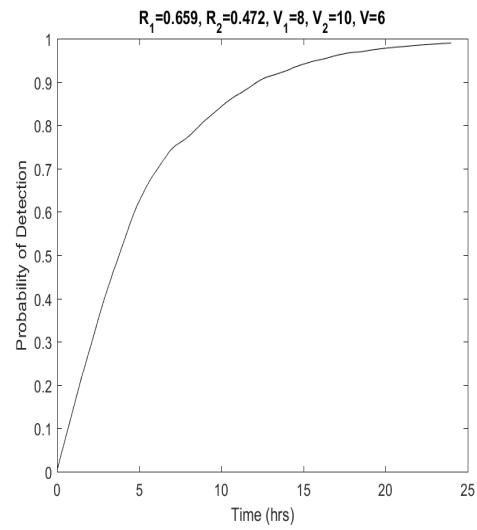


Figure A.250.  $P_D$  vs Time

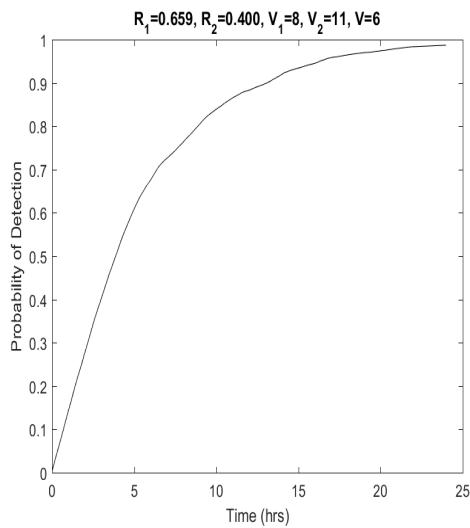


Figure A.251.  $P_D$  vs Time

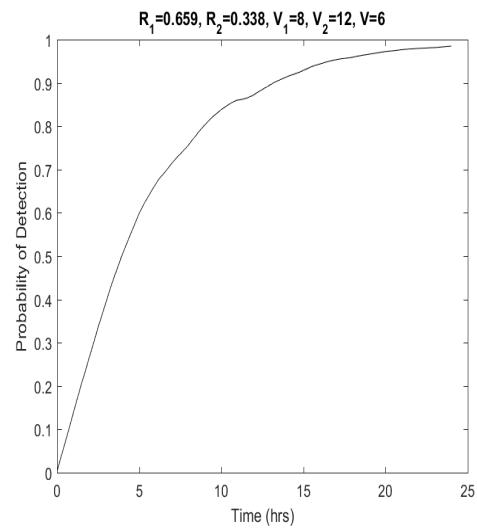


Figure A.252.  $P_D$  vs Time

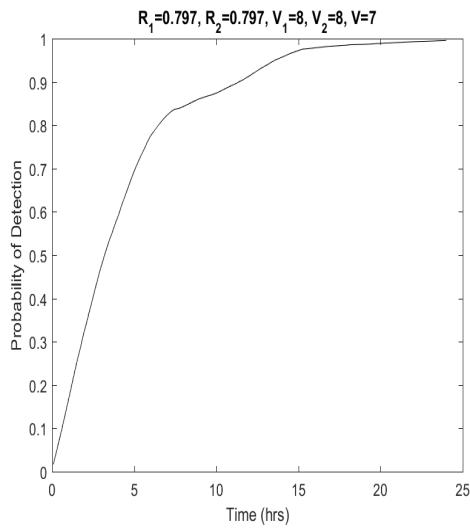


Figure A.253.  $P_D$  vs Time

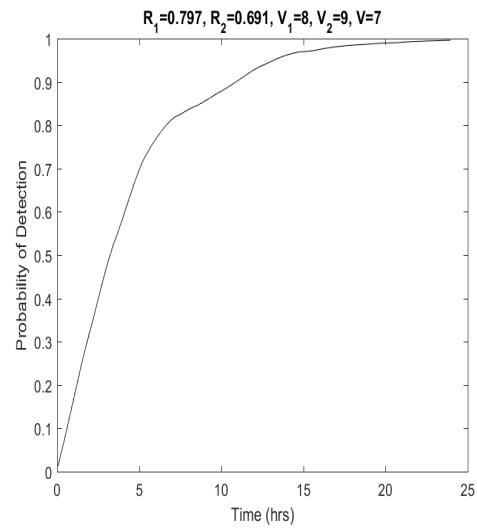


Figure A.254.  $P_D$  vs Time

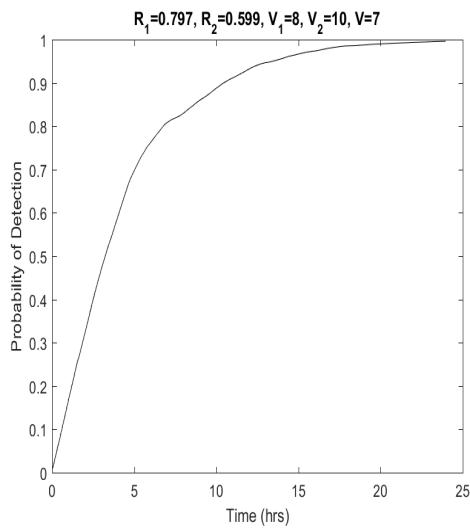


Figure A.255.  $P_D$  vs Time

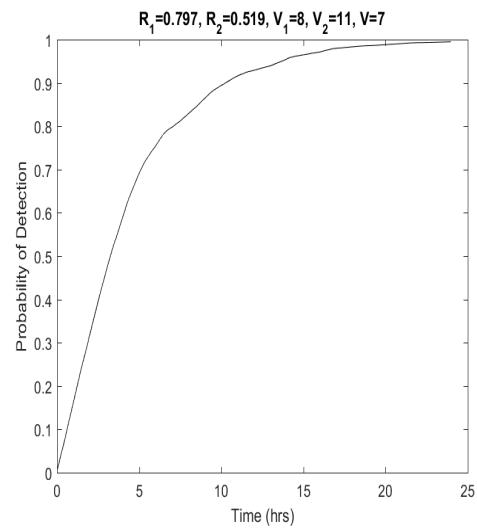


Figure A.256.  $P_D$  vs Time

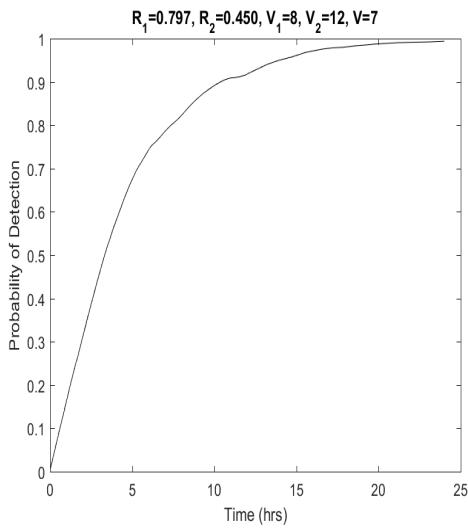


Figure A.257.  $P_D$  vs Time

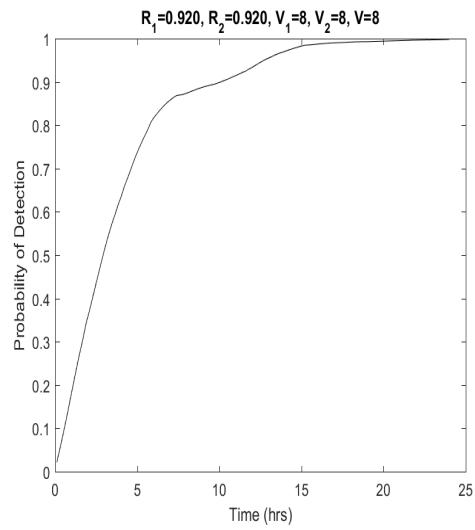


Figure A.258.  $P_D$  vs Time

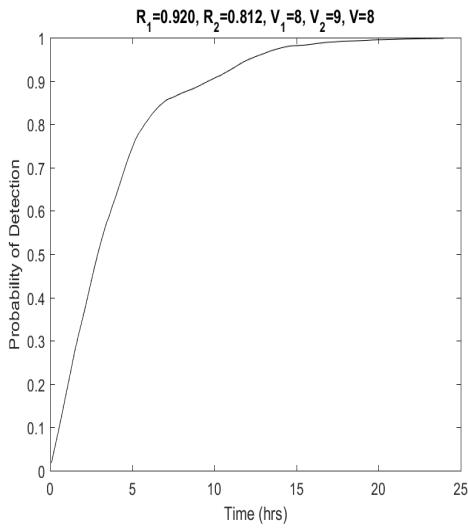


Figure A.259.  $P_D$  vs Time

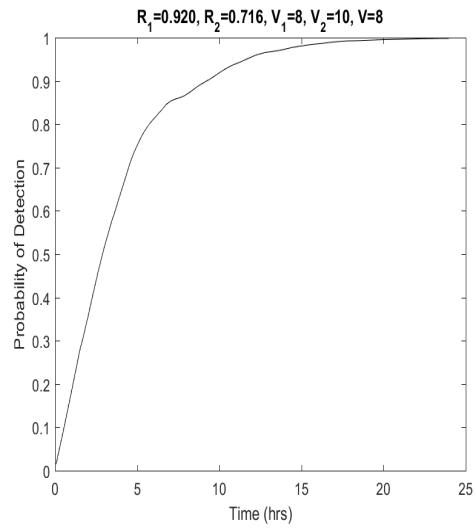


Figure A.260.  $P_D$  vs Time

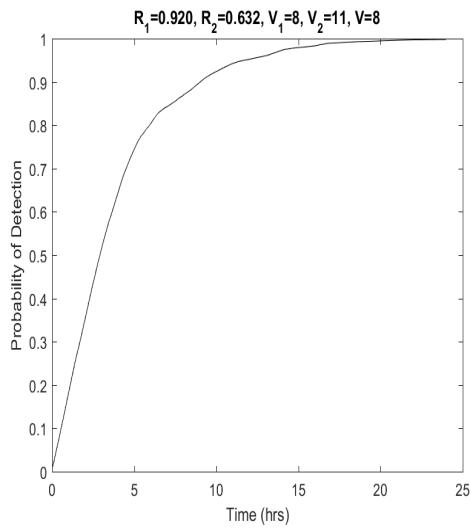


Figure A.261.  $P_D$  vs Time

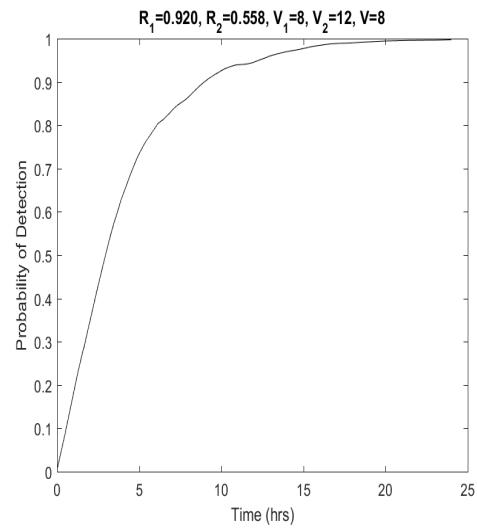


Figure A.262.  $P_D$  vs Time

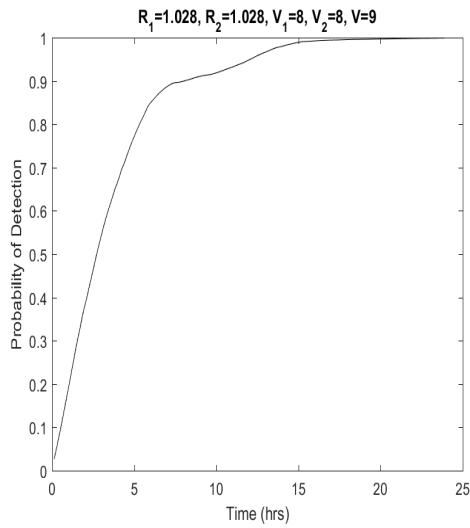


Figure A.263.  $P_D$  vs Time

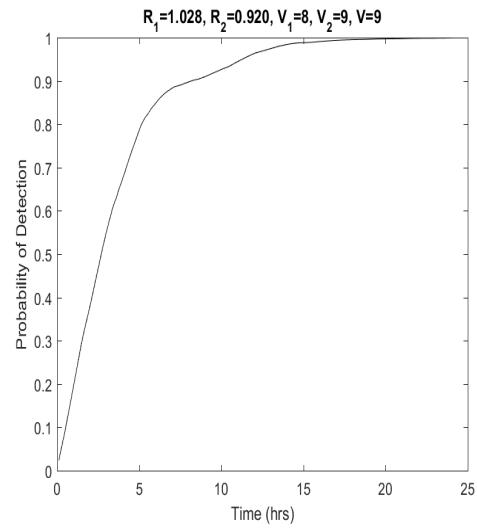


Figure A.264.  $P_D$  vs Time

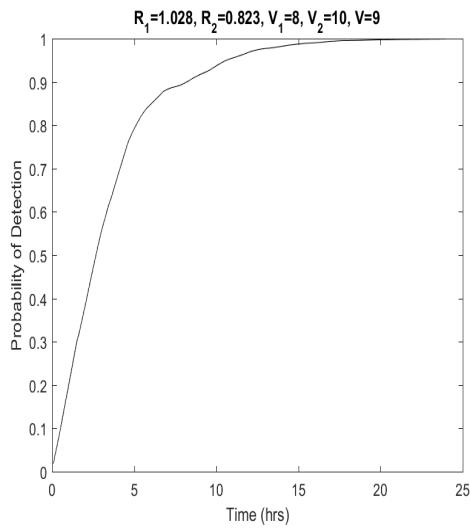


Figure A.265.  $P_D$  vs Time

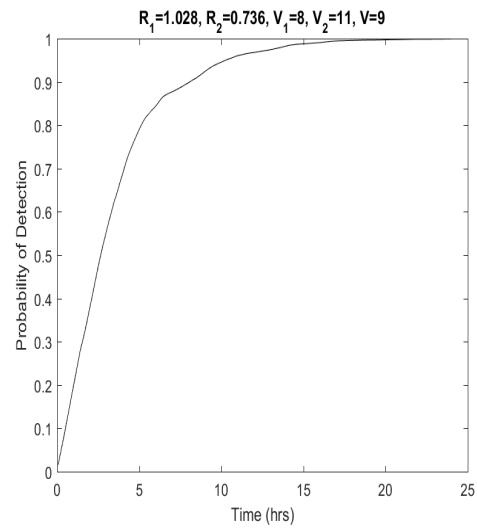


Figure A.266.  $P_D$  vs Time

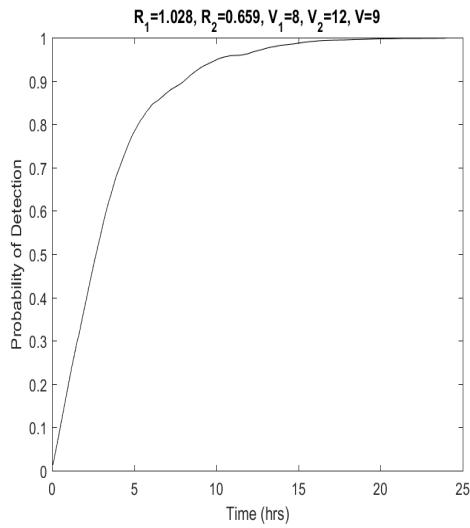


Figure A.267.  $P_D$  vs Time

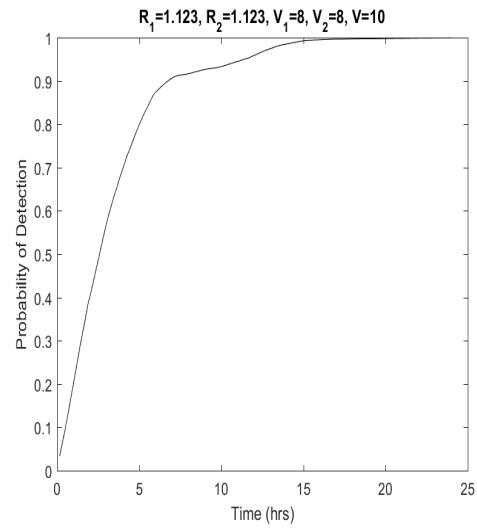


Figure A.268.  $P_D$  vs Time

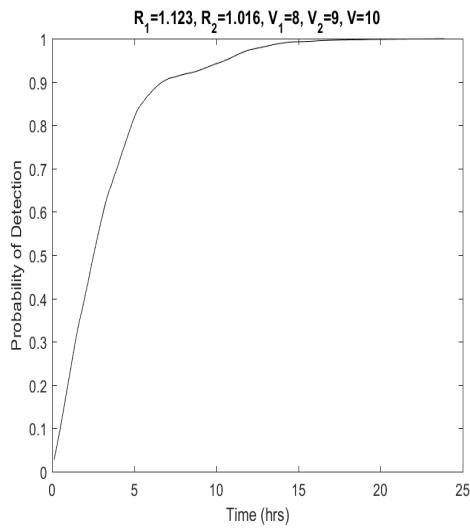


Figure A.269.  $P_D$  vs Time

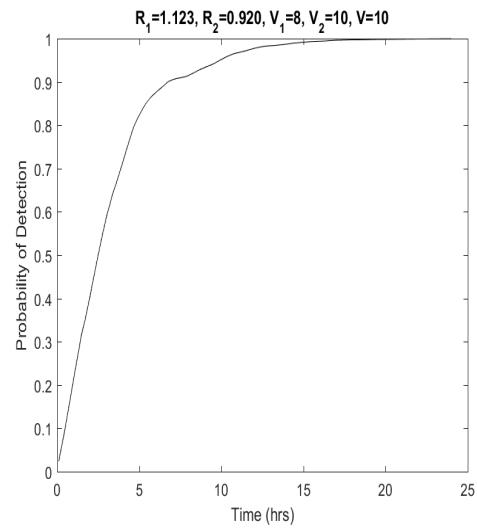


Figure A.270.  $P_D$  vs Time

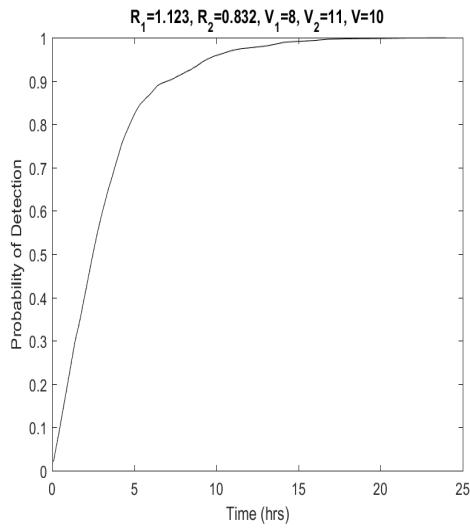


Figure A.271.  $P_D$  vs Time

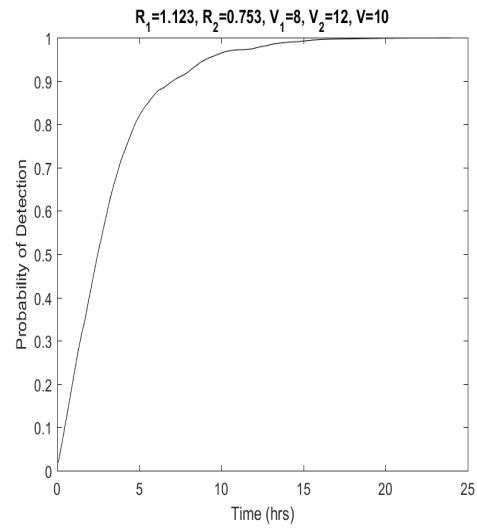


Figure A.272.  $P_D$  vs Time

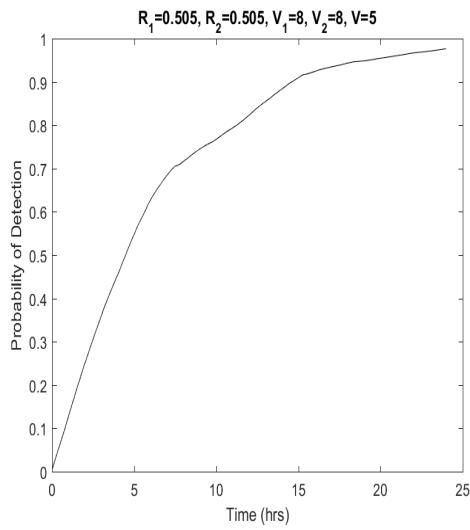


Figure A.273.  $P_D$  vs Time

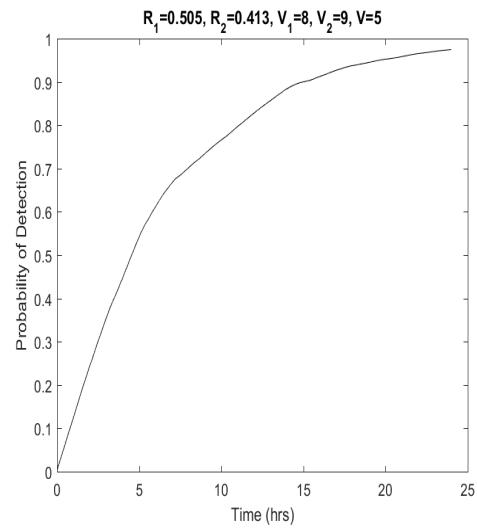


Figure A.274.  $P_D$  vs Time

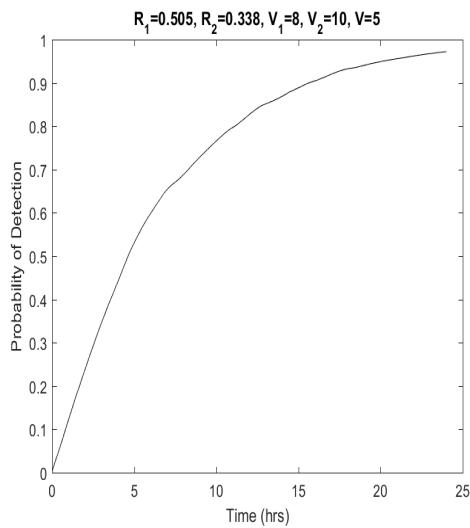


Figure A.275.  $P_D$  vs Time

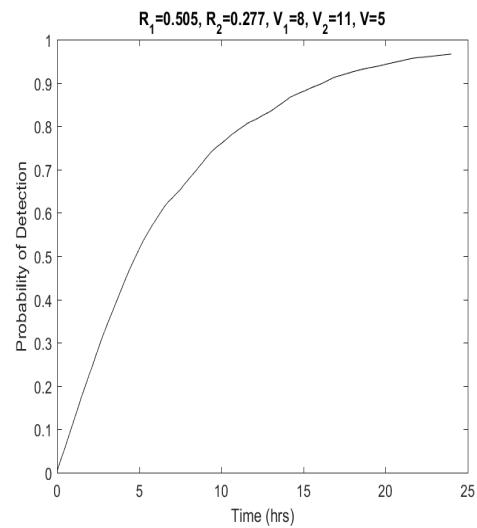


Figure A.276.  $P_D$  vs Time

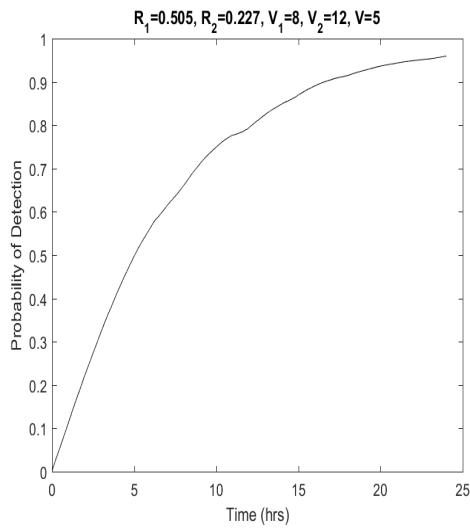


Figure A.277.  $P_D$  vs Time

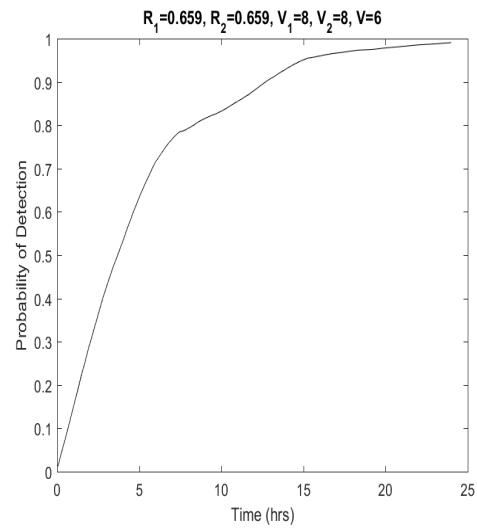


Figure A.278.  $P_D$  vs Time

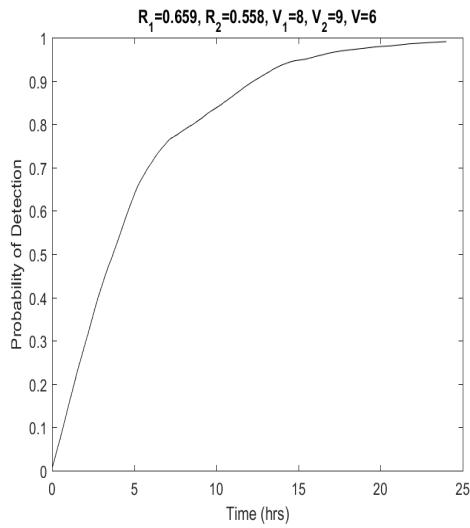


Figure A.279.  $P_D$  vs Time

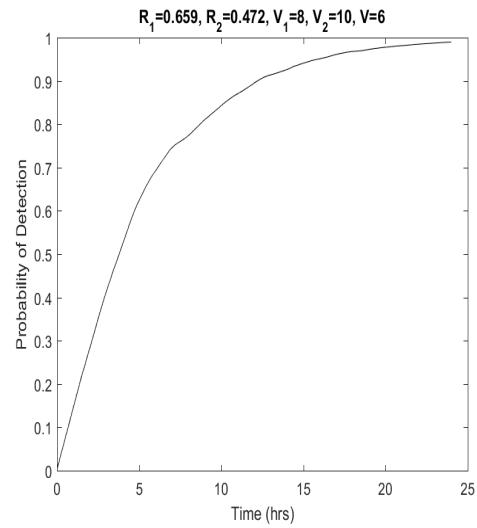


Figure A.280.  $P_D$  vs Time

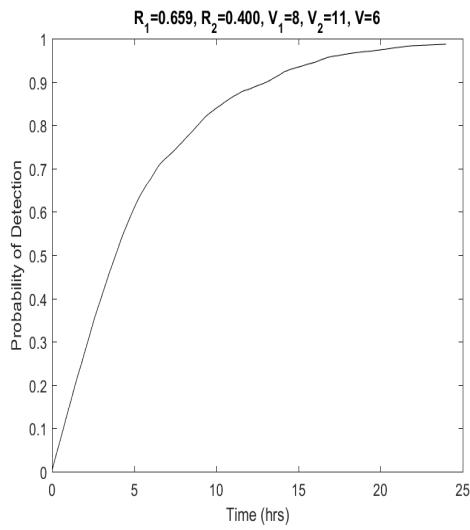


Figure A.281.  $P_D$  vs Time

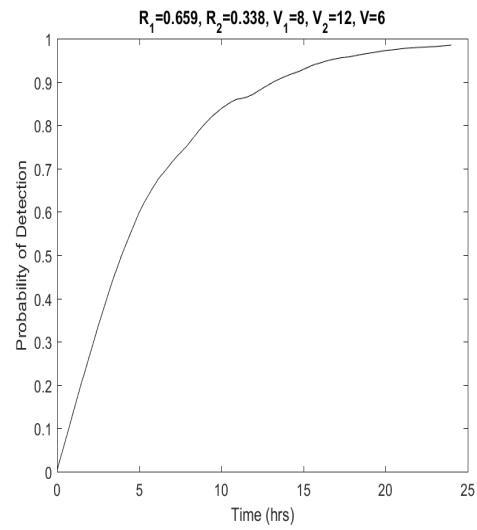


Figure A.282.  $P_D$  vs Time

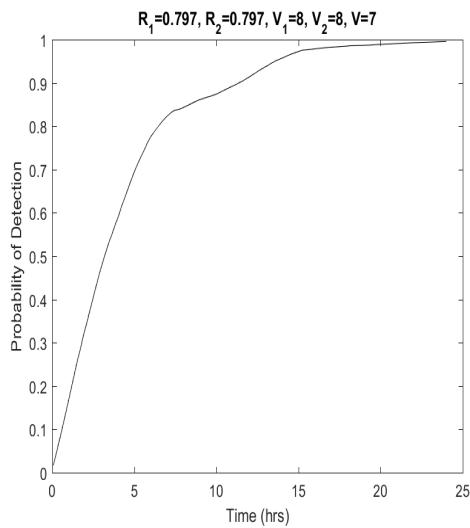


Figure A.283.  $P_D$  vs Time

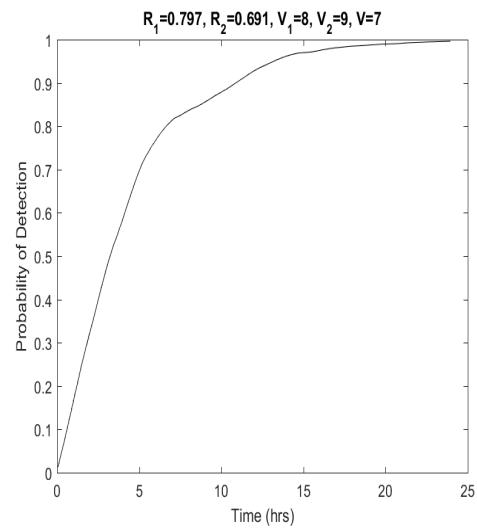


Figure A.284.  $P_D$  vs Time

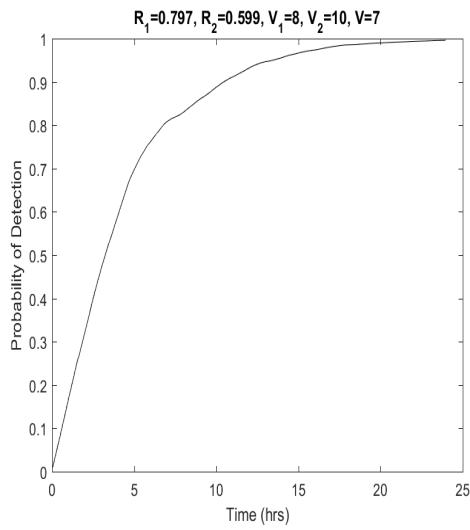


Figure A.285.  $P_D$  vs Time

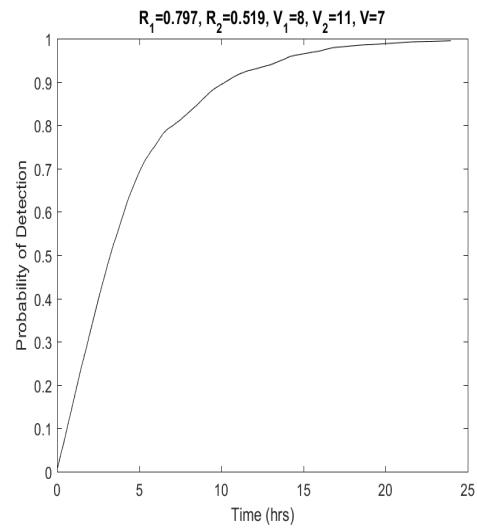


Figure A.286.  $P_D$  vs Time

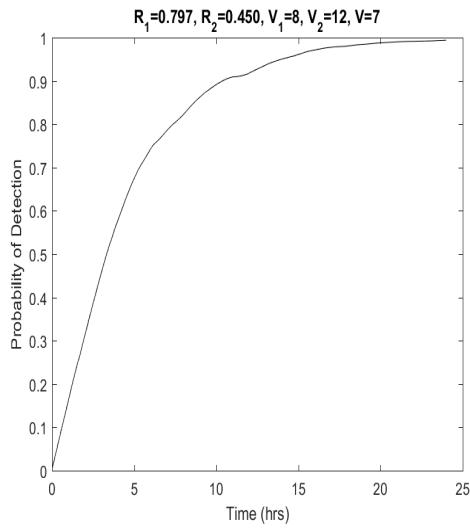


Figure A.287.  $P_D$  vs Time

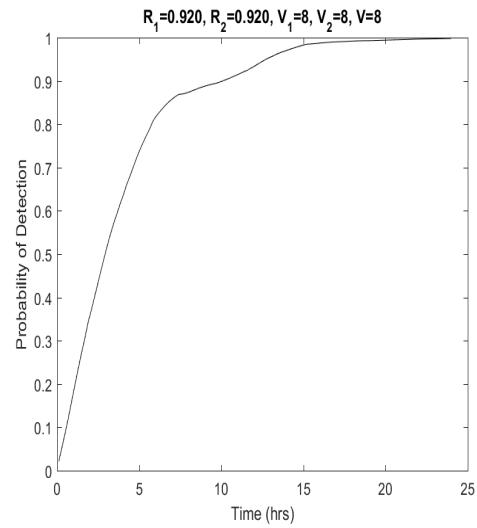


Figure A.288.  $P_D$  vs Time

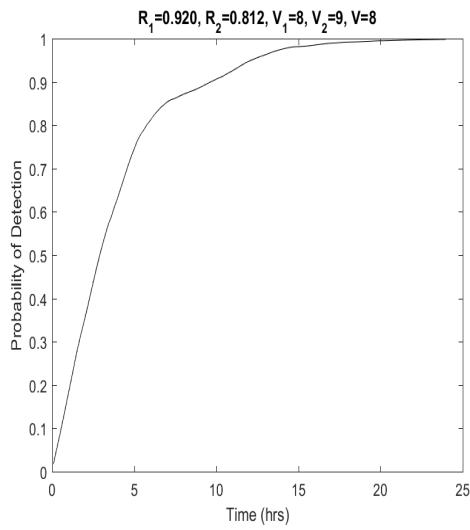


Figure A.289.  $P_D$  vs Time

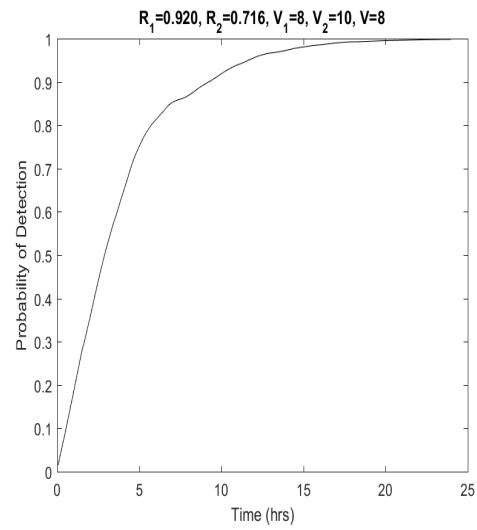


Figure A.290.  $P_D$  vs Time

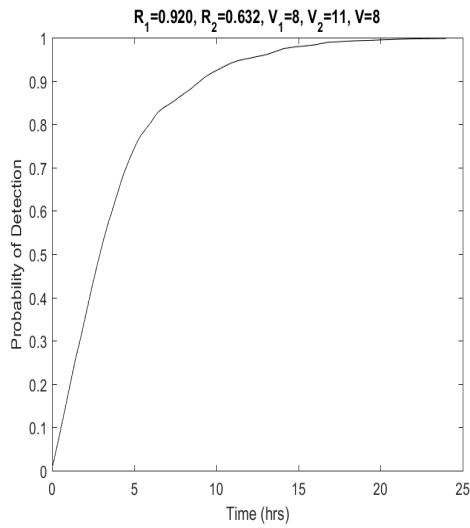


Figure A.291.  $P_D$  vs Time

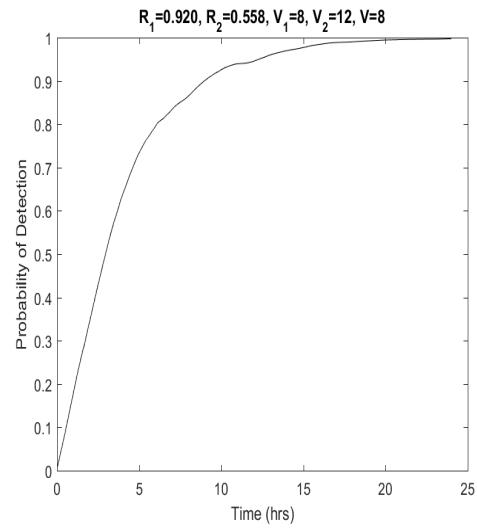


Figure A.292.  $P_D$  vs Time

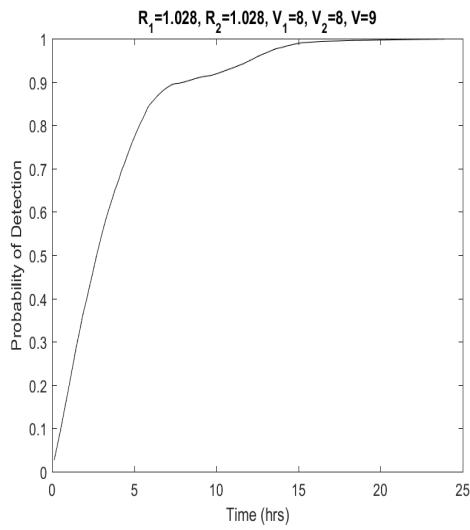


Figure A.293.  $P_D$  vs Time

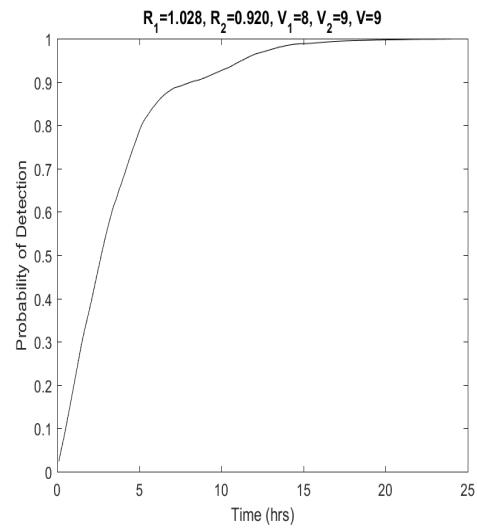


Figure A.294.  $P_D$  vs Time

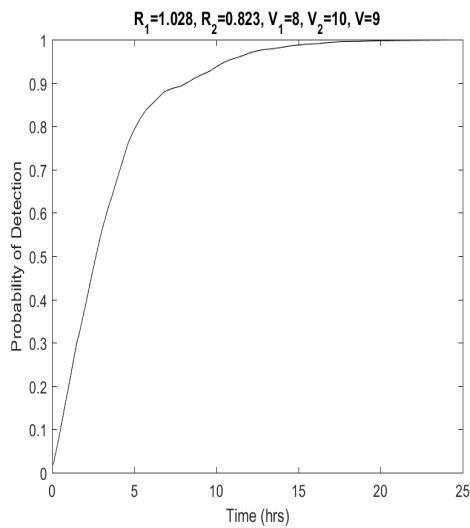


Figure A.295.  $P_D$  vs Time

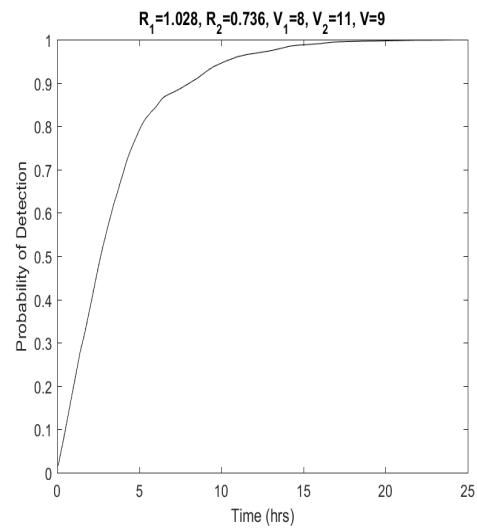


Figure A.296.  $P_D$  vs Time

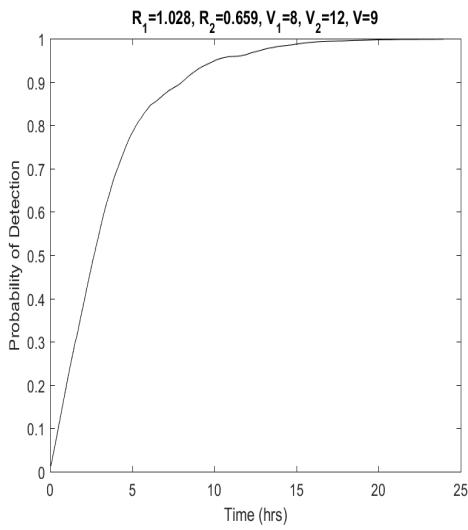


Figure A.297.  $P_D$  vs Time

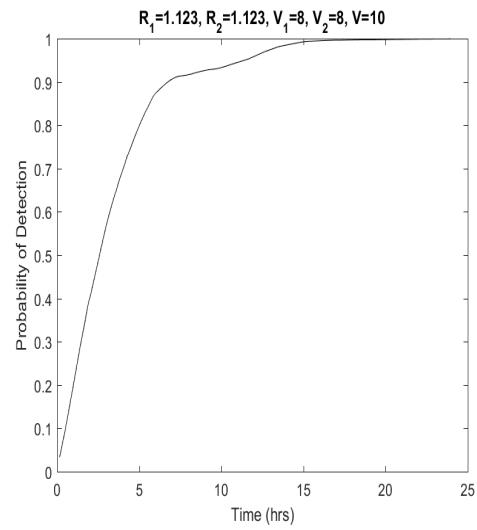


Figure A.298.  $P_D$  vs Time

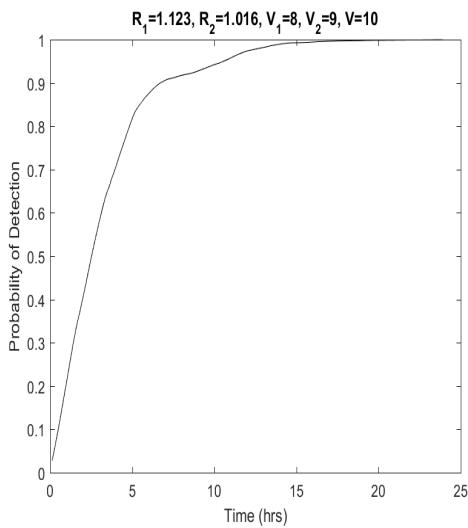


Figure A.299.  $P_D$  vs Time

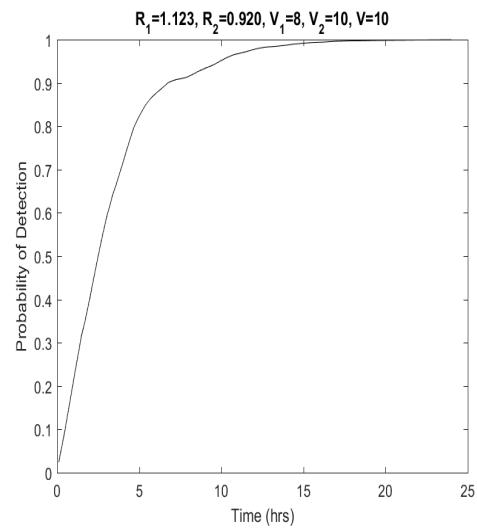


Figure A.300.  $P_D$  vs Time

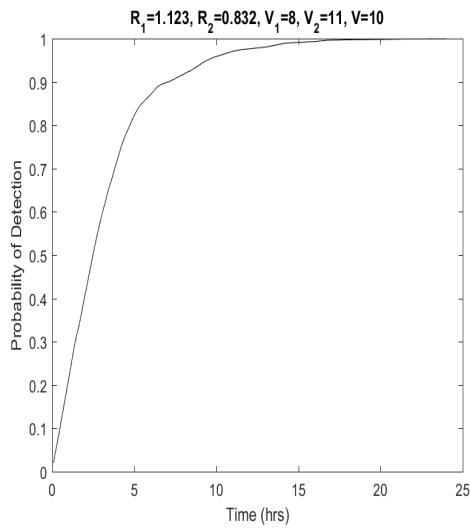


Figure A.301.  $P_D$  vs Time

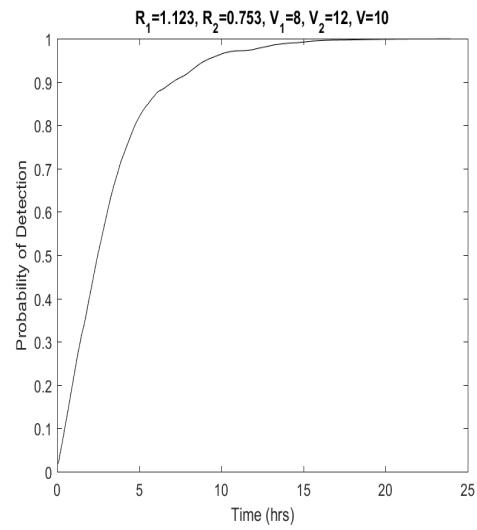


Figure A.302.  $P_D$  vs Time

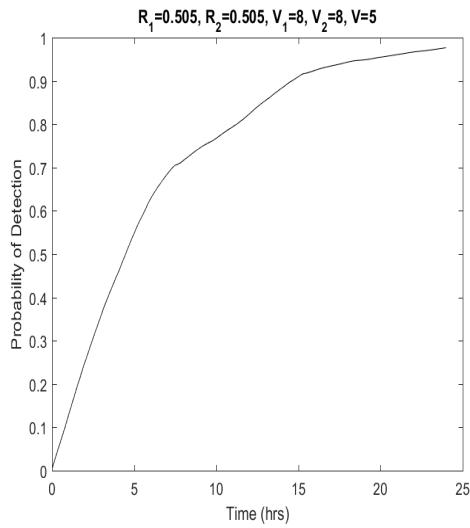


Figure A.303.  $P_D$  vs Time

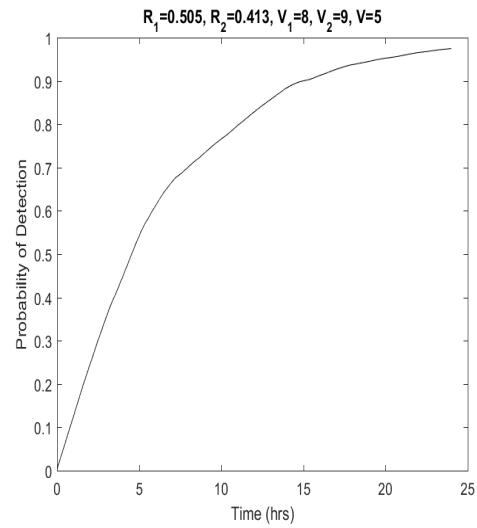


Figure A.304.  $P_D$  vs Time

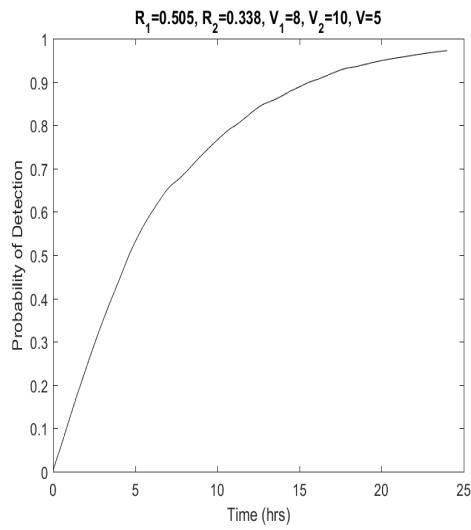


Figure A.305.  $P_D$  vs Time

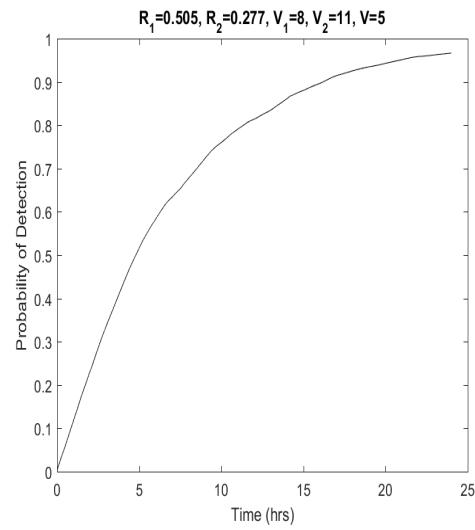


Figure A.306.  $P_D$  vs Time

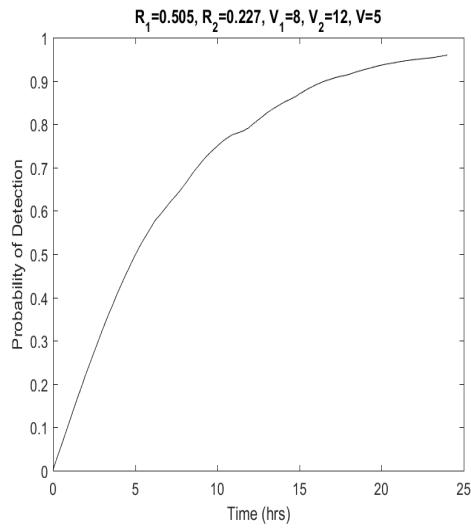


Figure A.307.  $P_D$  vs Time

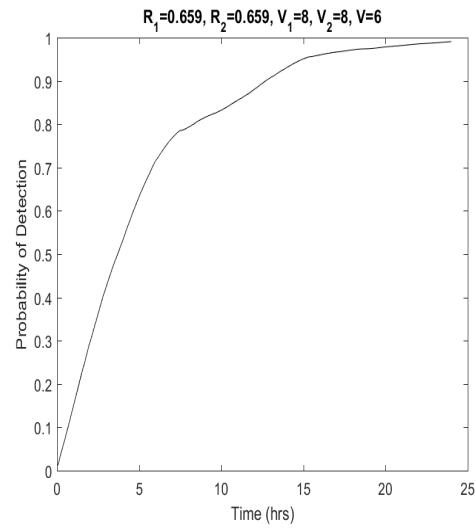


Figure A.308.  $P_D$  vs Time

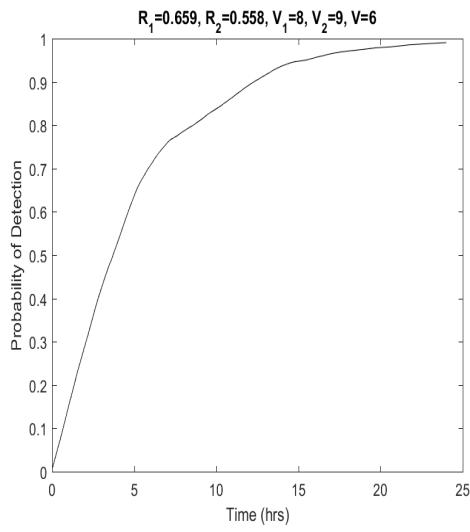


Figure A.309.  $P_D$  vs Time

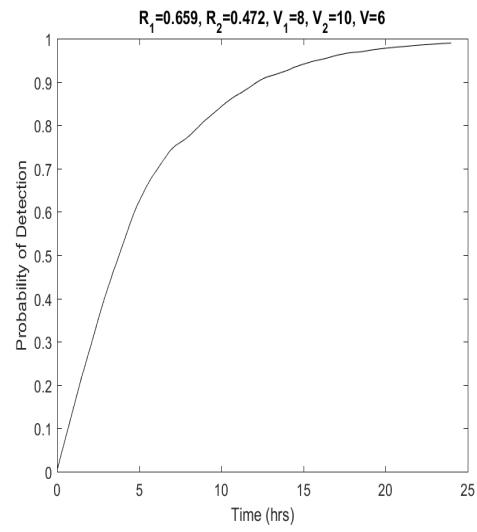


Figure A.310.  $P_D$  vs Time

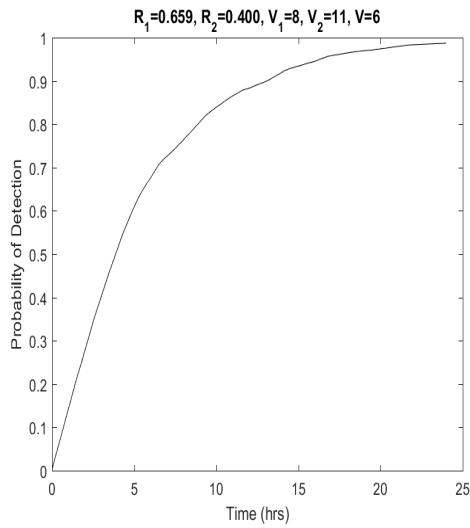


Figure A.311.  $P_D$  vs Time

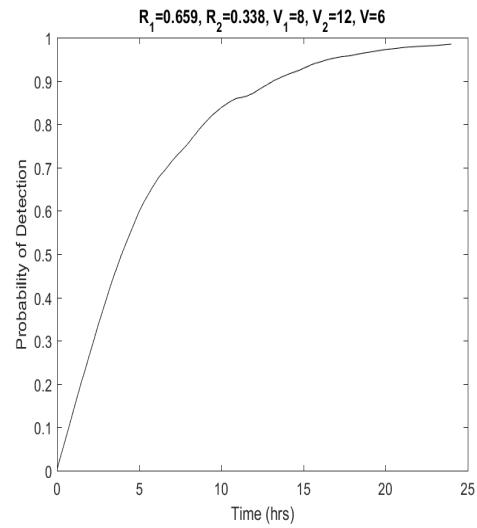


Figure A.312.  $P_D$  vs Time

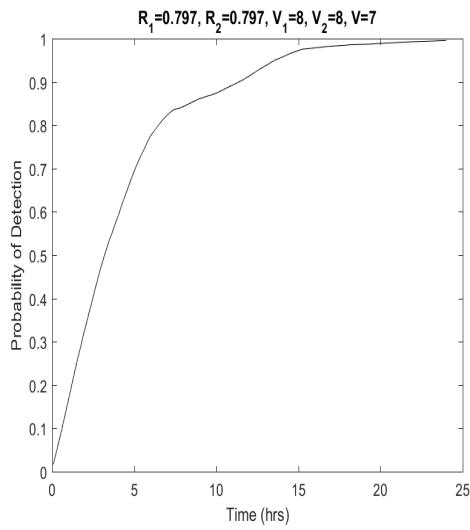


Figure A.313.  $P_D$  vs Time

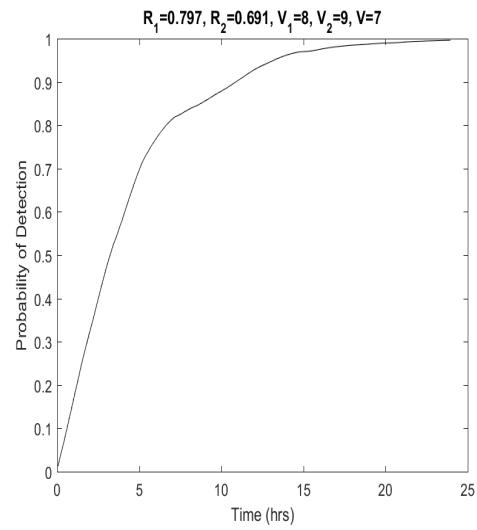


Figure A.314.  $P_D$  vs Time

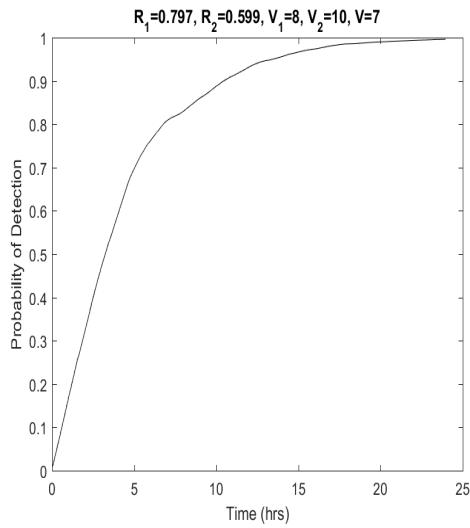


Figure A.315.  $P_D$  vs Time

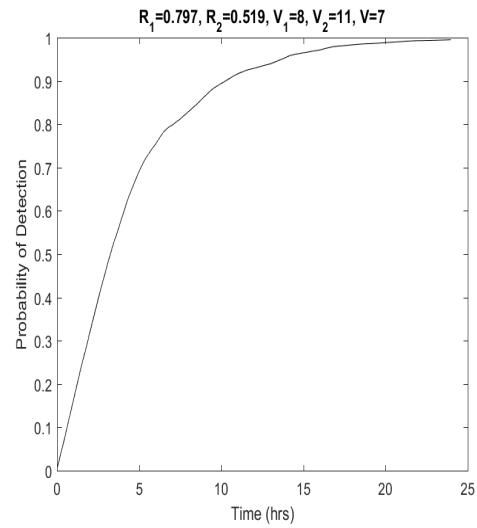


Figure A.316.  $P_D$  vs Time

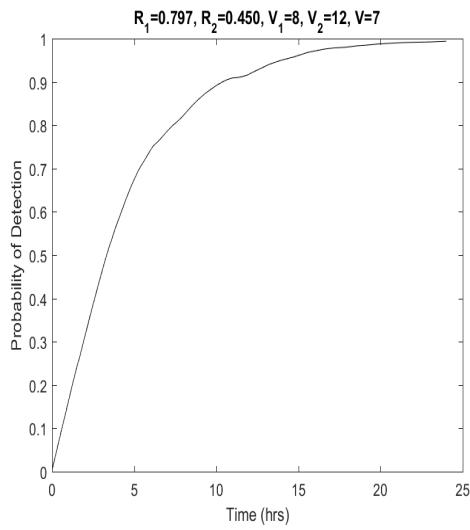


Figure A.317.  $P_D$  vs Time

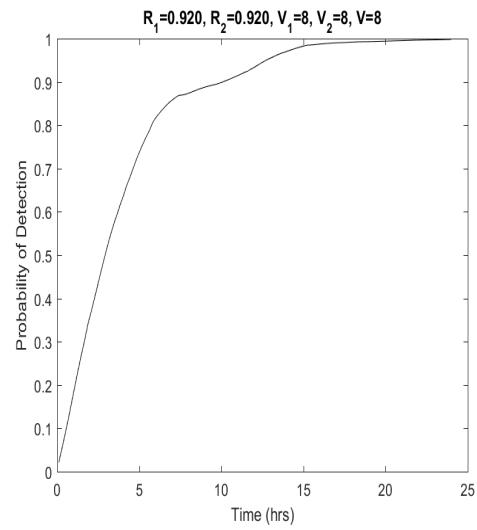


Figure A.318.  $P_D$  vs Time

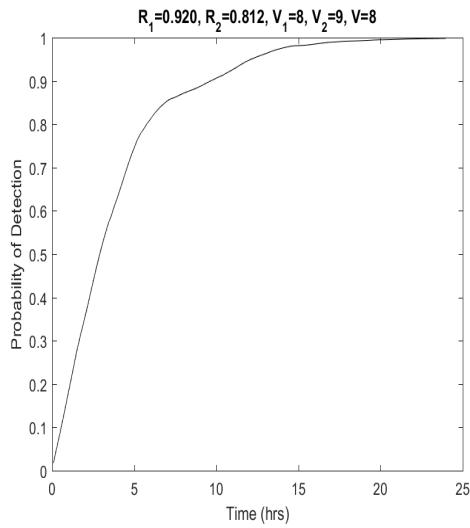


Figure A.319.  $P_D$  vs Time

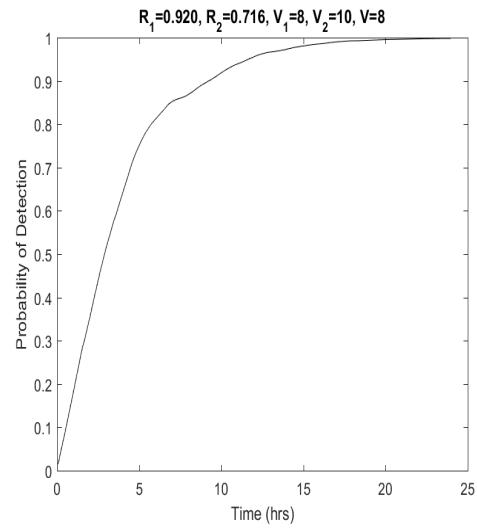


Figure A.320.  $P_D$  vs Time

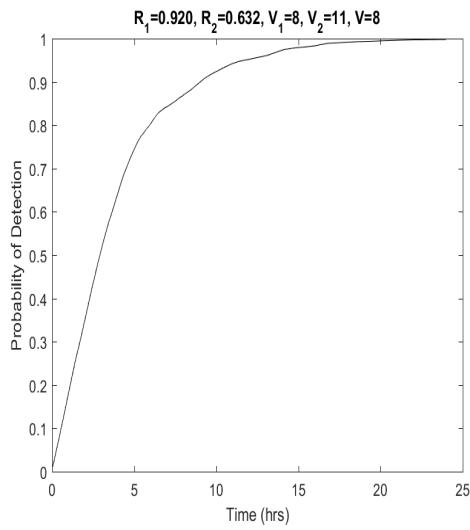


Figure A.321.  $P_D$  vs Time

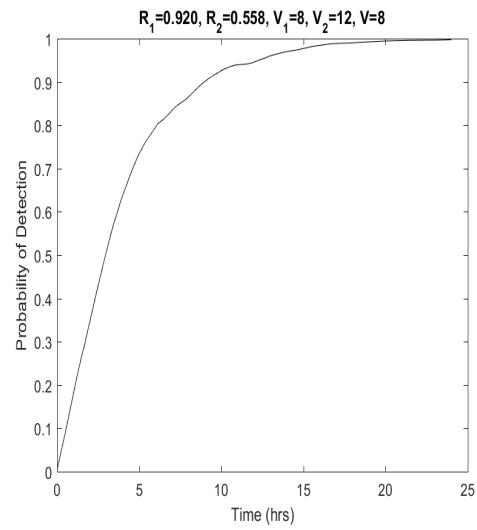


Figure A.322.  $P_D$  vs Time

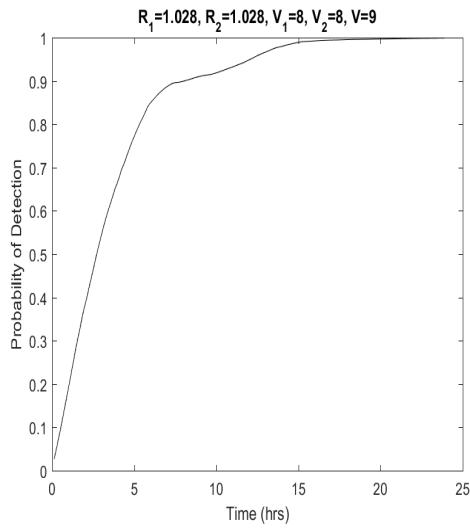


Figure A.323.  $P_D$  vs Time

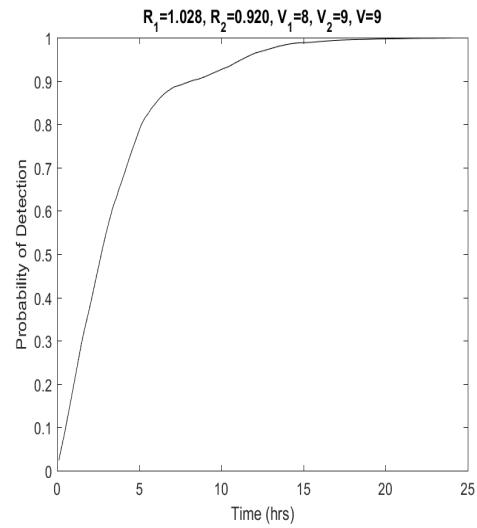


Figure A.324.  $P_D$  vs Time

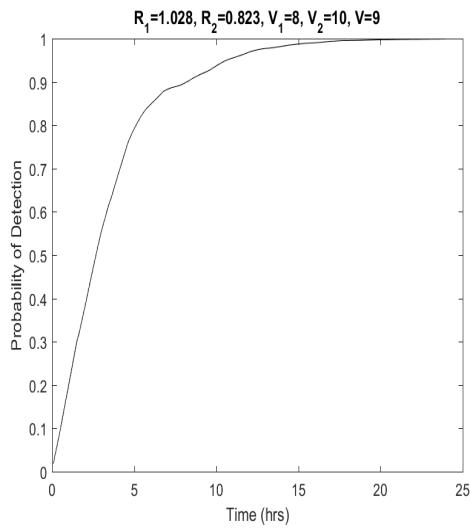


Figure A.325.  $P_D$  vs Time

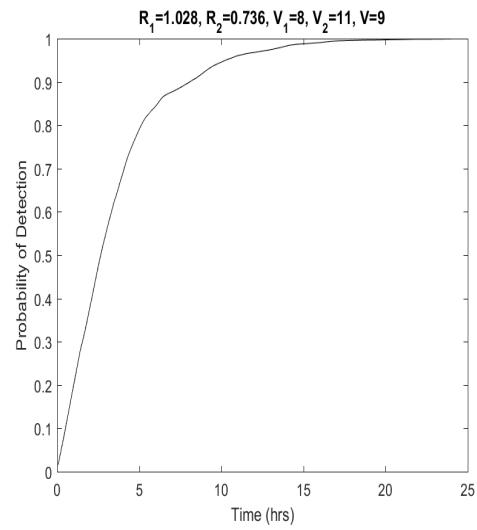


Figure A.326.  $P_D$  vs Time

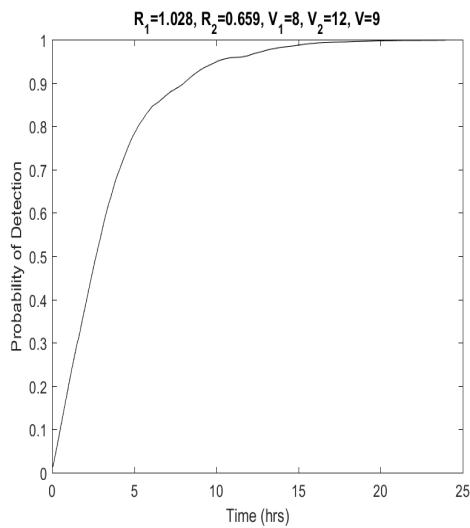


Figure A.327.  $P_D$  vs Time

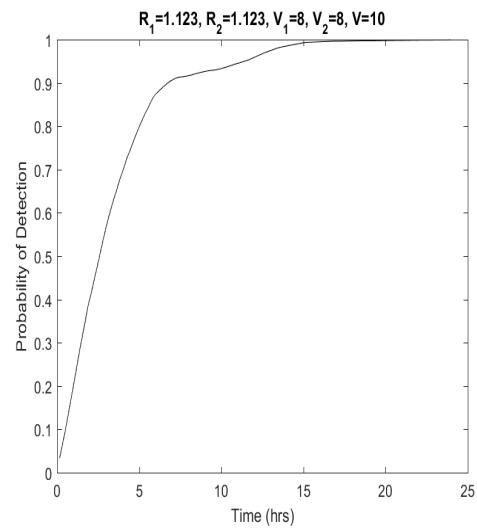


Figure A.328.  $P_D$  vs Time

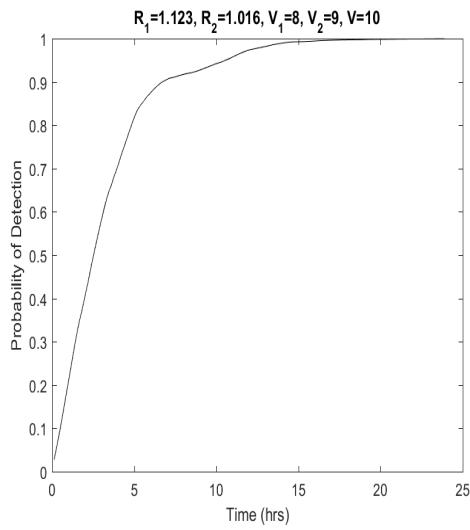


Figure A.329.  $P_D$  vs Time

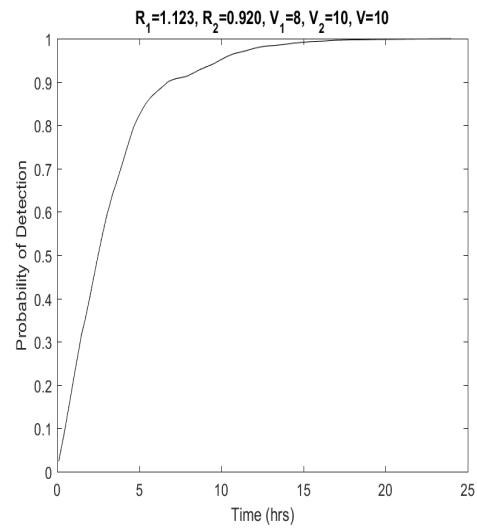


Figure A.330.  $P_D$  vs Time

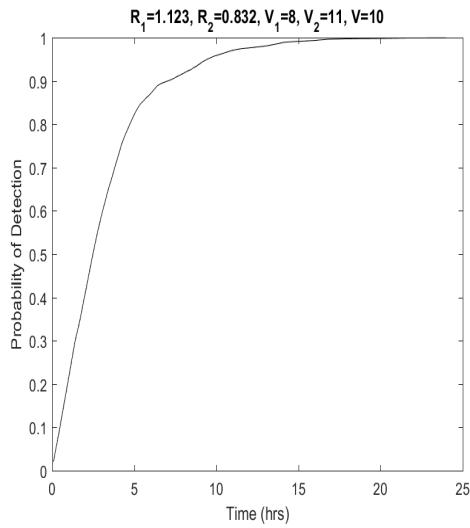


Figure A.331.  $P_D$  vs Time

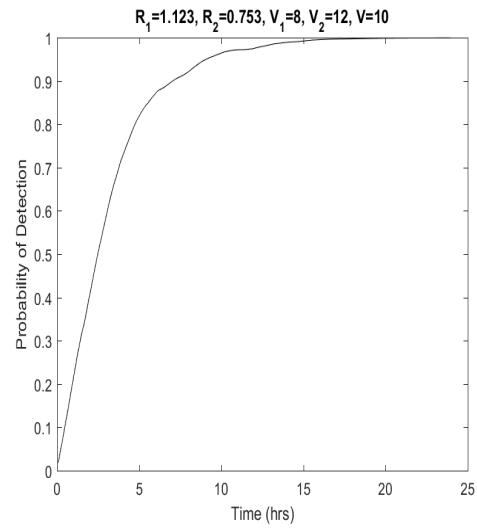
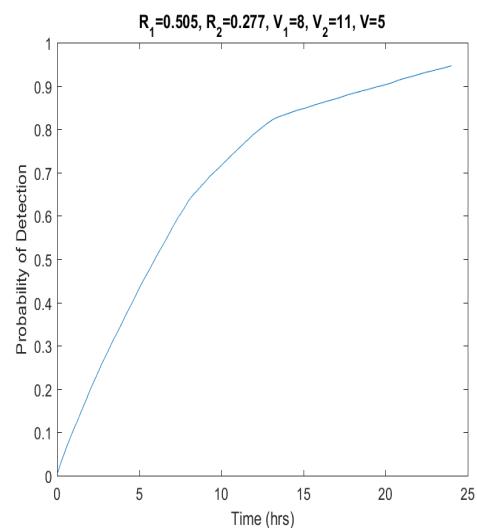
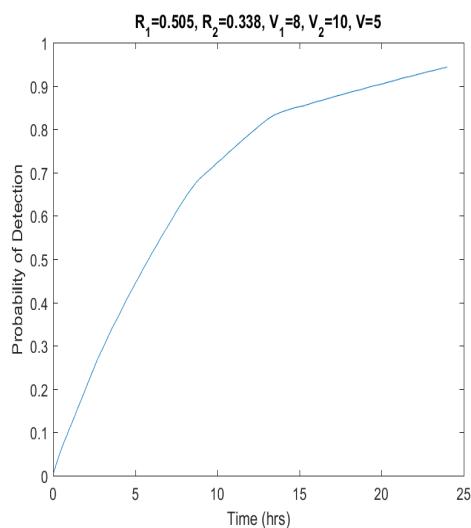
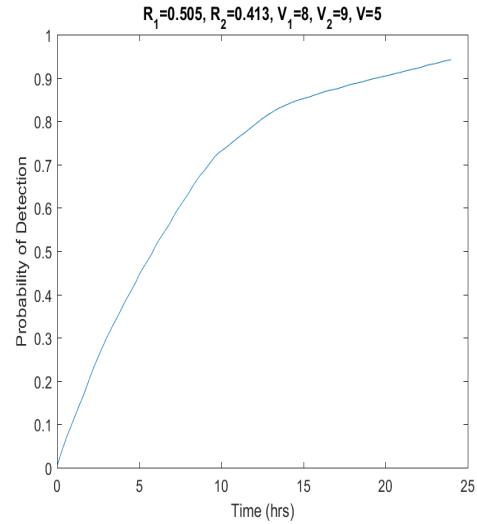
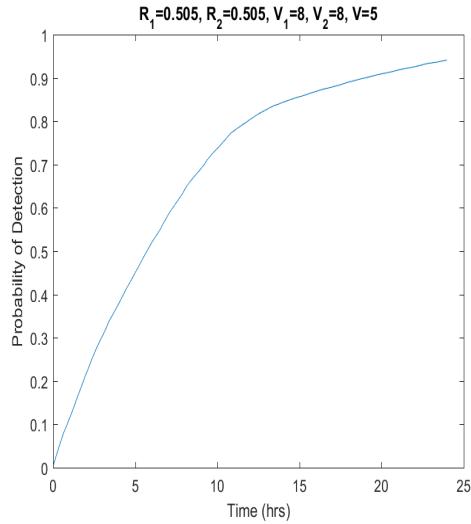


Figure A.332.  $P_D$  vs Time

## Spiral

### A.3.3 Spiral Search



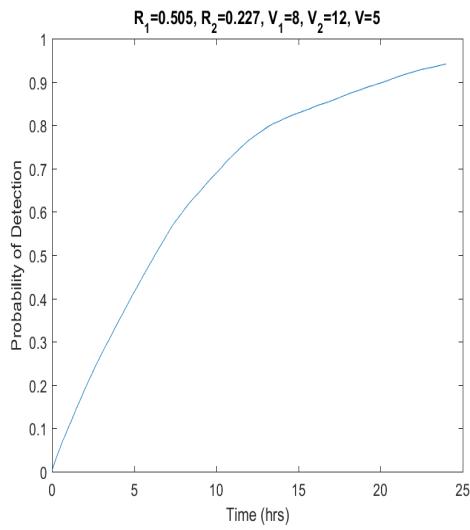


Figure A.337.  $P_D$  vs Time

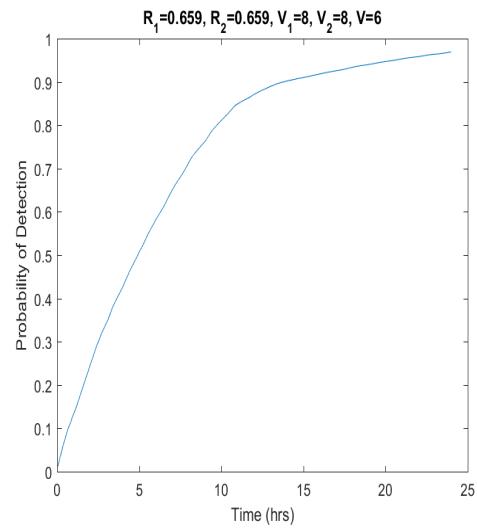


Figure A.338.  $P_D$  vs Time

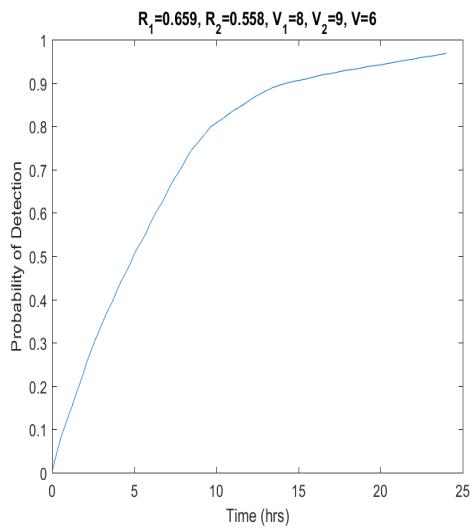


Figure A.339.  $P_D$  vs Time

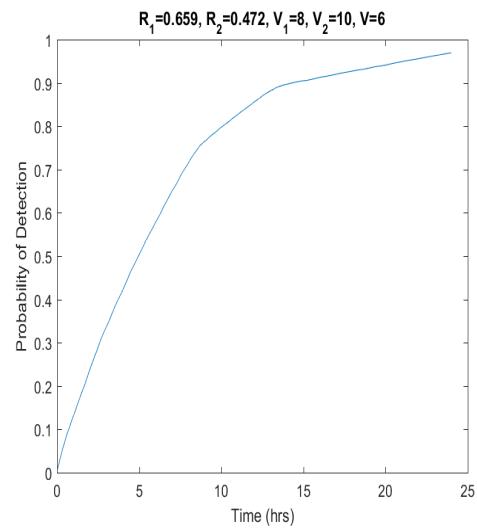


Figure A.340.  $P_D$  vs Time

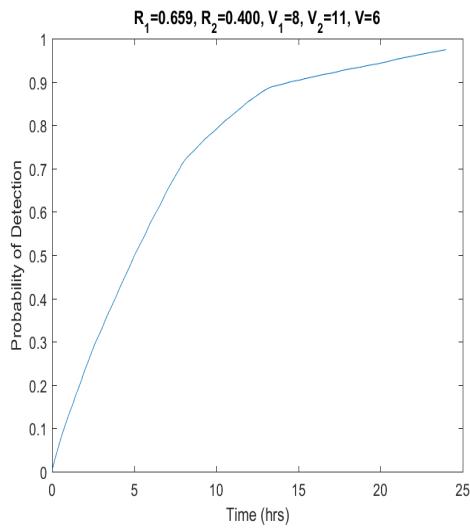


Figure A.341.  $P_D$  vs Time

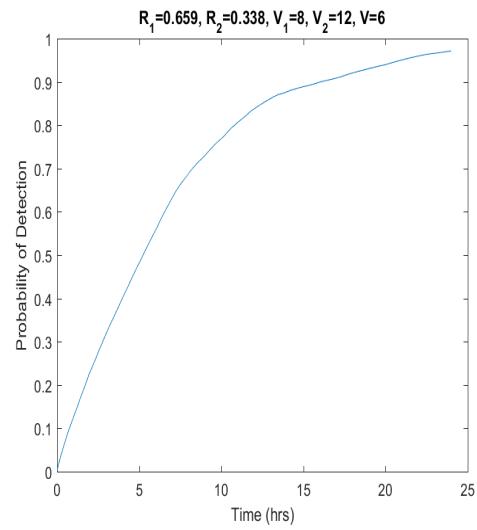


Figure A.342.  $P_D$  vs Time

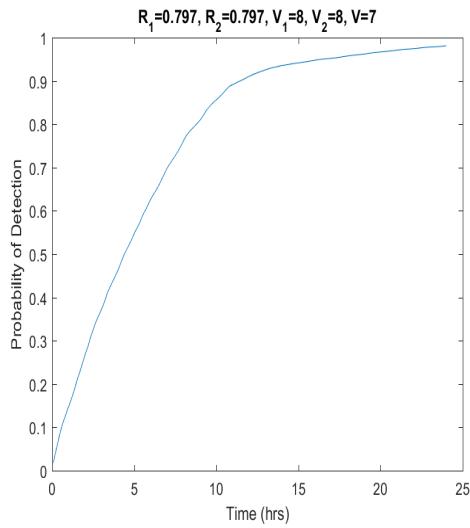


Figure A.343.  $P_D$  vs Time

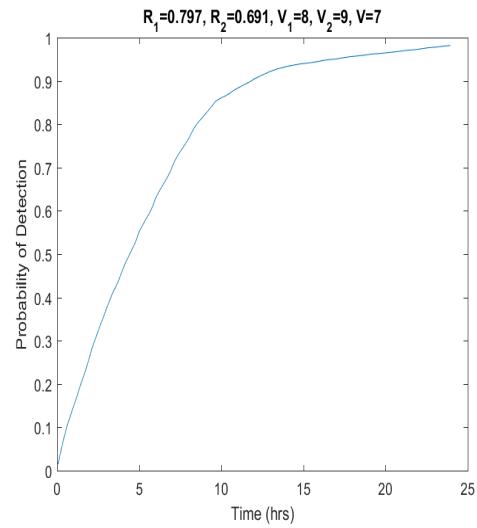


Figure A.344.  $P_D$  vs Time

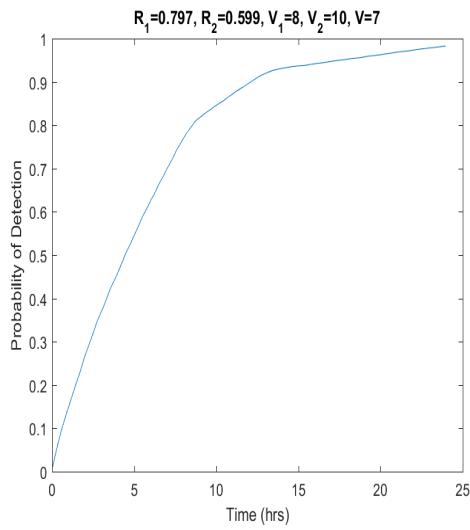


Figure A.345.  $P_D$  vs Time

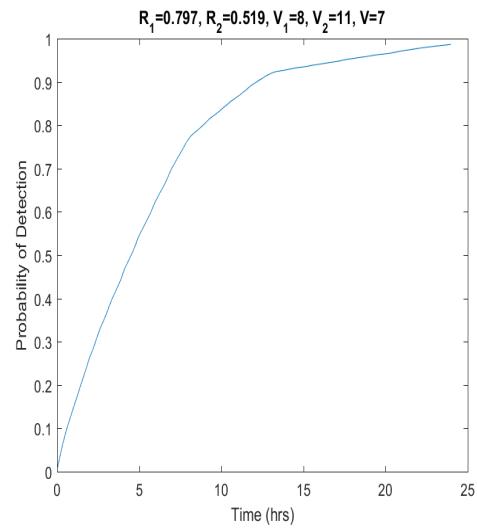


Figure A.346.  $P_D$  vs Time

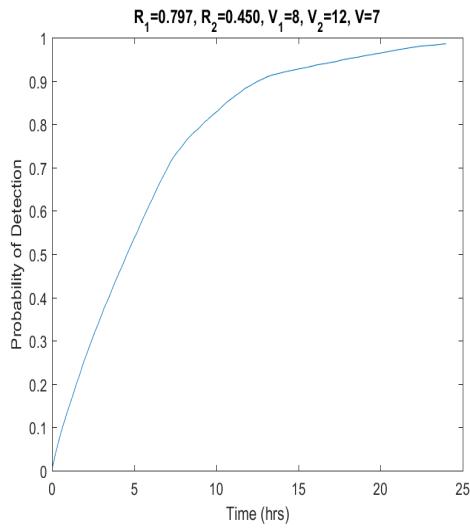


Figure A.347.  $P_D$  vs Time

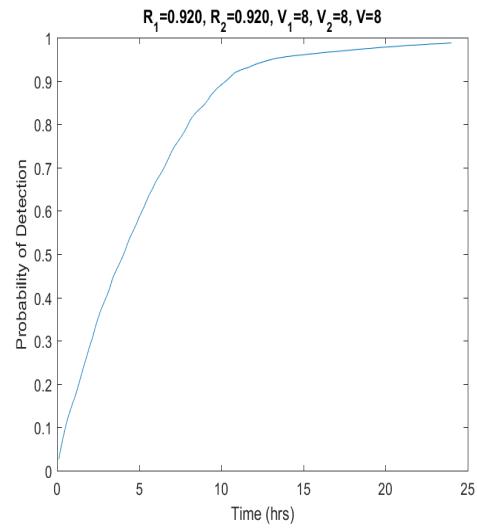


Figure A.348.  $P_D$  vs Time

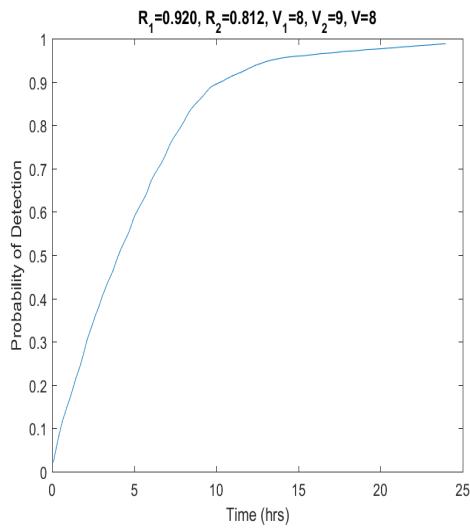


Figure A.349.  $P_D$  vs Time

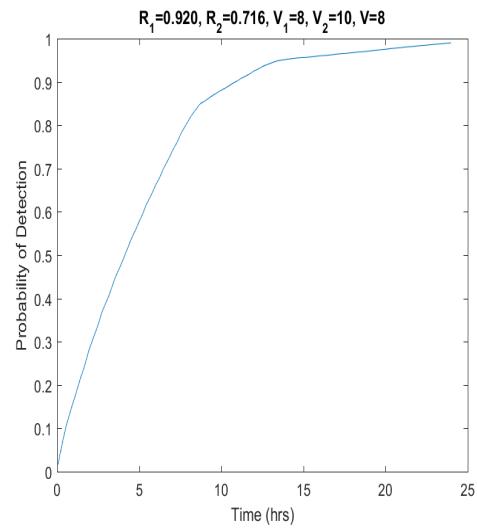


Figure A.350.  $P_D$  vs Time

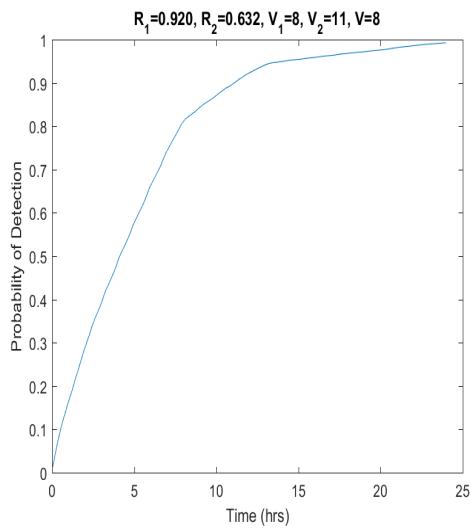


Figure A.351.  $P_D$  vs Time

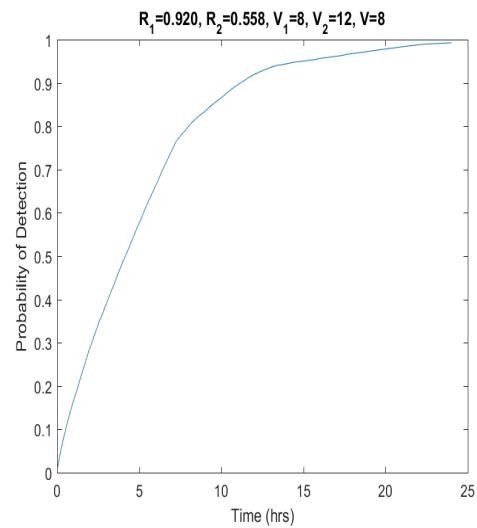


Figure A.352.  $P_D$  vs Time

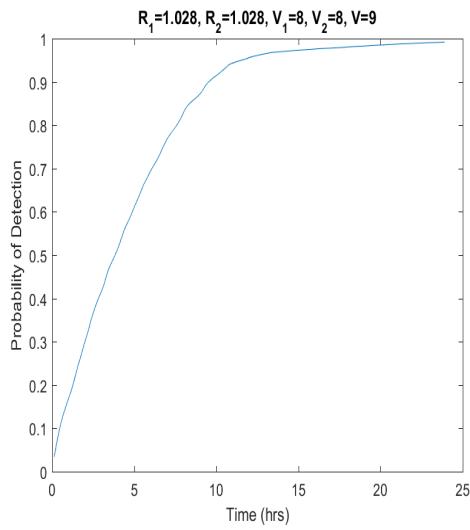


Figure A.353.  $P_D$  vs Time

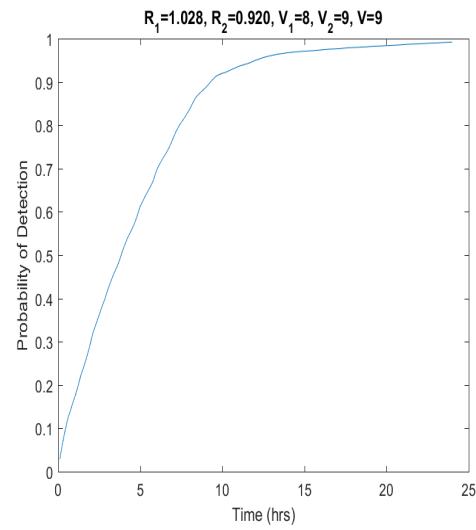


Figure A.354.  $P_D$  vs Time

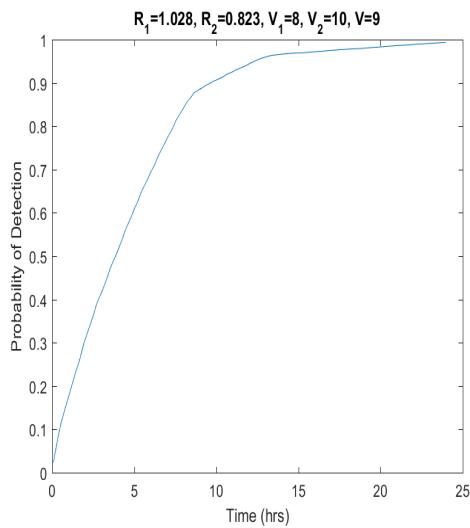


Figure A.355.  $P_D$  vs Time

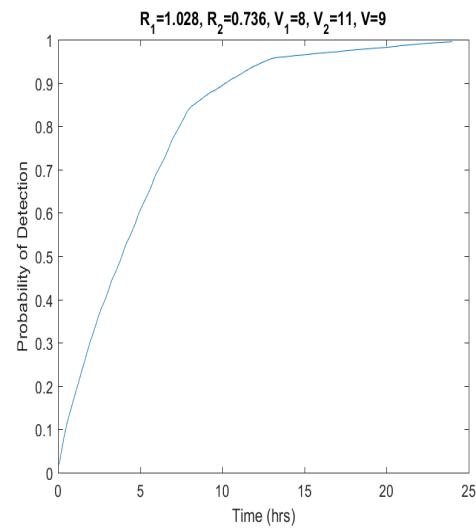


Figure A.356.  $P_D$  vs Time

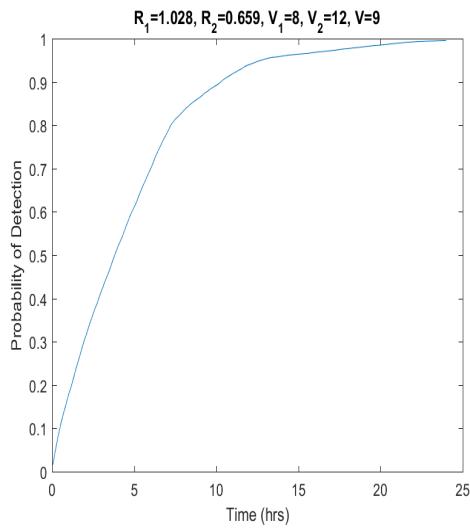


Figure A.357.  $P_D$  vs Time

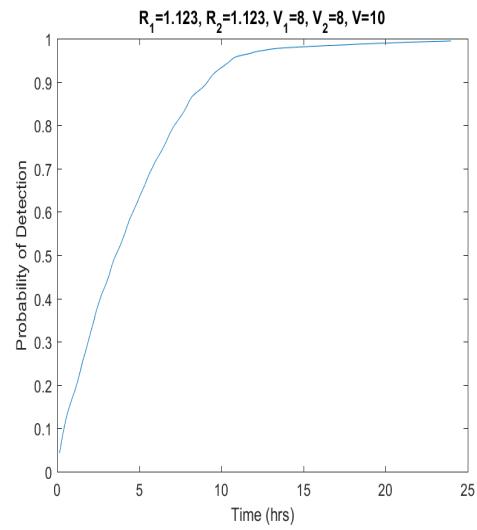


Figure A.358.  $P_D$  vs Time

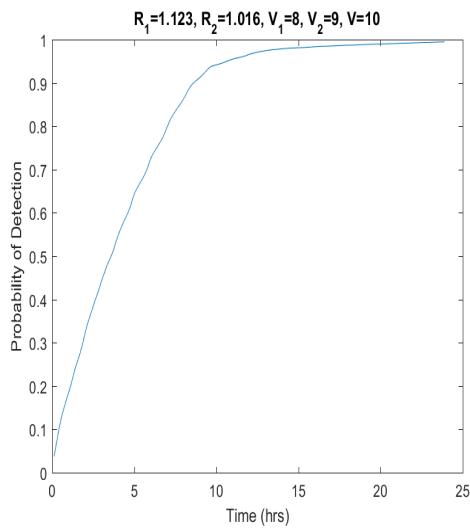


Figure A.359.  $P_D$  vs Time

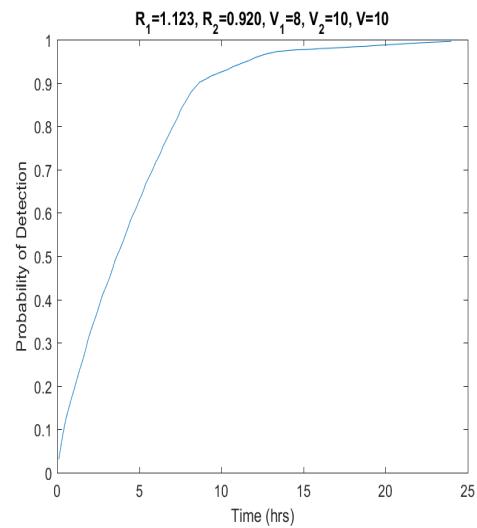


Figure A.360.  $P_D$  vs Time

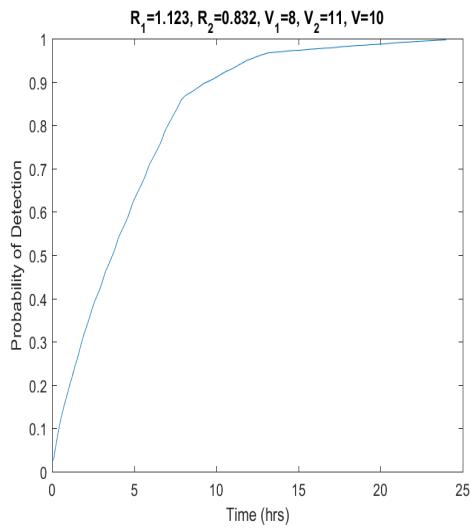


Figure A.361.  $P_D$  vs Time

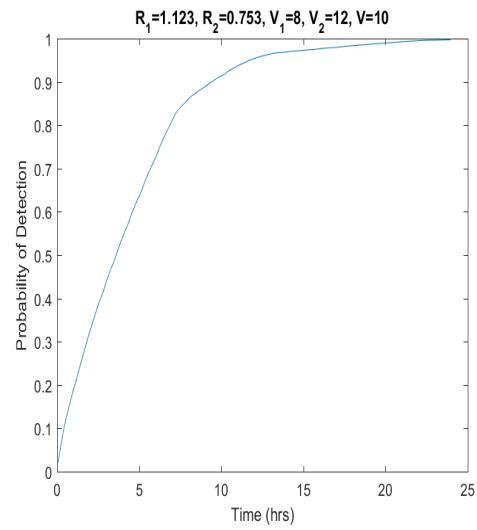


Figure A.362.  $P_D$  vs Time

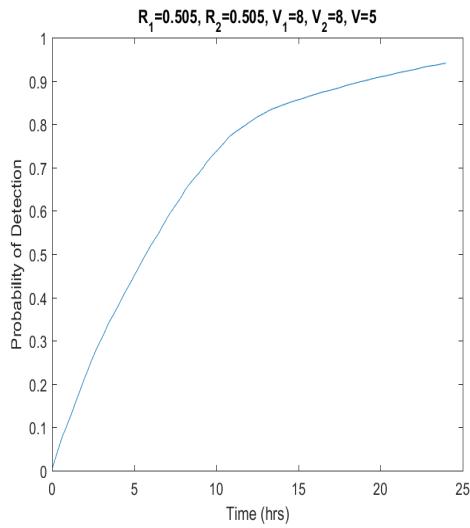


Figure A.363.  $P_D$  vs Time

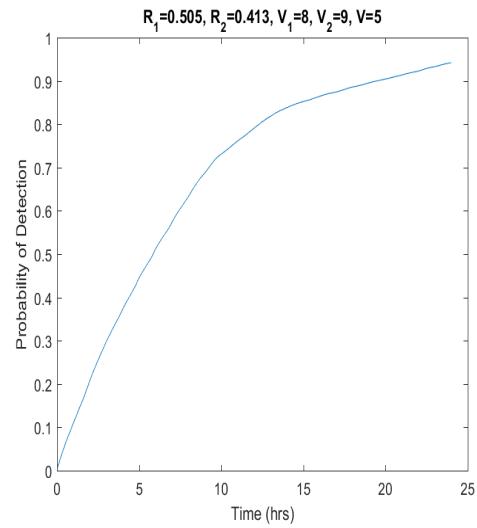


Figure A.364.  $P_D$  vs Time

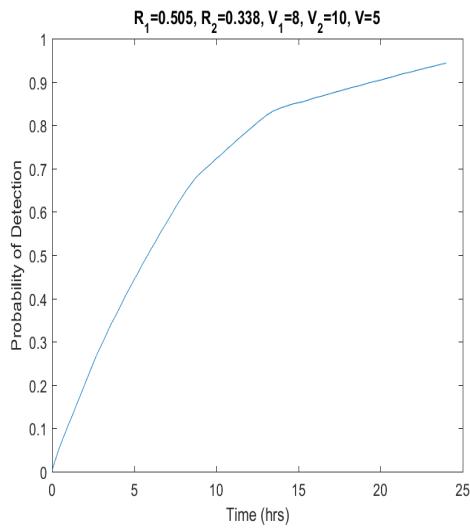


Figure A.365.  $P_D$  vs Time

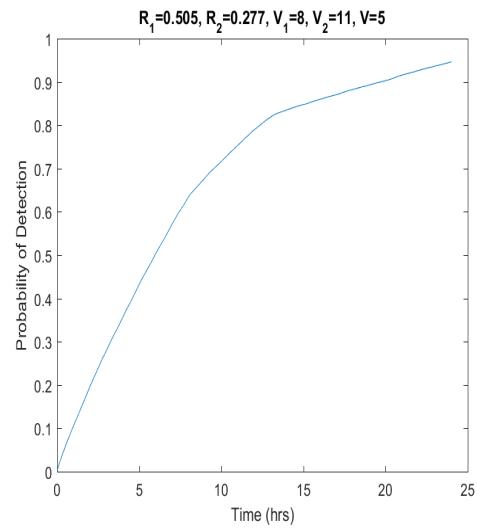


Figure A.366.  $P_D$  vs Time

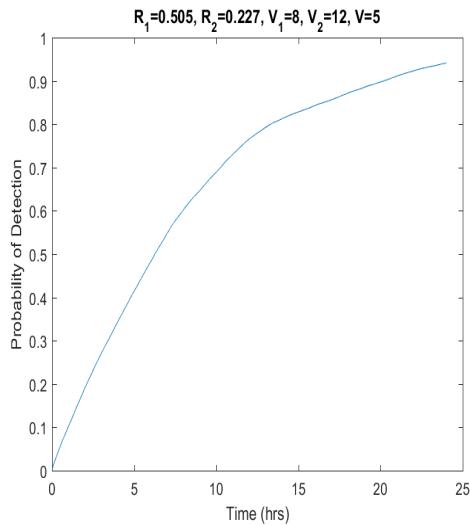


Figure A.367.  $P_D$  vs Time

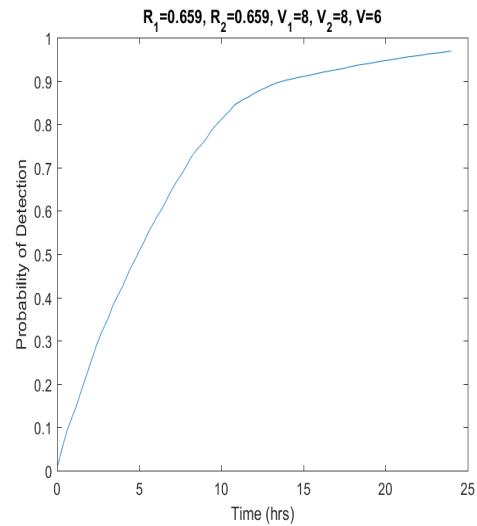


Figure A.368.  $P_D$  vs Time

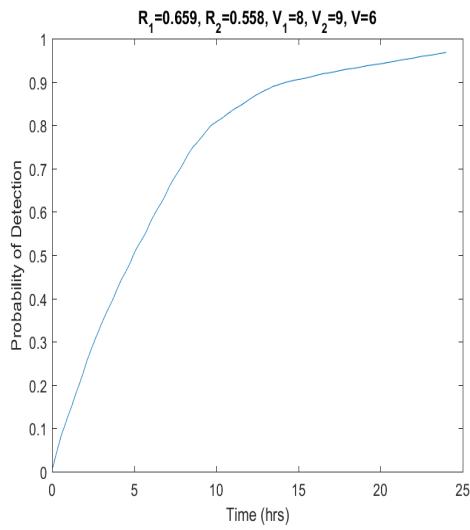


Figure A.369.  $P_D$  vs Time

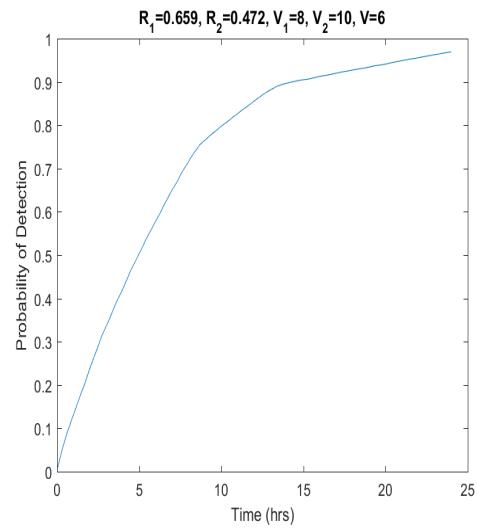


Figure A.370.  $P_D$  vs Time

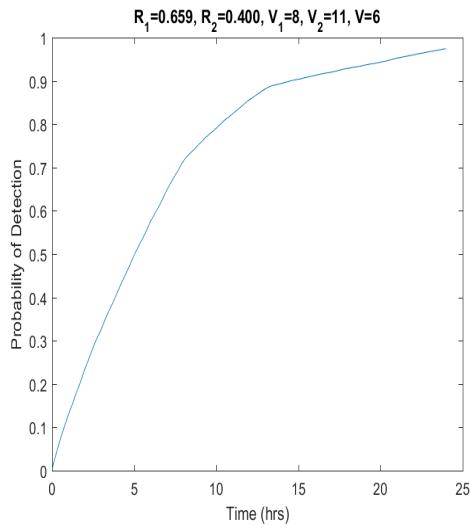


Figure A.371.  $P_D$  vs Time

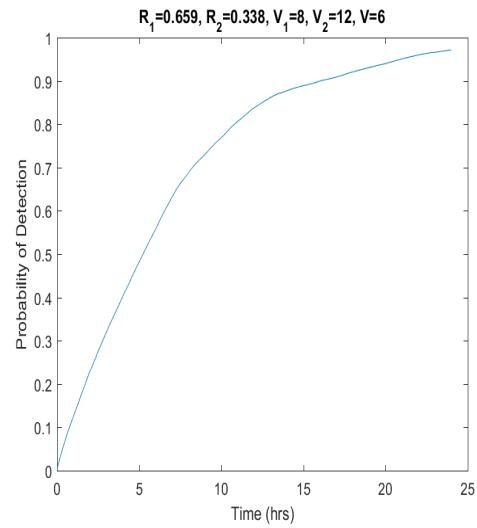


Figure A.372.  $P_D$  vs Time

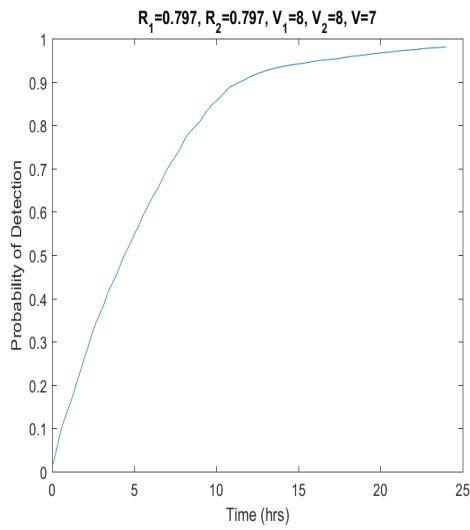


Figure A.373.  $P_D$  vs Time

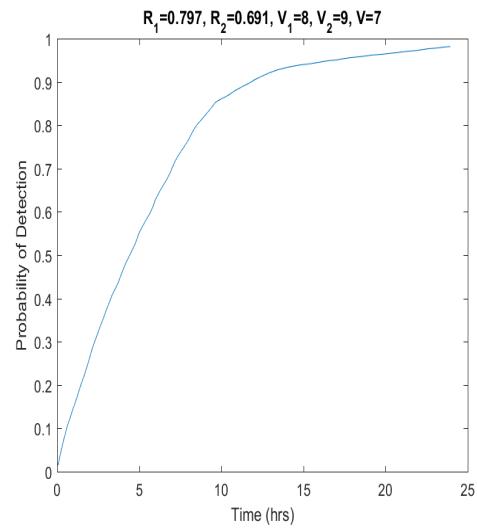


Figure A.374.  $P_D$  vs Time

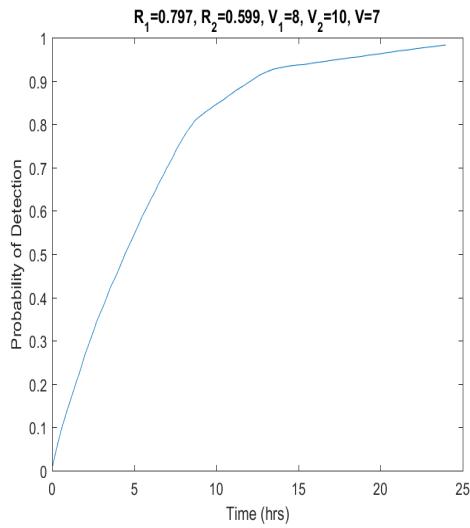


Figure A.375.  $P_D$  vs Time

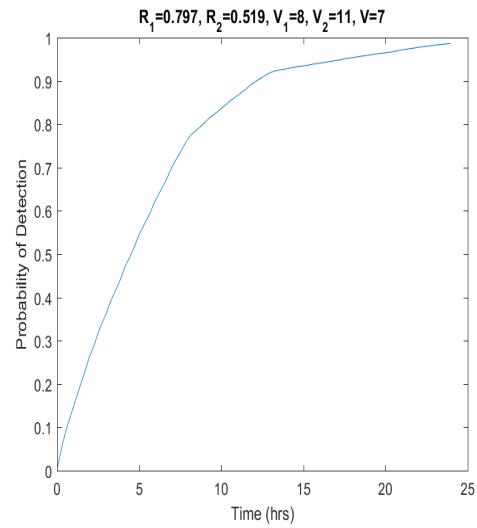


Figure A.376.  $P_D$  vs Time

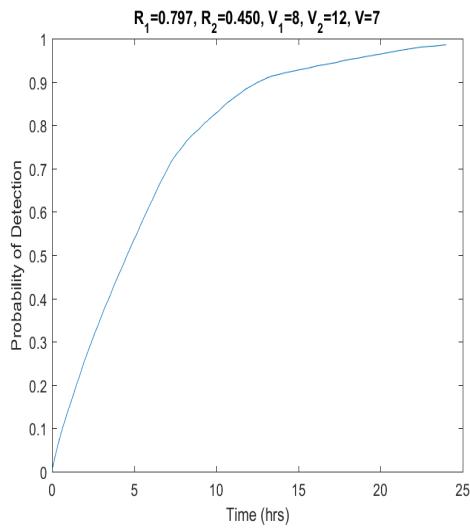


Figure A.377.  $P_D$  vs Time

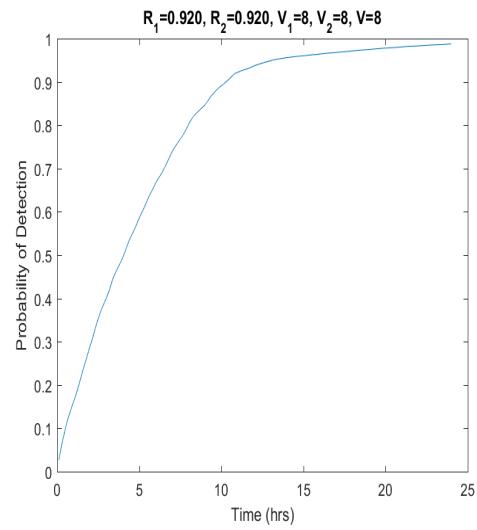


Figure A.378.  $P_D$  vs Time

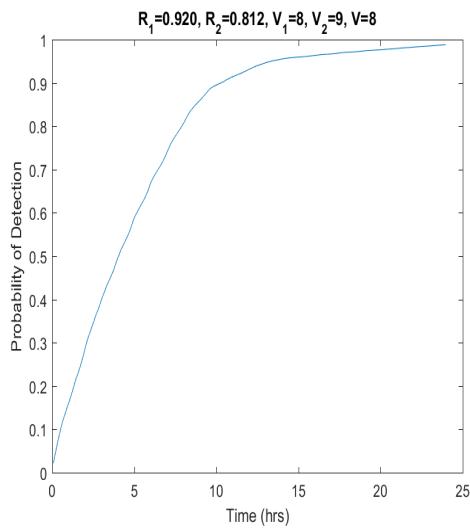


Figure A.379.  $P_D$  vs Time

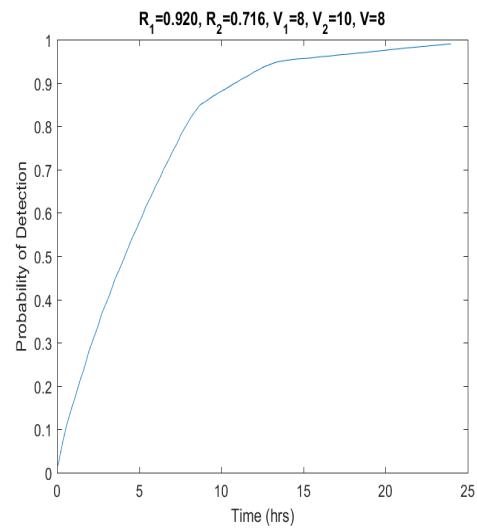


Figure A.380.  $P_D$  vs Time

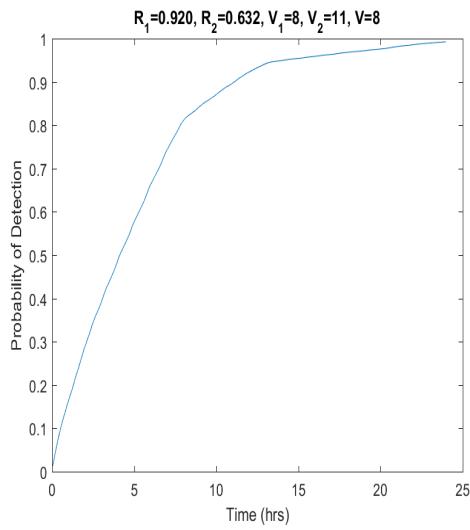


Figure A.381.  $P_D$  vs Time

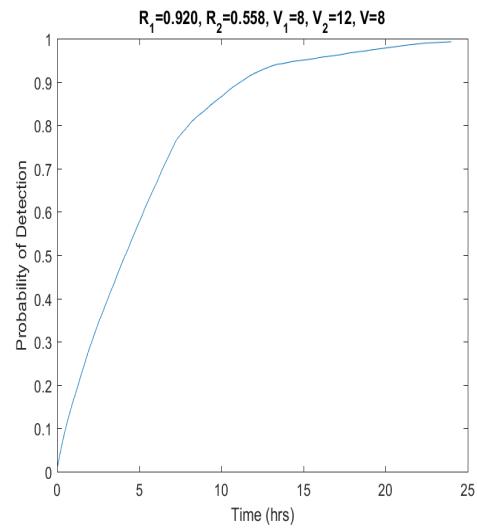


Figure A.382.  $P_D$  vs Time

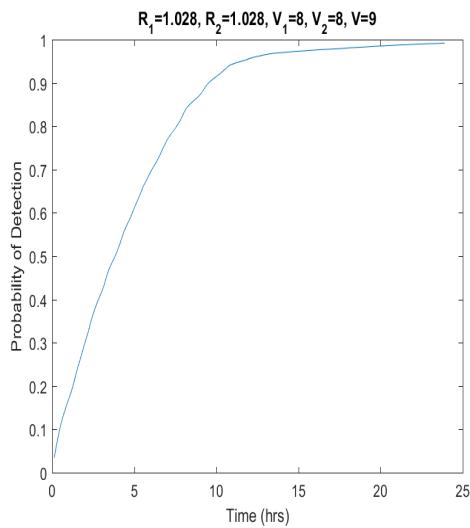


Figure A.383.  $P_D$  vs Time

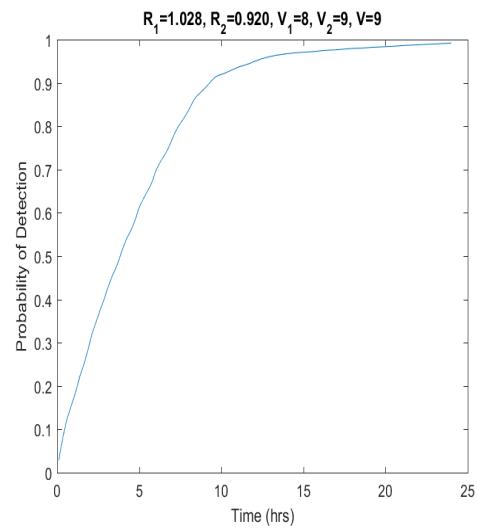


Figure A.384.  $P_D$  vs Time

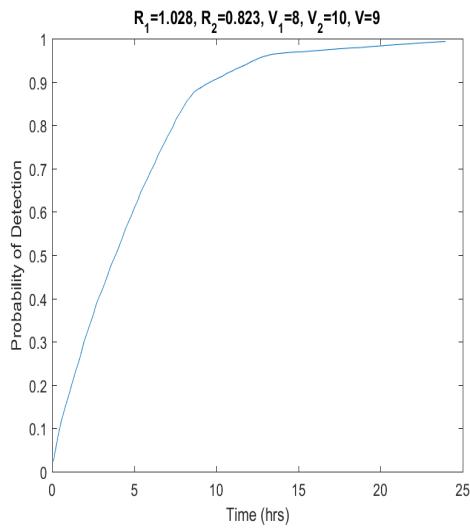


Figure A.385.  $P_D$  vs Time

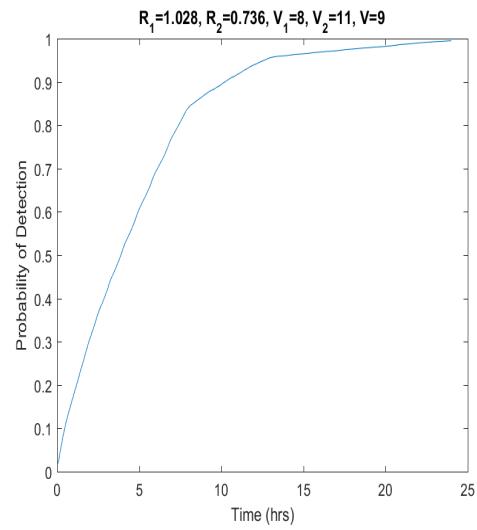


Figure A.386.  $P_D$  vs Time

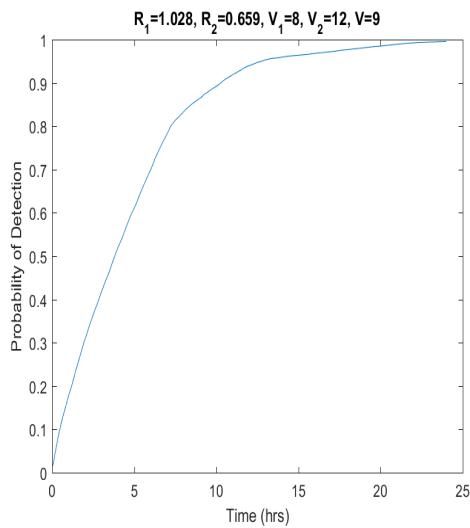


Figure A.387.  $P_D$  vs Time

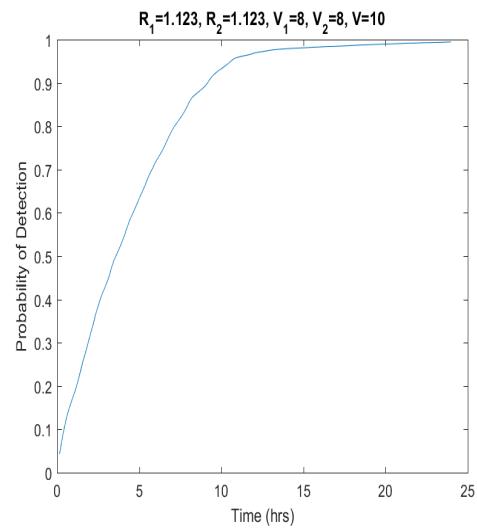


Figure A.388.  $P_D$  vs Time

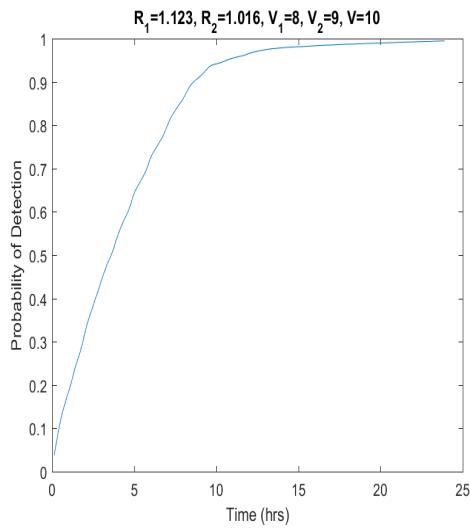


Figure A.389.  $P_D$  vs Time

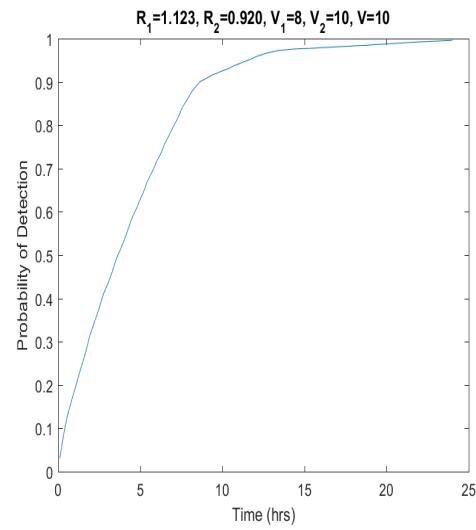


Figure A.390.  $P_D$  vs Time

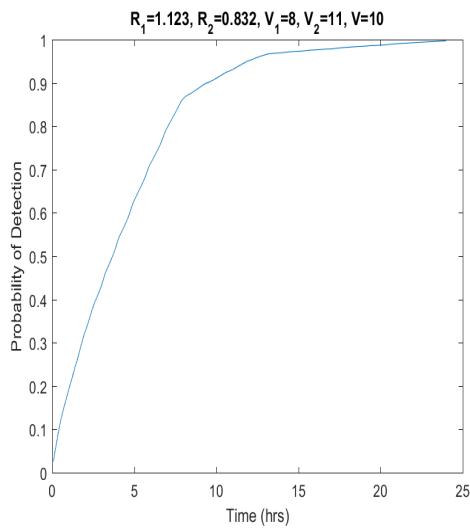


Figure A.391.  $P_D$  vs Time

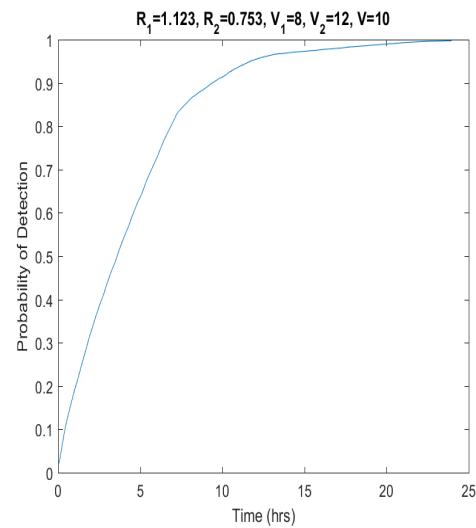


Figure A.392.  $P_D$  vs Time

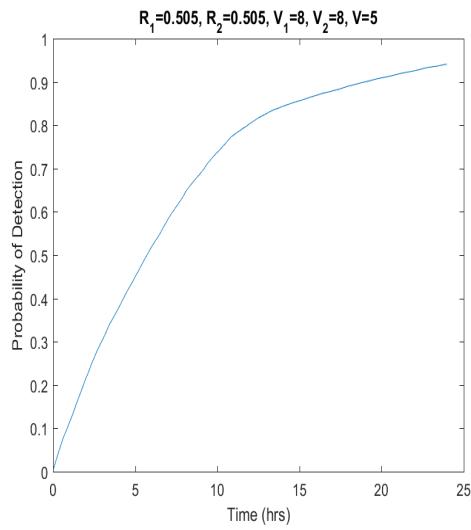


Figure A.393.  $P_D$  vs Time

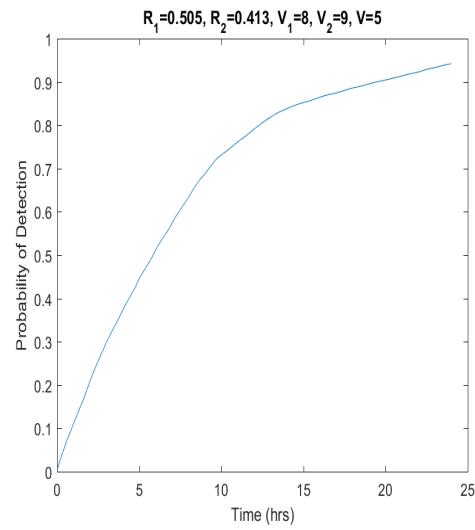


Figure A.394.  $P_D$  vs Time

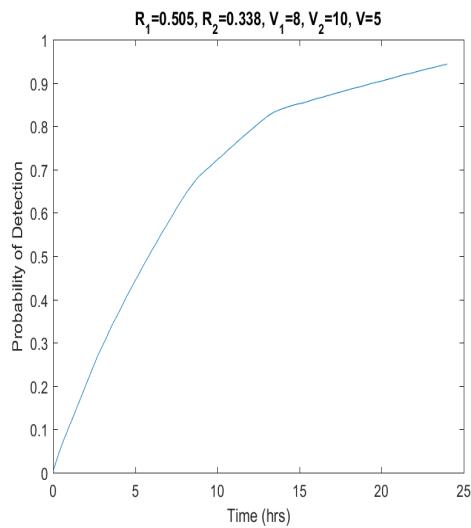


Figure A.395.  $P_D$  vs Time

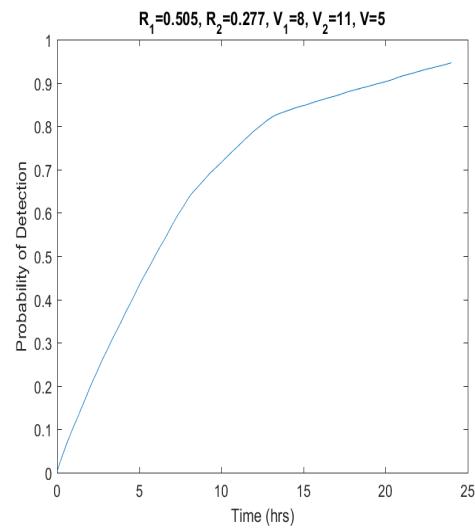


Figure A.396.  $P_D$  vs Time

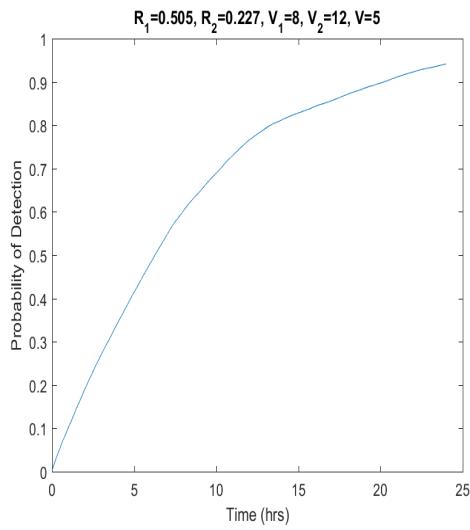


Figure A.397.  $P_D$  vs Time

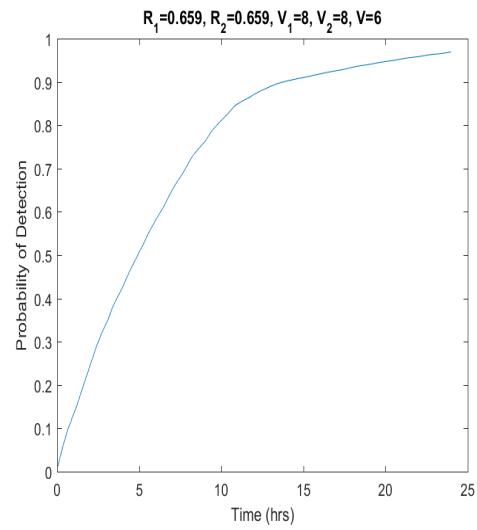


Figure A.398.  $P_D$  vs Time

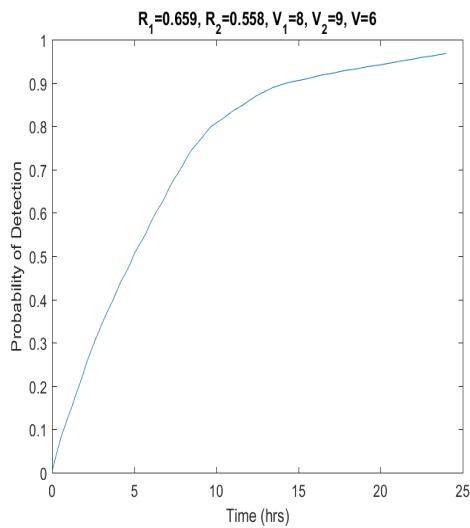


Figure A.399.  $P_D$  vs Time

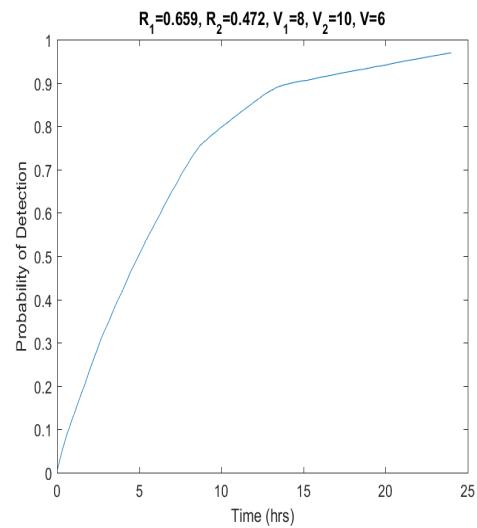


Figure A.400.  $P_D$  vs Time

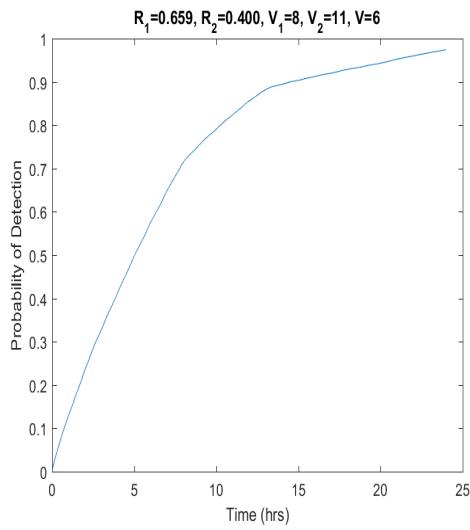


Figure A.401.  $P_D$  vs Time

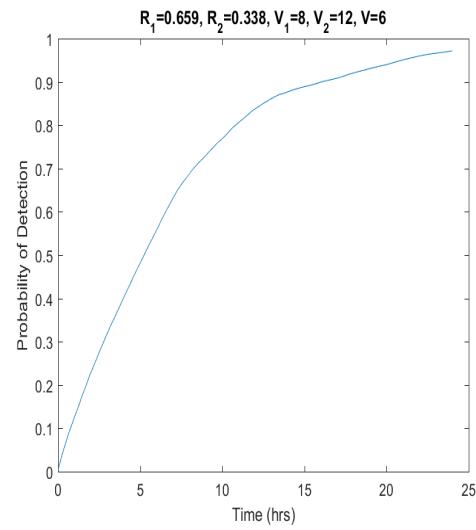


Figure A.402.  $P_D$  vs Time

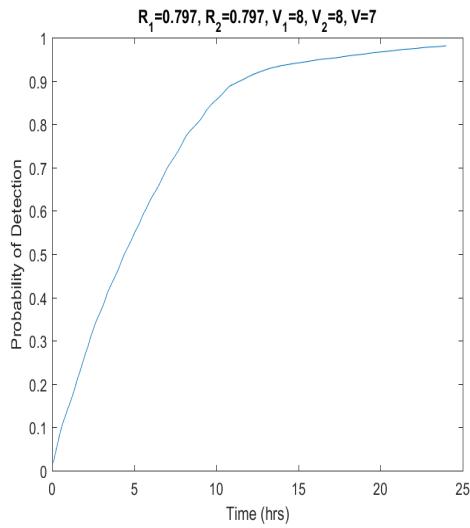


Figure A.403.  $P_D$  vs Time

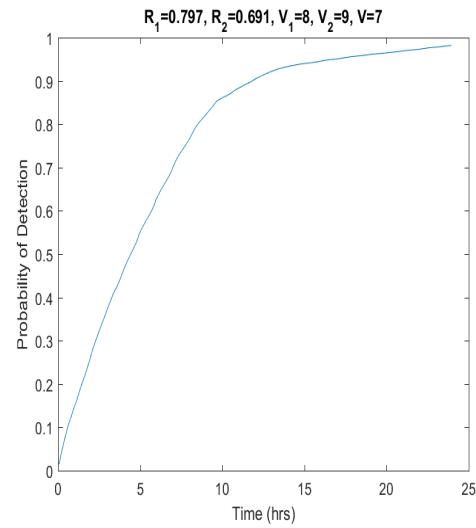


Figure A.404.  $P_D$  vs Time

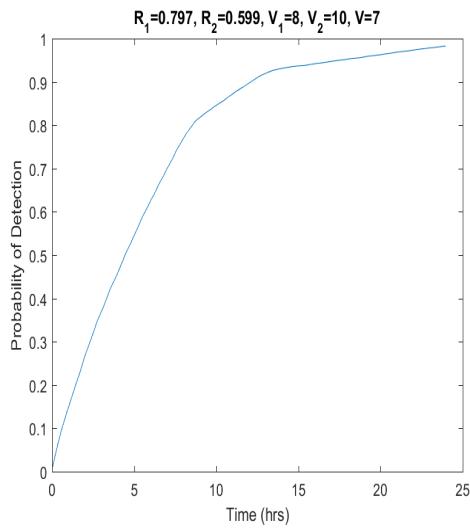


Figure A.405.  $P_D$  vs Time

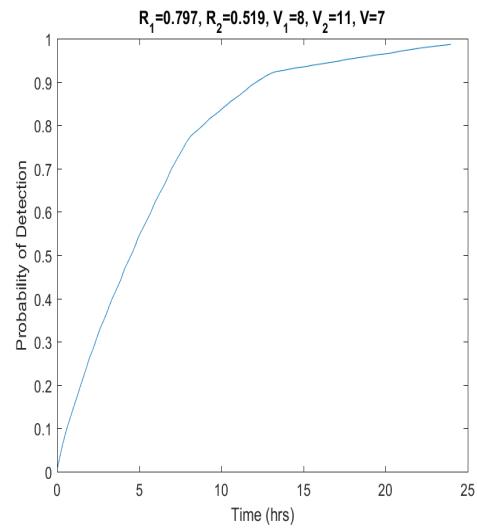


Figure A.406.  $P_D$  vs Time

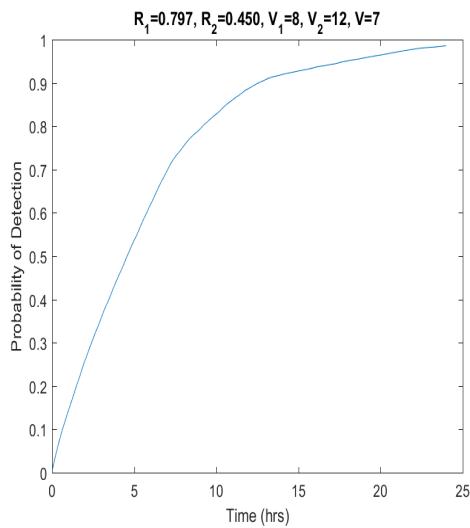


Figure A.407.  $P_D$  vs Time

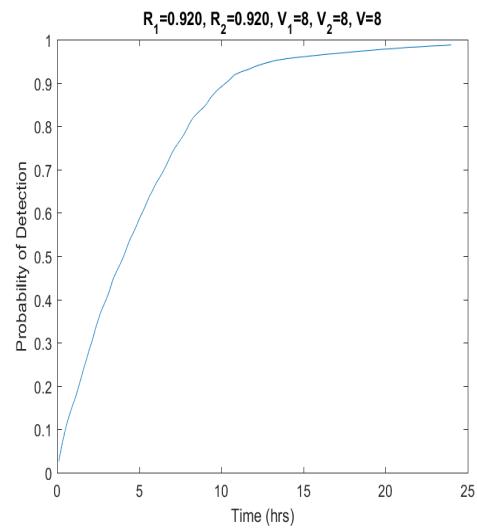


Figure A.408.  $P_D$  vs Time

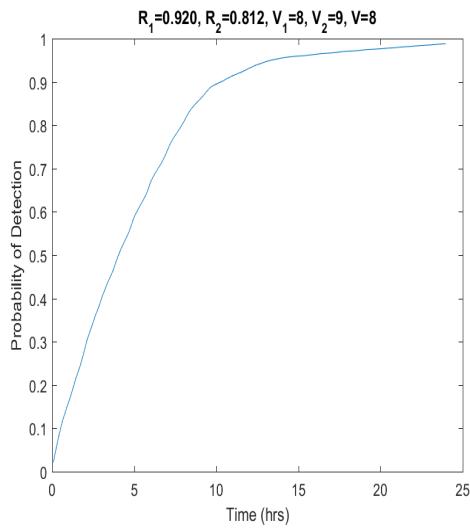


Figure A.409.  $P_D$  vs Time

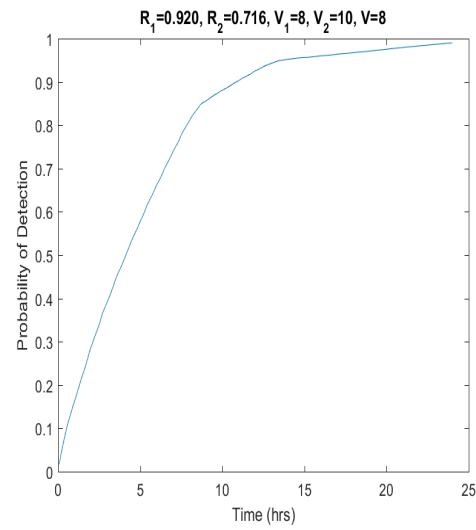


Figure A.410.  $P_D$  vs Time

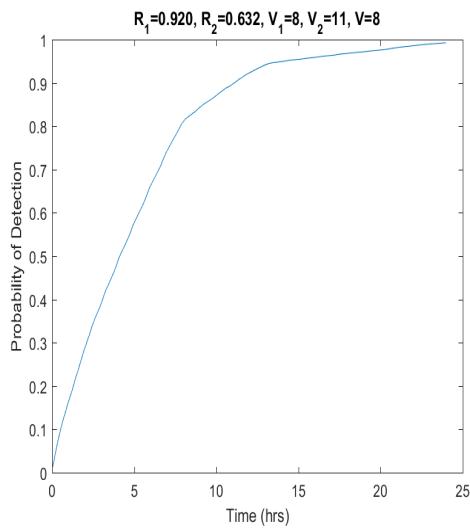


Figure A.411.  $P_D$  vs Time

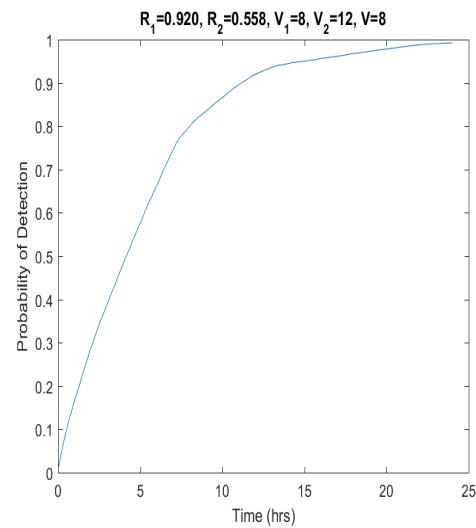


Figure A.412.  $P_D$  vs Time

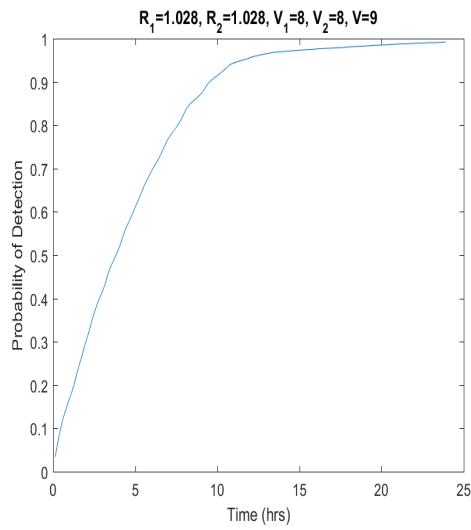


Figure A.413.  $P_D$  vs Time

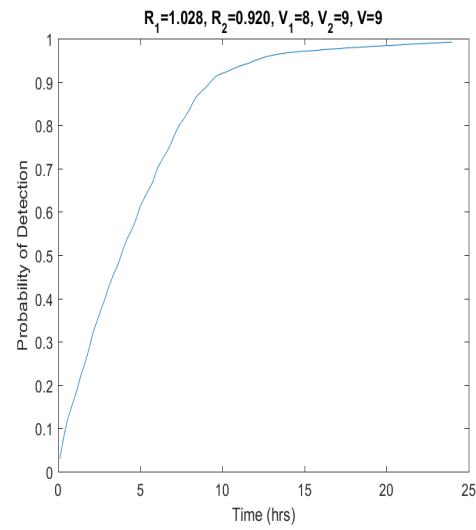


Figure A.414.  $P_D$  vs Time

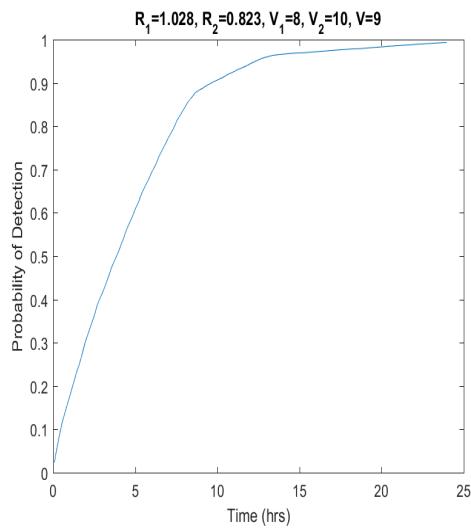


Figure A.415.  $P_D$  vs Time

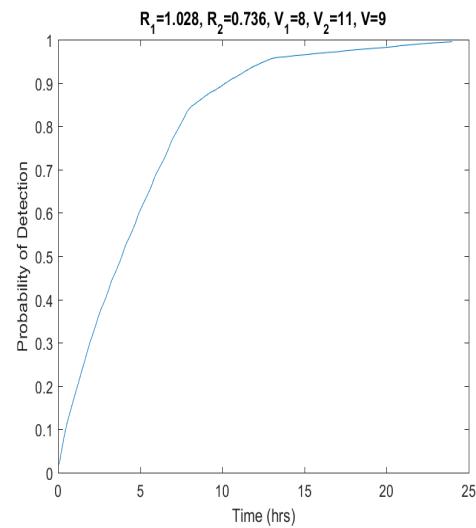


Figure A.416.  $P_D$  vs Time

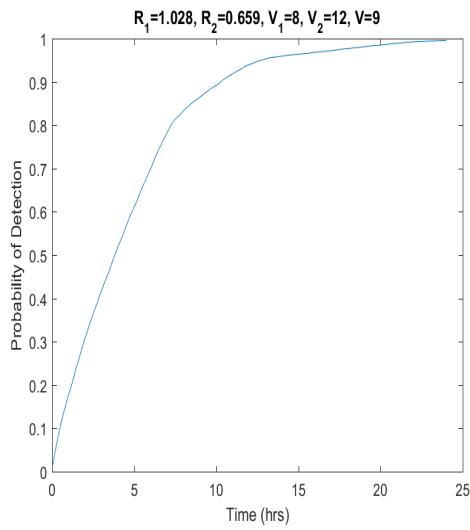


Figure A.417.  $P_D$  vs Time

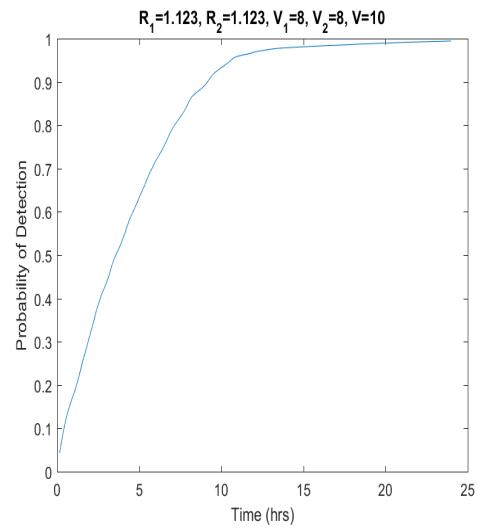


Figure A.418.  $P_D$  vs Time

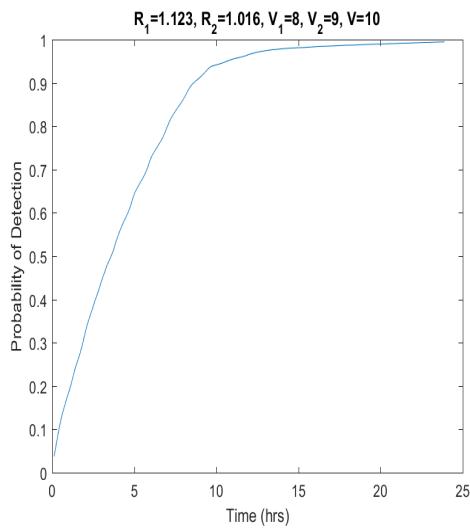


Figure A.419.  $P_D$  vs Time

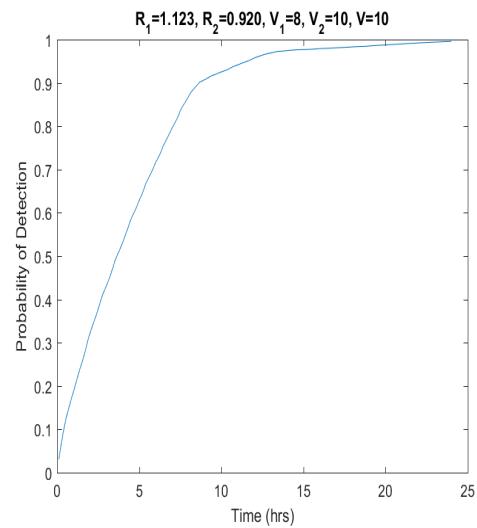


Figure A.420.  $P_D$  vs Time

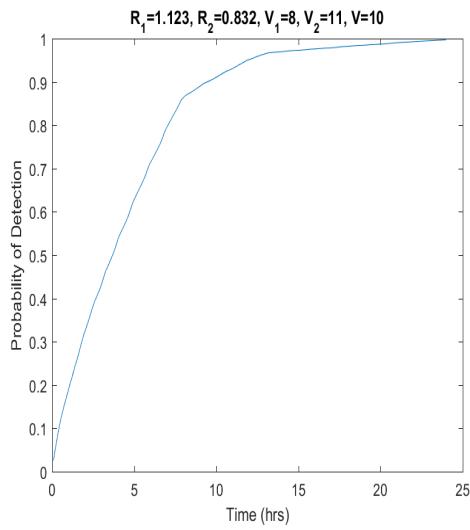


Figure A.421.  $P_D$  vs Time

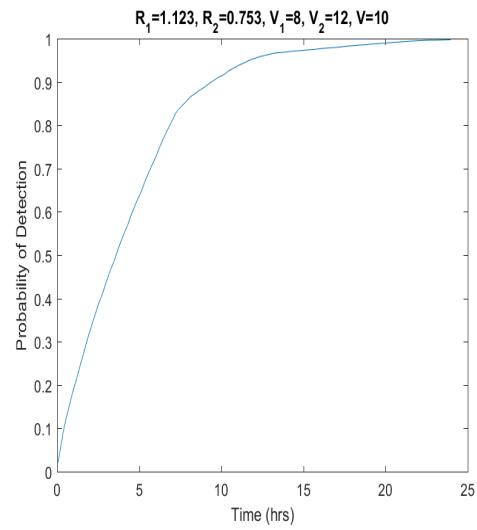


Figure A.422.  $P_D$  vs Time

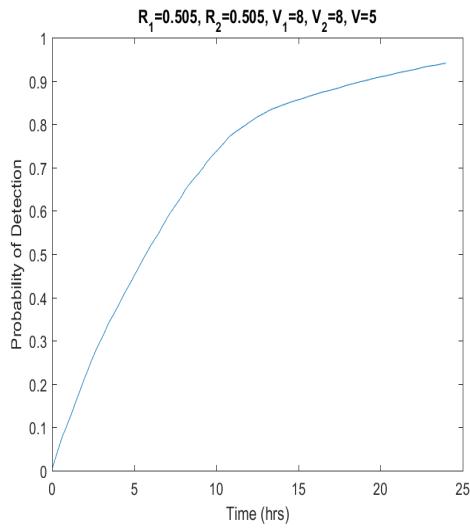


Figure A.423.  $P_D$  vs Time

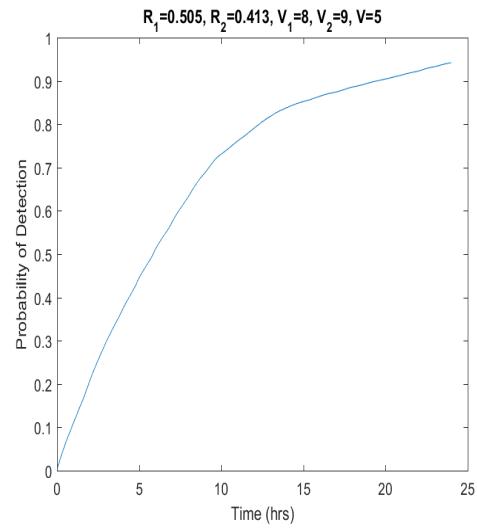


Figure A.424.  $P_D$  vs Time

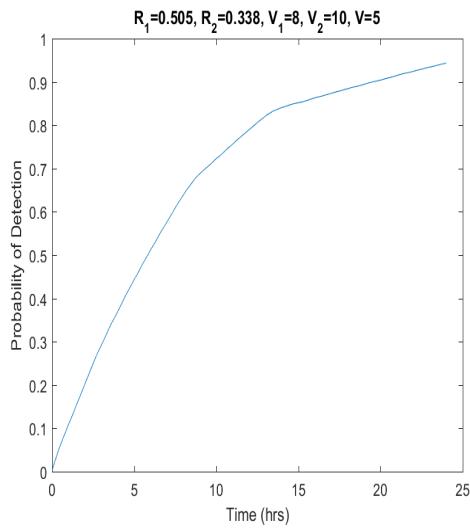


Figure A.425.  $P_D$  vs Time

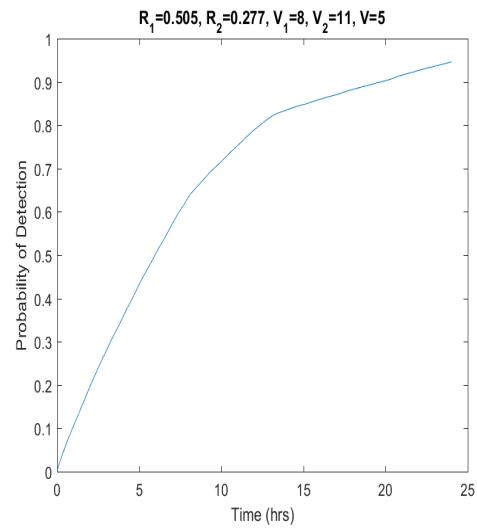


Figure A.426.  $P_D$  vs Time

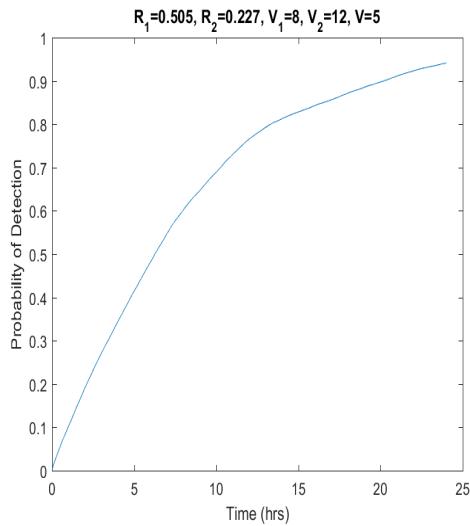


Figure A.427.  $P_D$  vs Time

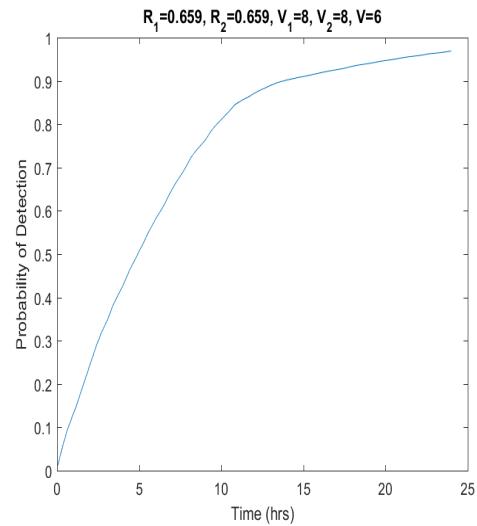


Figure A.428.  $P_D$  vs Time

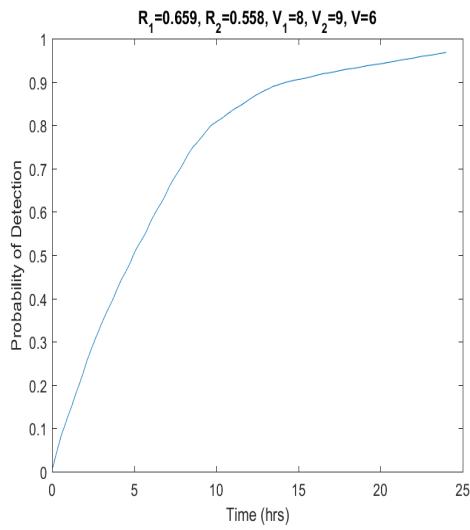


Figure A.429.  $P_D$  vs Time

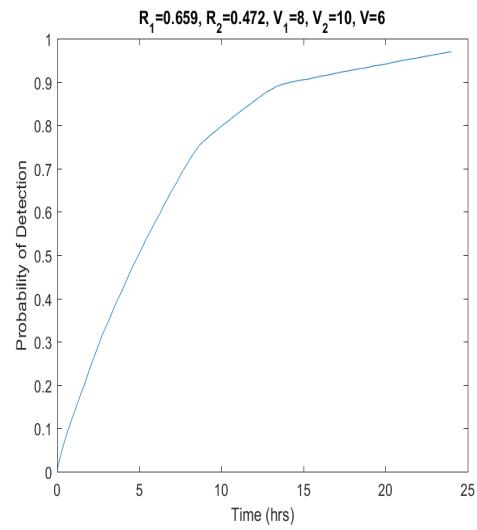


Figure A.430.  $P_D$  vs Time

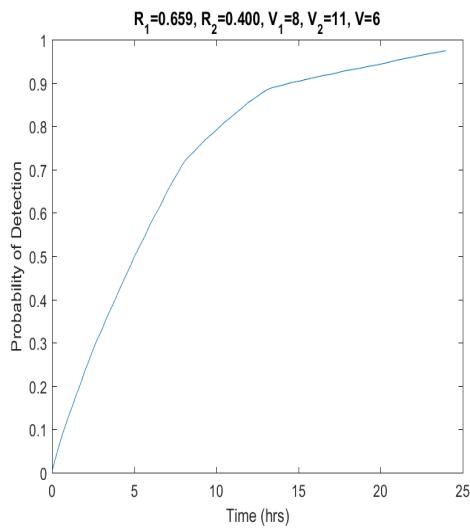


Figure A.431.  $P_D$  vs Time

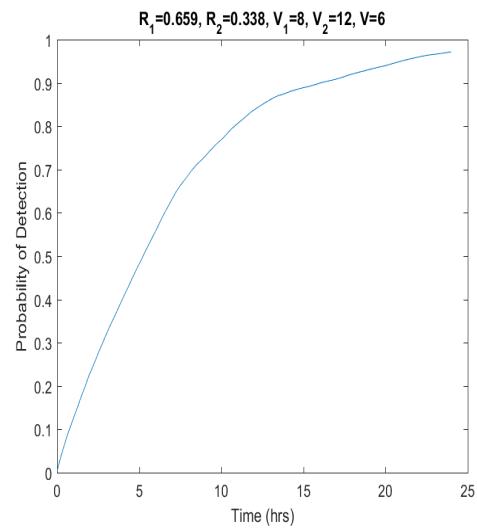


Figure A.432.  $P_D$  vs Time

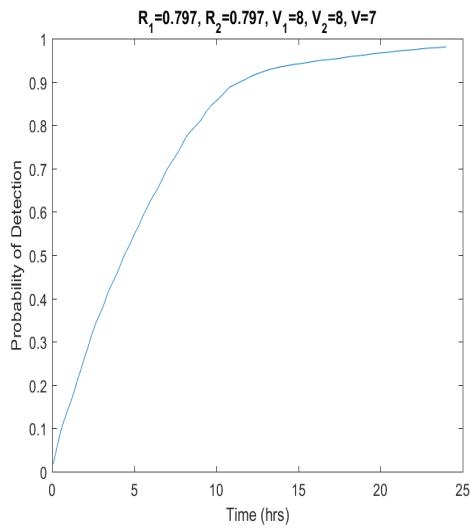


Figure A.433.  $P_D$  vs Time

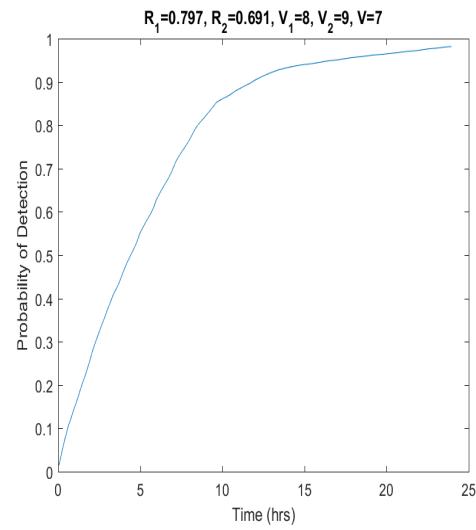


Figure A.434.  $P_D$  vs Time

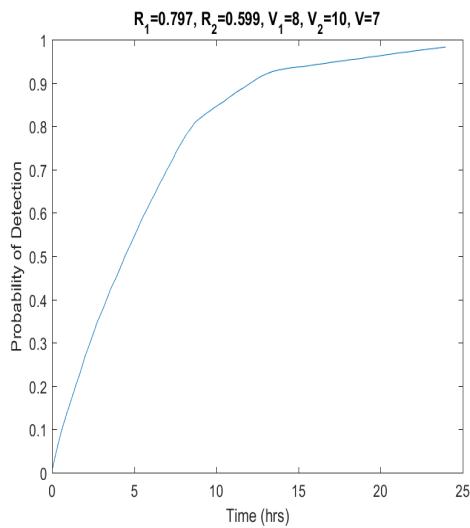


Figure A.435.  $P_D$  vs Time

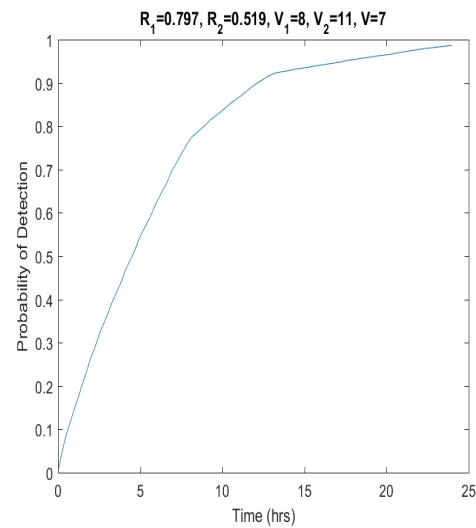


Figure A.436.  $P_D$  vs Time

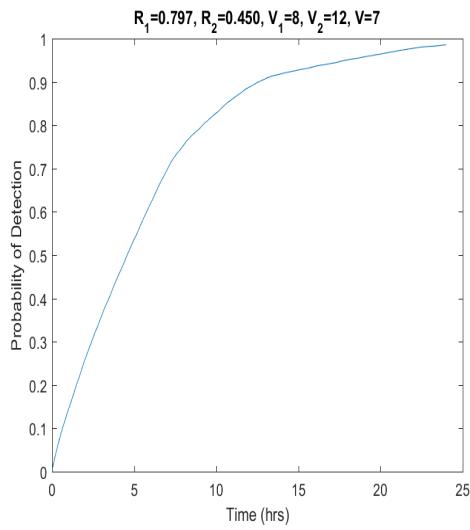


Figure A.437.  $P_D$  vs Time

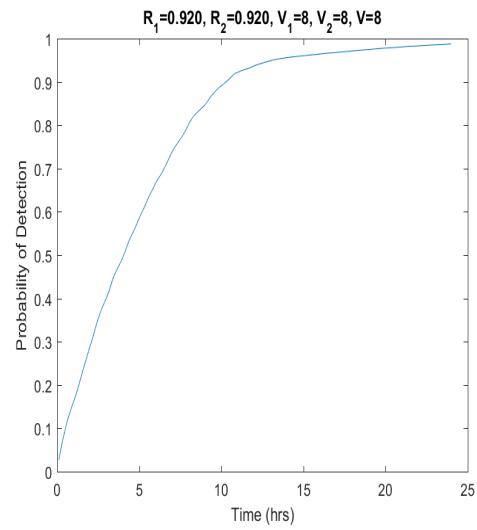


Figure A.438.  $P_D$  vs Time

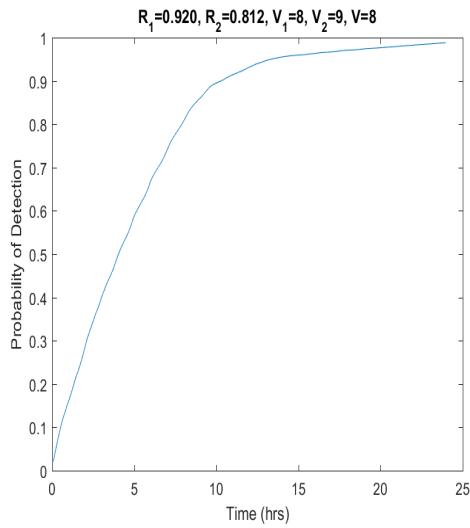


Figure A.439.  $P_D$  vs Time

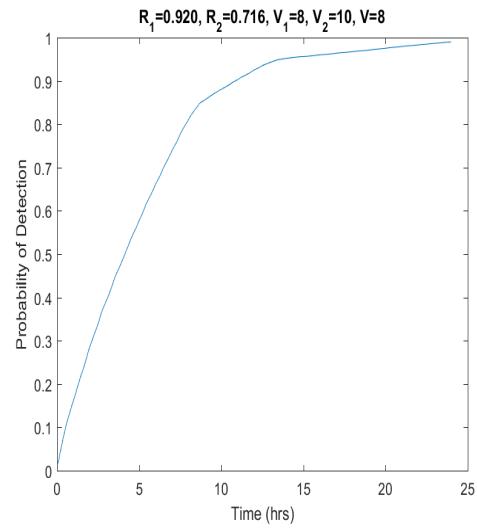


Figure A.440.  $P_D$  vs Time

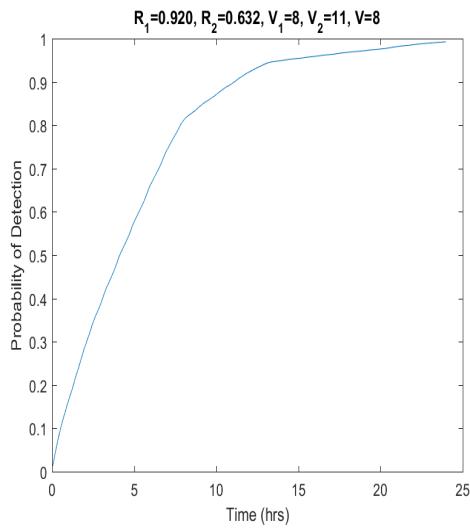


Figure A.441.  $P_D$  vs Time

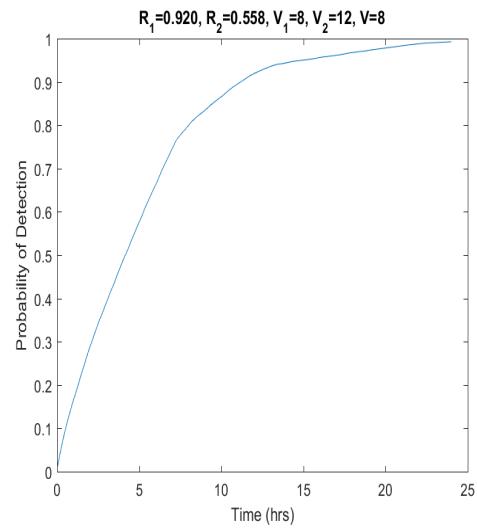


Figure A.442.  $P_D$  vs Time

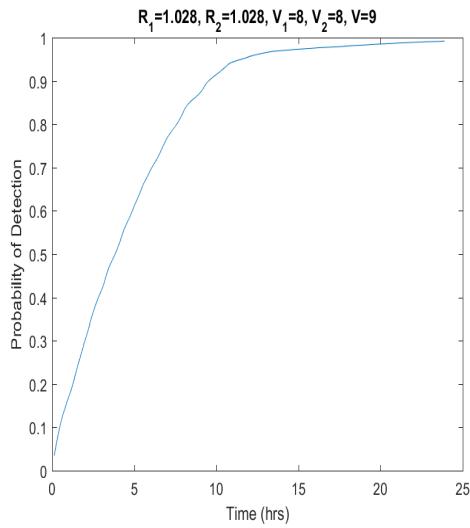


Figure A.443.  $P_D$  vs Time

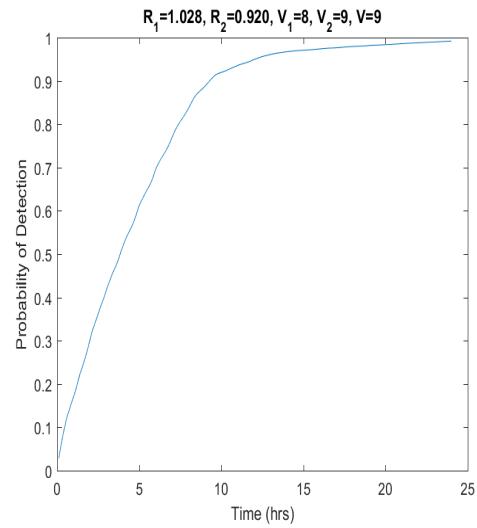


Figure A.444.  $P_D$  vs Time

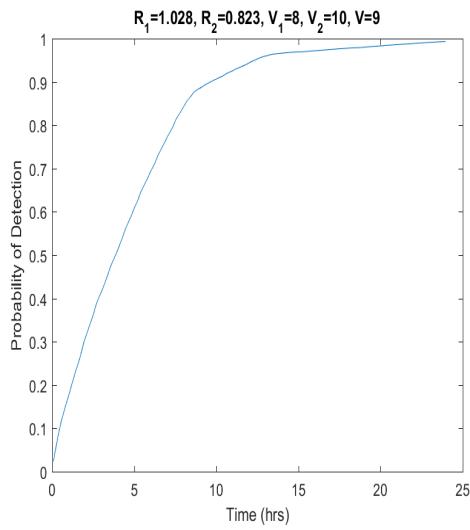


Figure A.445.  $P_D$  vs Time

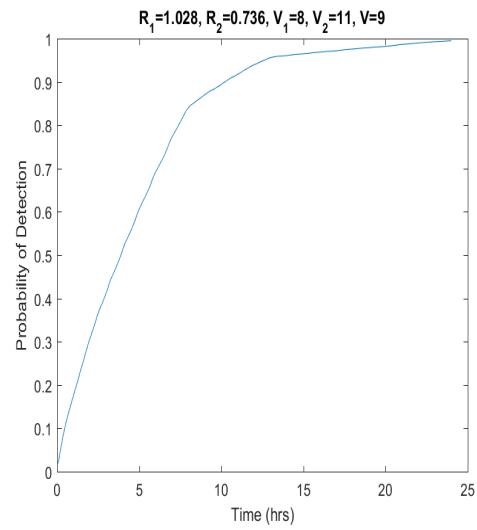


Figure A.446.  $P_D$  vs Time

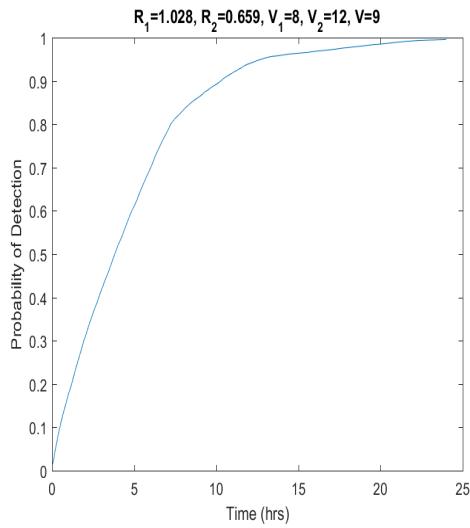


Figure A.447.  $P_D$  vs Time

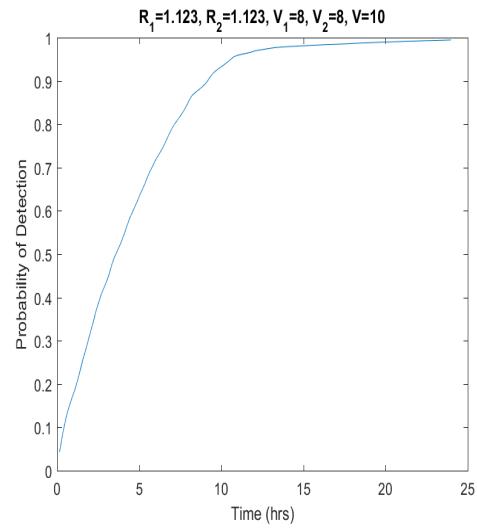


Figure A.448.  $P_D$  vs Time

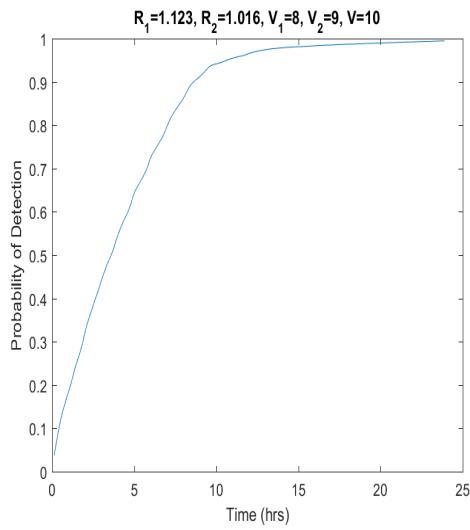


Figure A.449.  $P_D$  vs Time

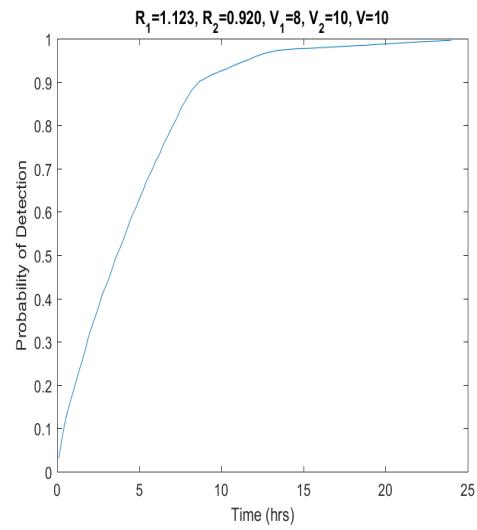


Figure A.450.  $P_D$  vs Time

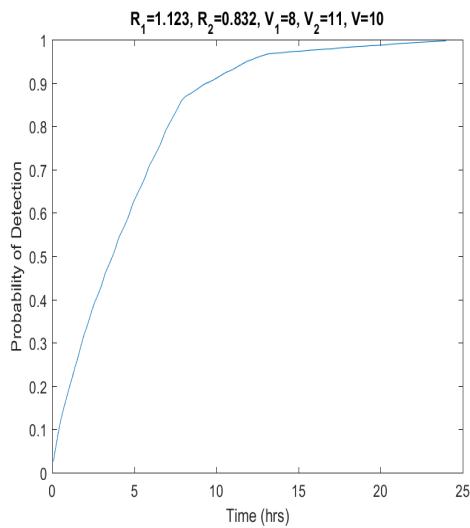


Figure A.451.  $P_D$  vs Time

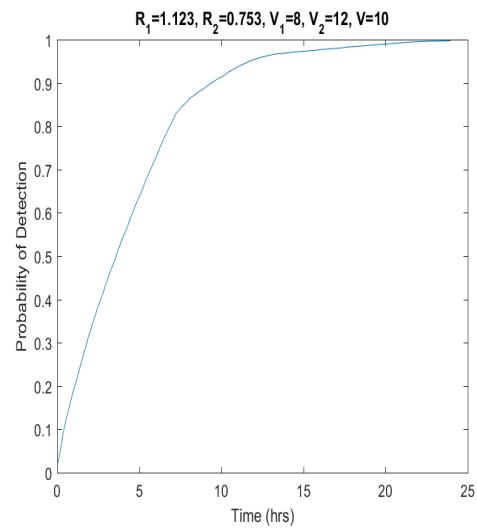


Figure A.452.  $P_D$  vs Time

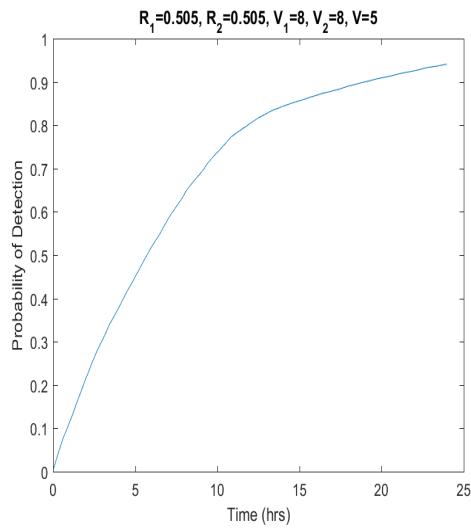


Figure A.453.  $P_D$  vs Time

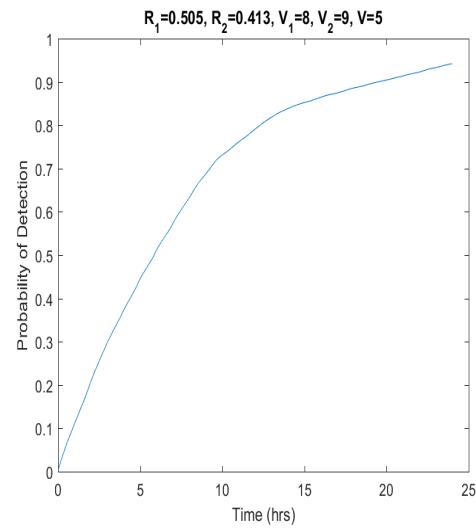


Figure A.454.  $P_D$  vs Time

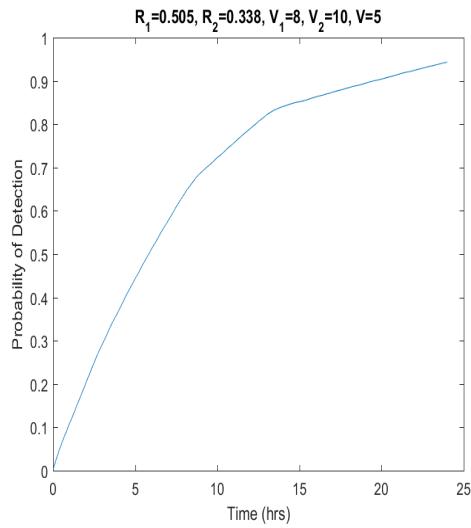


Figure A.455.  $P_D$  vs Time

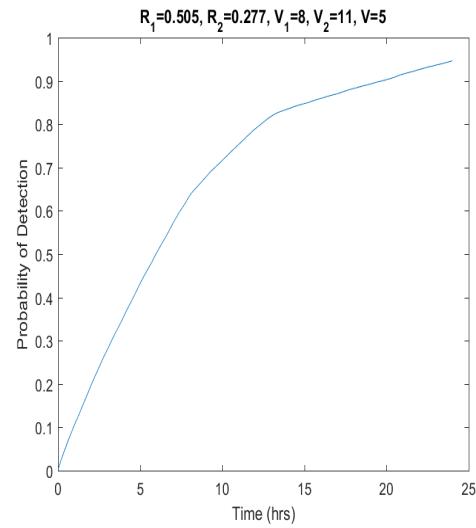


Figure A.456.  $P_D$  vs Time

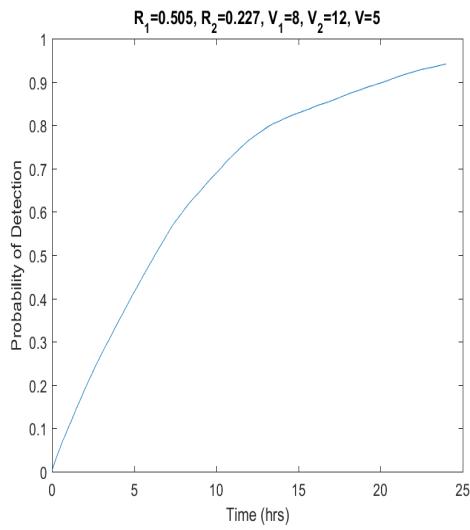


Figure A.457.  $P_D$  vs Time

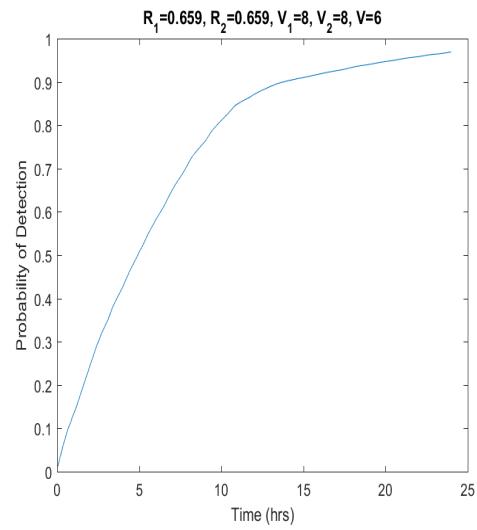


Figure A.458.  $P_D$  vs Time

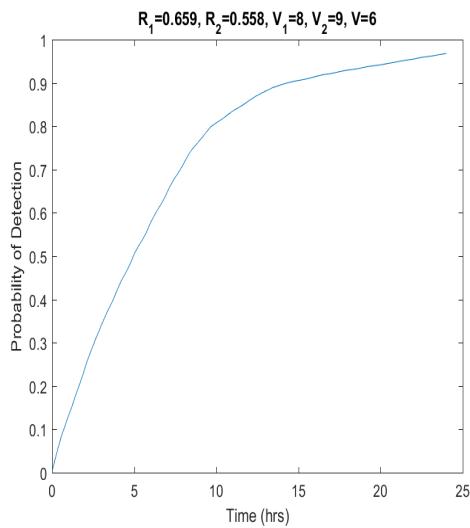


Figure A.459.  $P_D$  vs Time

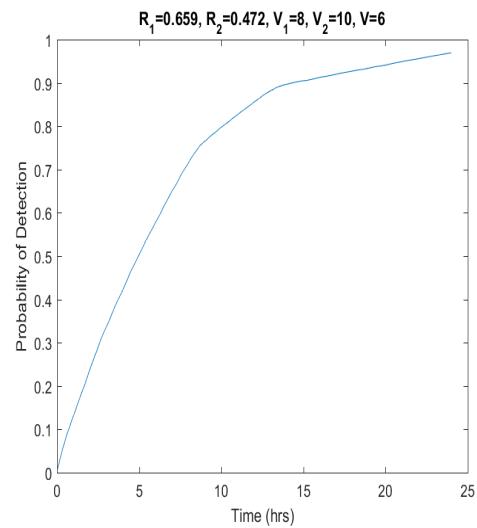


Figure A.460.  $P_D$  vs Time

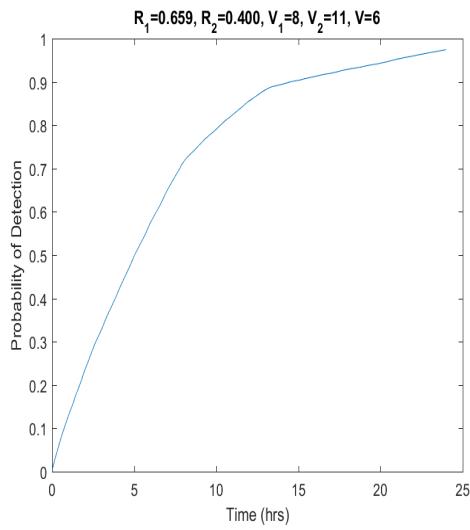


Figure A.461.  $P_D$  vs Time

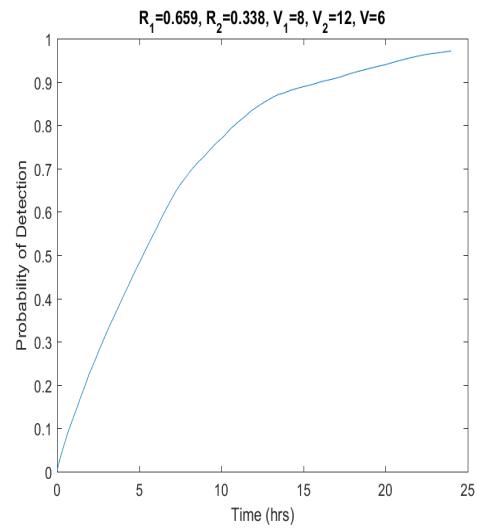


Figure A.462.  $P_D$  vs Time

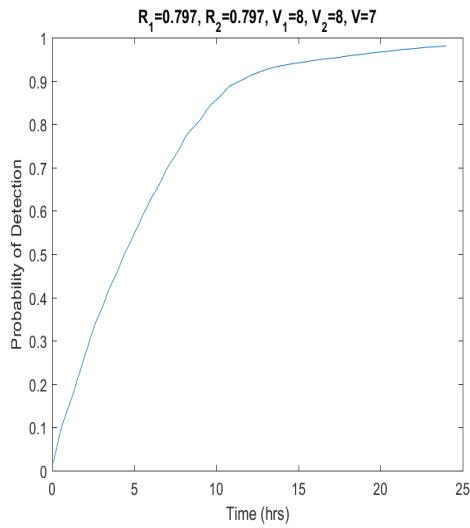


Figure A.463.  $P_D$  vs Time

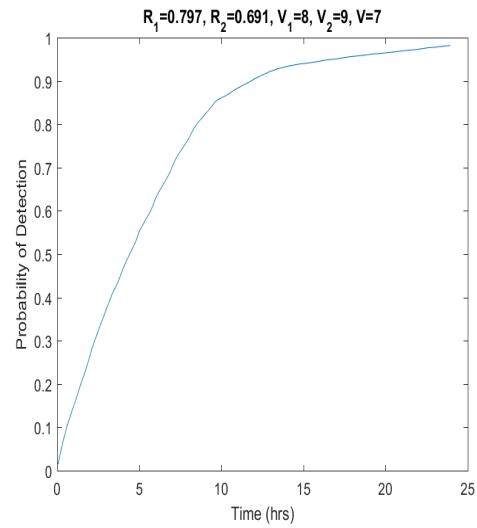


Figure A.464.  $P_D$  vs Time

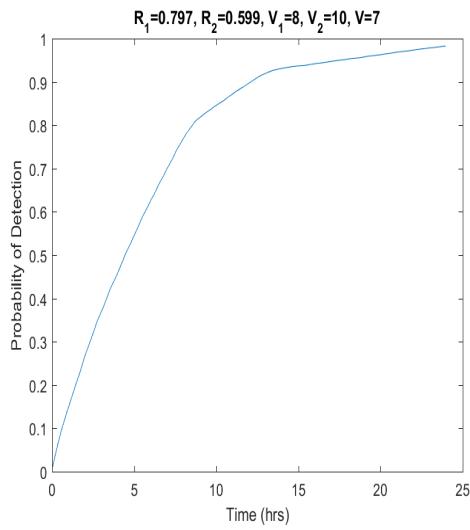


Figure A.465.  $P_D$  vs Time

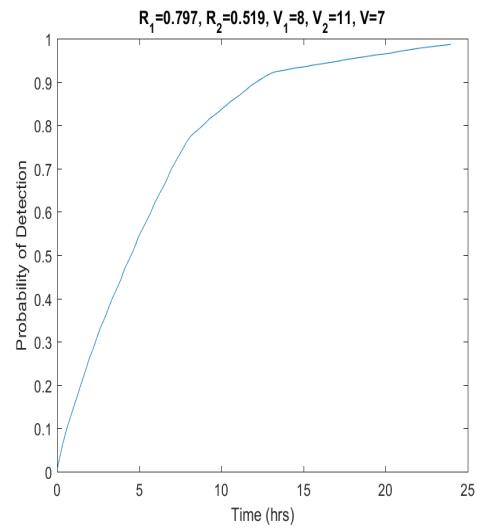


Figure A.466.  $P_D$  vs Time

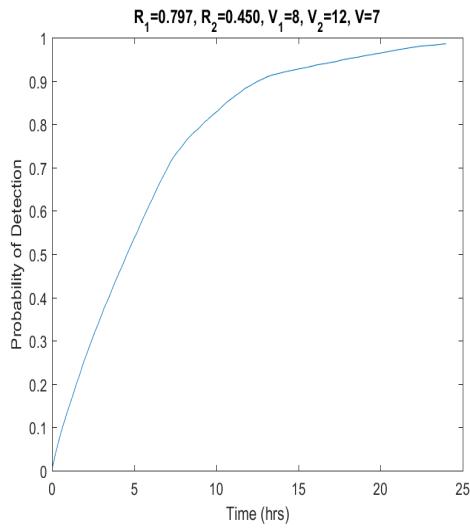


Figure A.467.  $P_D$  vs Time

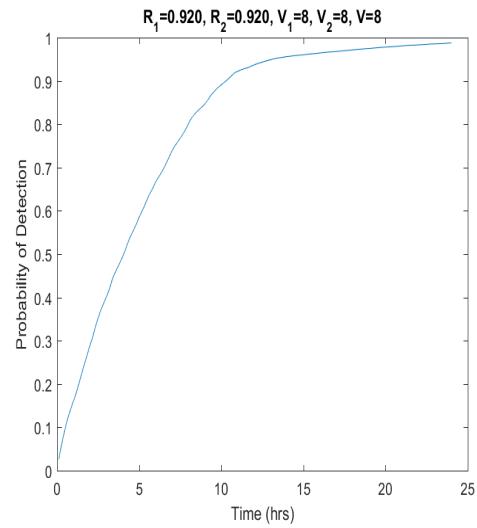


Figure A.468.  $P_D$  vs Time

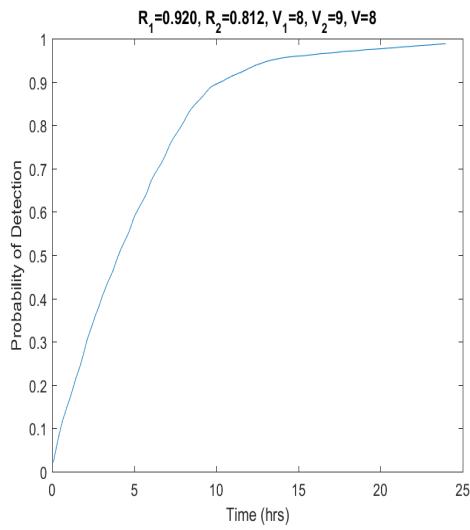


Figure A.469.  $P_D$  vs Time

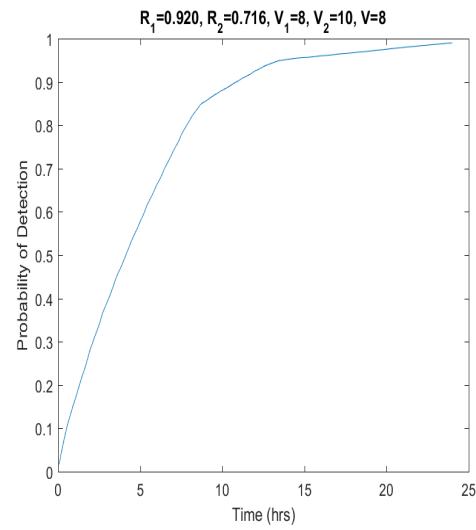


Figure A.470.  $P_D$  vs Time

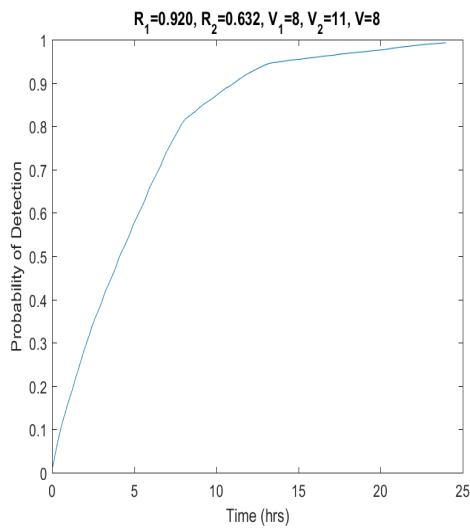


Figure A.471.  $P_D$  vs Time

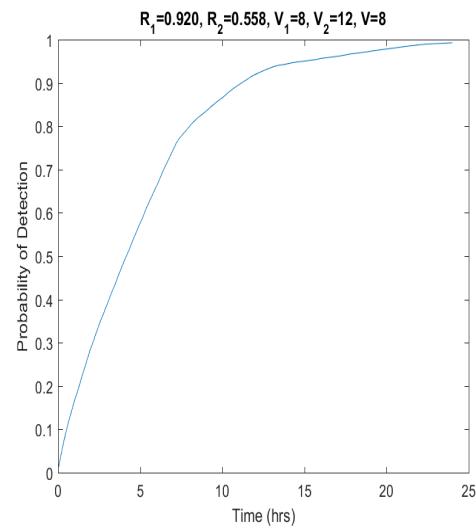


Figure A.472.  $P_D$  vs Time

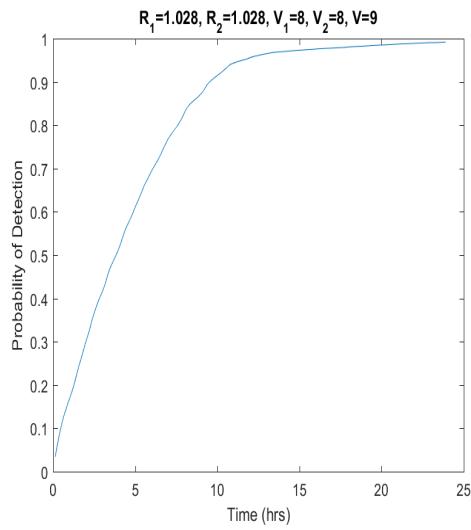


Figure A.473.  $P_D$  vs Time

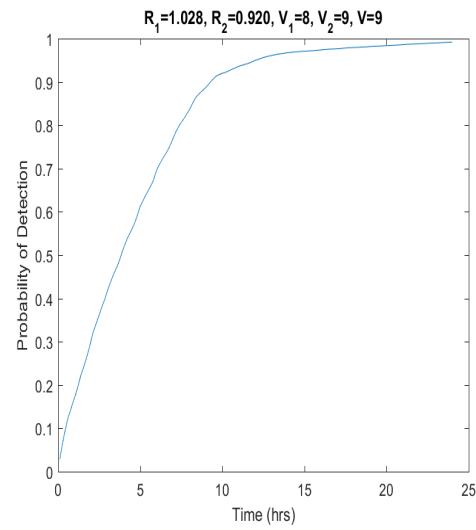


Figure A.474.  $P_D$  vs Time

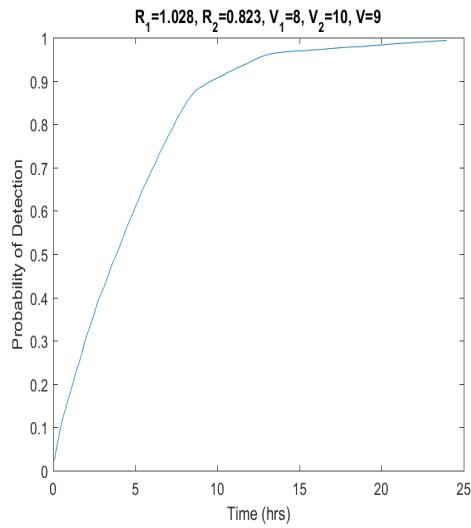


Figure A.475.  $P_D$  vs Time

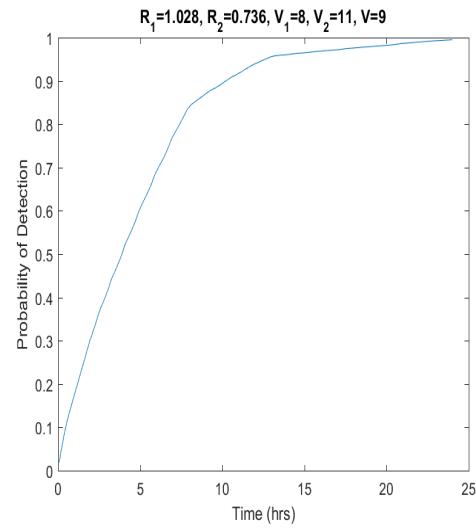


Figure A.476.  $P_D$  vs Time

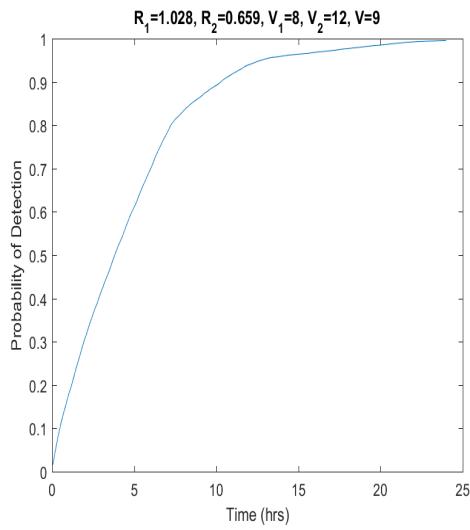


Figure A.477.  $P_D$  vs Time

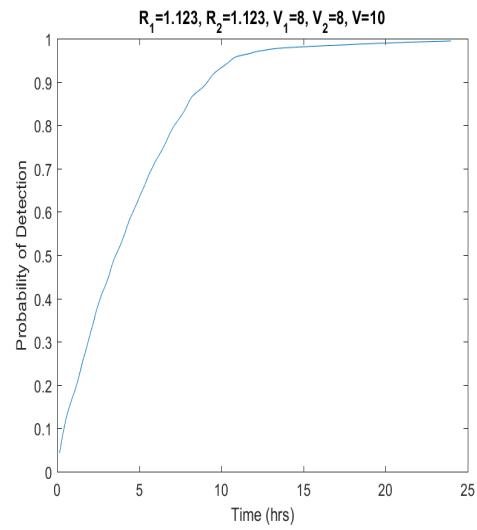


Figure A.478.  $P_D$  vs Time

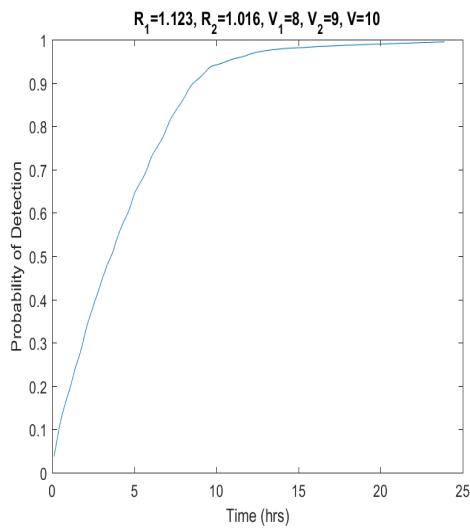


Figure A.479.  $P_D$  vs Time

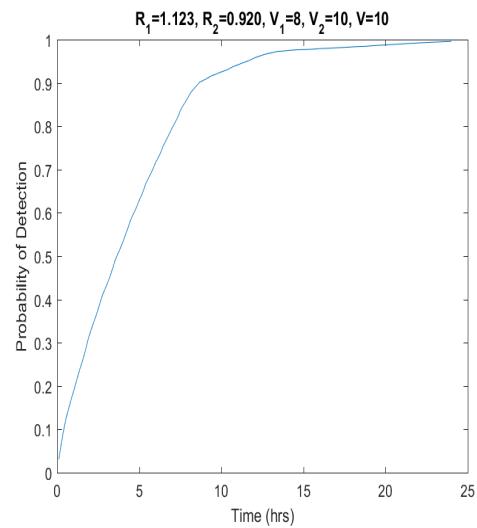


Figure A.480.  $P_D$  vs Time

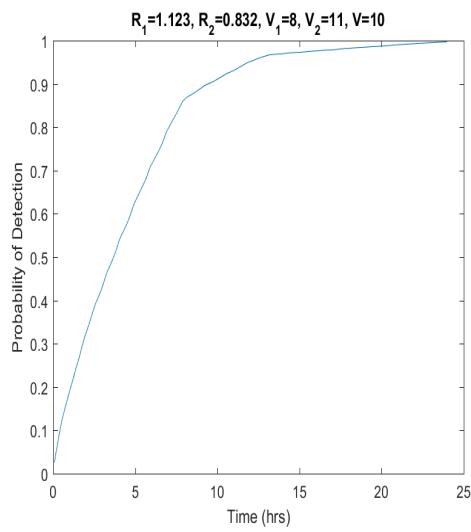


Figure A.481.  $P_D$  vs Time

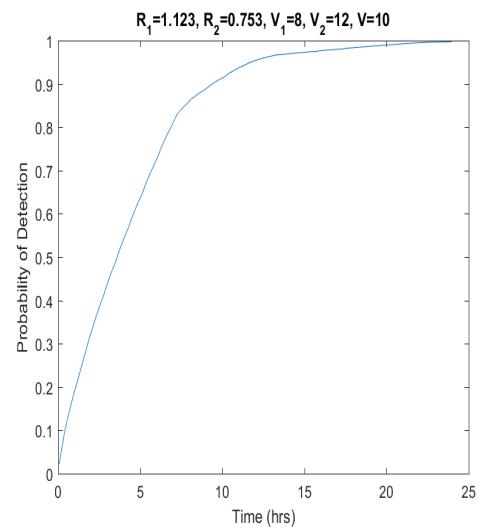


Figure A.482.  $P_D$  vs Time

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## APPENDIX B: Tables

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### B.1 Discrete

Table B.1. Parameter Variation for  $p_i$  for Uniform Distribution

Parameter	$b$	$r^2$	SSE	RMSE
$p_{uniform}, q_{uniform}$	0.01061	0.99242	0.006608	0.027096
$p_{uniform}, q_{long}$	0.009988	0.99935	0.00049844	0.007442
$p_{uniform}, q_{corner}$	0.01198	0.99795	0.0015533	0.013137

Best fit line parameters for a uniformly distributed location probability density of the form  $f(x) = 1 - e^{-bx}$

Table B.2. Parameter Variation for  $p_i$  for Corner Stratification

Parameter	$b$	$r^2$	SSE	RMSE
$p_{cornerstrat}, q_{uniform}$	0.01321	0.99908	0.00065853	0.008554
$p_{cornerstrat}, q_{long}$	0.00896	0.9992	0.00066179	0.0085751
$p_{cornerstrat}, q_{corner}$	0.01119	0.99808	0.0013515	0.012254

Best fit line parameters for a corner stratified distribution location probability density of the form  $f(x) = 1 - e^{-bx}$

Table B.3. Parameter Variation for  $p_i$  for Centered Gaussian

Parameter	$b$	$r^2$	SSE	RMSE
$p_{Gauss}, q_{uniform}$	0.0147	0.99781	0.0014554	0.012717
$p_{Gauss}, q_{long}$	0.01315	0.99609	0.0026167	0.017051
$p_{Gauss}, q_{corner}$	0.01589	0.9963	0.00234	0.016124

Best fit line parameters for a centered Gaussian distribution location probability density of the form  $f(x) = 1 - e^{-bx}$

Table B.4. Parameter Variation for  $p_i$  for Side Gaussian

Parameter	$b$	$r^2$	SSE	RMSE
$p_{Gauss}, q_{uniform}$	0.02405	0.9944	0.0027756	0.017561
$p_{Gauss}, q_{long}$	0.01428	0.99225	0.0047039	0.0228621
$p_{Gauss}, q_{corner}$	0.01895	0.98981	0.0054218	0.024544

Best fit line parameters for a offset Gaussian distribution location probability density of the form  $f(x) = 1 - e^{-bx}$

## B.2 Bayesian

Table B.5. Parameter Variation for  $p_i$  for Uniform Distribution

Parameter	$a$	$b$	$r^2$	SSE	RMSE
$p_{strat}, q_{uniform}$	0.90643	-0.01139	0.99645	0.004106	0.015103
$p_{strat}, q_{long}$	0.89218	-0.011534	0.98964	0.0104	0.024037
$p_{strat}, q_{corner}$	0.8876	-0.014151	0.99384	0.0058881	0.018086

Best fit line parameters for a uniformly distributed location probability density of the form  $f(x) = a - e^{-bx}$

Table B.6. Parameter Variation for  $p_i$  for Corner Stratification

Parameter	$a$	$b$	$r^2$	SSE	RMSE
$p_{strat}, q_{uniform}$	0.88918	-0.018268	0.976	0.0175	0.031181
$p_{strat}, q_{long}$	0.89054	-0.012323	0.98204	0.016665	0.030427
$p_{strat}, q_{corner}$	0.89926	-0.011051	0.99147	0.008804	0.022116

Best fit line parameters for a corner stratified distribution location probability density of the form  $f(x) = a - e^{-bx}$

Table B.7. Parameter Variation for  $p_i$  for Centered Gaussian

Parameter	$a$	$b$	$r^2$	SSE	RMSE
$p_{strat}, q_{uniform}$	0.88971	-0.017384	0.9847	0.012037	0.025859
$p_{strat}, q_{long}$	0.88274	-0.016095	0.98036	0.01587	0.029693
$p_{strat}, q_{corner}$	0.88519	-0.019317	0.98268	0.01265	0.02651

Best fit line parameters for a centered Gaussian distribution location probability density of the form  $f(x) = a - e^{-bx}$

Table B.8. Parameter Variation for  $p_i$  for Side Gaussian

Parameter	$a$	$b$	$r^2$	SSE	RMSE
$p_{strat}, q_{uniform}$	0.88957	-0.029116	0.97623	0.012426	0.026275
$p_{strat}, q_{long}$	0.88243	-0.01768	0.96822	0.023033	0.035772
$p_{strat}, q_{corner}$	0.88301	-0.023588	0.96697	0.019693	0.0330771

Best fit line parameters for a offset Gaussian distribution location probability density of the form  $f(x) = a - e^{-bx}$

## B.3 Continuous

### B.3.1 Inline Ladder Search

Table B.9. Inline Ladder Search: Time to  $P_D = 90\%(V = 5)$

Velocities		Ship 1				
		8	9	10	11	12
Ship 2	8	14.8081	14.5798	15.6685	16.1428	16.6395
	9	15.7111	15.6258	16.6558	16.8813	17.6232
	10	18.5017	18.0867	17.8148	18.2146	18.7484
	11	18.2434	19.5095	19.5958	18.8285	20.0086
	12	18.1569	19.1277	20.5486	21.9053	21.3588

Inline Ladder Search

Table B.10. Inline Ladder Search: Time to  $P_D = 90\%(V = 6)$

Velocities		Ship 1				
		8	9	10	11	12
Ship 2	8	12.7567	12.2523	12.1515	12.4612	13.1644
	9	13.3414	12.6024	12.3187	12.5611	13.1930
	10	14.0466	13.0154	12.6810	13.2800	13.5936
	11	13.7793	15.2770	14.1587	14.0189	14.6239
	12	13.9657	15.0961	16.4268	15.6398	15.5540

Inline Ladder Search

Table B.11. Inline Ladder Search: Time to  $P_D = 90\% (V = 7)$

Velocities		Ship 1				
		8	9	10	11	12
Ship 2	8	10.9647	10.6280	10.4097	10.2503	10.2367
	9	11.7028	11.2848	10.7237	10.5538	10.5661
	10	12.7877	11.5763	11.0378	10.8235	10.7182
	11	12.2397	12.4420	11.3968	11.0259	10.9716
	12	11.8331	12.0358	12.8213	11.4023	11.4783

Inline Ladder Search

Table B.12. Inline Ladder Search: Time to  $P_D = 90\% (V = 8)$

Velocities		Ship 1				
		8	9	10	11	12
Ship 2	8	8.7756	8.6460	8.8498	8.7901	8.8294
	9	10.1282	9.7988	9.2987	9.2396	9.0239
	10	11.4149	10.2606	9.8759	9.5892	9.3740
	11	11.1874	11.3872	10.2884	9.8888	9.6073
	12	10.9687	11.0076	11.0854	10.1519	9.7240

Inline Ladder Search

Table B.13. Inline Ladder Search: Time to  $P_D = 90\% (V = 9)$

Velocities		Ship 1				
		8	9	10	11	12
Ship 2	8	6.8662	6.9782	7.1117	7.5250	7.7083
	9	7.5069	7.8241	7.7043	7.8639	7.9255
	10	9.7362	9.0589	8.7202	8.4741	8.3054
	11	10.1011	10.0333	9.2198	8.9486	8.6854
	12	10.0425	10.1511	10.0968	9.2826	8.9569

Inline Ladder Search

Table B.14. Inline Ladder Search: Time to  $P_D = 90\% (V = 10)$

Velocities		Ship 1				
		8	9	10	11	12
Ship 2	8	6.6247	6.4570	6.3438	6.4058	6.5204
	9	6.5861	6.3278	6.3438	6.4924	6.8038
	10	7.2954	6.9782	7.0840	7.0118	7.1582
	11	9.1759	8.6565	8.2237	7.8774	7.6543
	12	9.2135	9.2844	9.0009	8.3631	8.1504

Inline Ladder Search

### B.3.2 Multipath Ladder

### B.3.3 Multipath Ladder Search

Table B.15. Multipath Ladder Search: Time to  $P_D = 90\% (V = 5)$

Velocities		Ship 1				
		8	9	10	11	12
Ship 2	8	14.6171	14.9854	15.6398	16.1907	16.5752
	9	14.9427	15.8392	16.3410	16.9964	17.6940
	10	18.2155	17.4714	17.5716	18.2338	18.7098
	11	18.2913	19.0874	18.5983	18.9148	19.9443
	12	18.3240	19.1792	20.6129	20.6580	21.1080

Multipath Ladder Search

Table B.16. Multipath Ladder Search: Time to  $P_D = 90\% (V = 6)$

Velocities		Ship 1				
		8	9	10	11	12
Ship 2	8	12.7024	12.2912	12.2072	13.0803	13.1644
	9	12.9135	12.4857	12.4580	12.9206	13.2073
	10	13.7122	12.7089	12.7089	13.2600	13.7510
	11	14.0389	14.0988	13.6395	14.2585	14.5953
	12	13.9371	15.6255	15.8545	15.2106	15.5254

Multipath Ladder Search

Table B.17. Multipath Ladder Search: Time to  $P_D = 90\% (V = 7)$

Velocities		Ship 1				
		8	9	10	11	12
Ship 2	8	11.4414	10.9266	10.4994	10.2503	10.3888
	9	11.8222	11.1654	10.7686	10.6212	10.7942
	10	12.3390	11.3070	10.9481	10.8235	11.0729
	11	12.4083	11.7002	11.1270	10.8910	11.3263
	12	12.0611	12.3145	11.5037	11.0729	11.630

Multipath Ladder Search

Table B.18. Multipath Ladder Search: Time to  $P_D = 90\% (V = 8)$

Velocities		Ship 1				
		8	9	10	11	12
Ship 2	8	10.1501	9.7165	9.2987	9.0398	8.9850
	9	10.7870	10.2105	9.7476	9.2895	9.1406
	10	11.2226	10.5813	9.9400	9.5392	9.45180
	11	11.5370	10.7379	10.0886	9.7890	9.6073
	12	11.0854	11.2021	10.3464	9.8407	9.6462

Multipath Ladder Search

Table B.19. Multipath Ladder Search: Time to  $P_D = 90\%$  ( $V = 9$ )

Velocities		Ship 1				
		8	9	10	11	12
Ship 2	8	8.1866	8.2470	8.2123	8.1351	8.0883
	9	9.3043	8.9871	8.7202	8.3385	8.1969
	10	10.3288	9.5669	9.2282	8.7452	8.4140
	11	10.7790	10.0333	9.4231	9.0164	8.6854
	12	10.4768	10.5311	9.7168	9.1197	8.7940

Multipath Ladder Search

Table B.20. Multipath Ladder Search: Time to  $P_D = 90\%$  ( $V = 10$ )

Velocities		Ship 1				
		8	9	10	11	12
Ship 2	8	6.7825	6.8444	6.7668	7.0983	7.0873
	9	6.9735	7.4901	7.4011	7.5312	7.3708
	10	8.7756	8.4585	8.2470	7.8774	7.6543
	11	9.9550	9.1759	8.6565	8.3102	8.0087
	12	9.8514	9.8514	9.0009	8.5757	8.221

Multipath Ladder Search

### B.3.4 Spiral

### B.3.5 Spiral Search

Table B.21. Spiral Search: Time to  $P_D = 90\% (V = 5)$

Velocities		Ship 1				
		8	9	10	11	12
Ship 2	8	19.0117	19.4895	19.5176	19.7109	20.2015
	9	21.6669	20.4074	20.8340	20.8907	20.0729
	10	22.2220	23.1092	21.5638	21.5909	21.67388
	11	21.8115	22.8090	25.0000	22.5980	22.65114
	12	21.7960	22.4003	23.4676	25.0000	25.0000

Spiral Search

Table B.22. Spiral Search: Time to  $P_D = 90\% (V = 6)$

Velocities		Ship 1				
		8	9	10	11	12
Ship 2	8	13.8424	14.3527	14.3253	14.5182	16.0405
	9	16.0252	15.7918	16.2205	16.5751	15.0818
	8	18.3108	17.6140	17.1124	17.3739	17.5859
	11	18.2326	18.7518	18.6519	18.2126	18.1725
	12	18.3013	18.9309	19.5462	19.8467	18.9309

Spiral Search

Table B.23. Spiral Search: Time to  $P_D = 90\% (V = 7)$

Velocities		Ship 1				
		8	9	10	11	12
Ship 2	8	11.5209	11.8819	12.1147	12.1723	12.5932
	9	11.7625	12.0013	12.3839	12.1723	11.4023
	10	15.1658	13.7749	13.5505	13.7233	14.0122
	11	15.0720	15.8476	15.0383	14.8697	15.1017
	12	14.8737	16.1153	16.5460	15.9886	15.7099

Spiral Search

Table B.24. Spiral Search: Time to  $P_D = 90\% (V = 8)$

Velocities		Ship 1				
		8	9	10	11	12
Ship 2	8	10.3616	10.2929	10.9019	11.0875	11.2021
	9	10.4576	10.2929	10.6454	10.6879	10.5409
	10	11.8638	10.5813	10.6454	10.7878	10.6575
	11	12.2362	13.2850	11.9365	11.8866	11.7855
	12	12.0189	13.6137	13.9637	13.1469	12.9524

Spiral Search

Table B.25. Spiral Search: Time to  $P_D = 90\% (V = 9)$

Velocities		Ship 1				
		8	9	10	11	12
Ship 2	8	9.6390	9.3043	9.6515	10.2367	10.3139
	9	9.8330	9.4100	9.3129	9.7621	9.8254
	10	10.4135	9.4822	9.3129	9.5587	9.6625
	11	10.5078	10.0333	9.5587	9.6943	9.71684
	12	10.3682	11.6710	11.5625	10.6397	10.5854

Spiral Search

Table B.26. Spiral Search: Time to  $P_D = 90\% (V = 10)$

Velocities		Ship 1				
		8	9	10	11	12
Ship 2	8	9.3062	8.6524	8.6699	9.5221	9.4262
	9	9.2981	8.7815	8.5642	8.8296	9.2135
	10	9.6215	8.8814	8.5642	8.5699	8.8592
	11	9.6087	9.1759	8.5699	8.6565	8.7883
	12	9.3553	9.7805	9.0009	8.7174	8.788

Spiral Search

---

## APPENDIX C: Codes

---

### C.1 Discrete

#### C.1.1 Main Code

```
1 clc
2 clear
3 discrete_search_prob
4 discrete_loc_probs2
5 close all
6 %p-uniform
7 dis_pUni_qUni
8 dis_pUni_qlongstrat
9 dis_pUni_qbothstrat
10 %p-long stratified
11 dis_plongstrat_qUni
12 dis_plongstrat_qlongstrat
13 dis_plongstrat_qbothstrat
14 %p-both stratified
15 dis_pbothstrat_qUni
16 dis_pbothstrat_qlongstrat
17 dis_pbothstrat_qbothstrat
18 %p-normal center
19 dis_pnormwide_qUni
20 dis_normwide_qlongstrat
21 dis_normwide_qbothstrat
22 %p-normal side
23 dis_pnormside_qUni
24 dis_normside_qlongstrat
25 dis_normside_qbothstrat
```

```

26 %p-normal corner
27 dis_pnormc_qUni
28 dis_normc_qlongstrat
29 dis_normc_qbothstrat
30 %%%
31
32 pd=[Pd_uni_uni(end),Pd_uni_ls(end),Pd_uni_bs(end),Pd_ls_uni(
33     end),...
34     Pd_ls_ls(end),Pd_ls_bs(end),Pd_bs_uni(end),Pd_bs_ls(end)
35     ,Pd_bs_bs(end)...
36     ,Pd_nw_uni(end),Pd_nw_ls(end),Pd_nw_bs(end),Pd_ns_uni(
37     end),...
38     Pd_ns_ls(end),Pd_ns_bs(end),Pd_nc_uni(end),Pd_nc_ls(end)
39     ,...
40     Pd_nc_bs(end)]
41
42 %%%
43 % close all
44 set(0,'defaulttextinterpreter','latex')
45 clear g
46 [~,~,~,Pd_uni_uni,k] = discrete_search2(P_uni,q_uni,500, 1)
47 ;
48 [~,~,~,Pd_uni_ls,k] = discrete_search2(P_uni,q_long_strat
49 ,500, 1 );
50 [~,~,~,Pd_uni_bs,k] = discrete_search2(P_uni,q_both_strat
51 ,500, 1 );
52 figure
53
54 step1=10:round(.05*length(Pd_uni_uni)):length(Pd_uni_uni);
55 step2=10:round(.05*length(Pd_uni_ls)):length(Pd_uni_ls);
56 step3=10:round(.05*length(Pd_uni_bs)):length(Pd_uni_bs);
57 plot(step1,Pd_uni_uni(step1),'xk',step2,Pd_uni_ls(step2),'xr'
58     ,step3,Pd_uni_bs(step3),'xg')

```

```

51 g = fittype('1-exp(b*x)', 'dependent', 'Prob', 'independent', 'x
   );
52
53 [fit_bb1, gof_bb1] = fit(step1', Pd_uni_uni(step1)', g, '
   StartPoint',[0], 'Upper',[0]);
54 [fit_bb2, gof_bb2] = fit(step2', Pd_uni_ls(step2)', g, 'StartPoint
  ',[0], 'Upper',[0]);
55 [fit_bb3, gof_bb3] = fit(step3', Pd_uni_bs(step3)', g, 'StartPoint
  ',[0], 'Upper',[0]);
56 hold on
57 plot(fit_bb1, 'k')
58 plot(fit_bb2, 'r')
59 plot(fit_bb3, 'g')
60 xlabel('n')
61 ylabel('$P_{\{D\}}$')
62 title('$P_{\{D\}}$ vs Number of Glimpses and Exponential Fitted
   Curves')
63 legend('(p_{uniform},q_{uniform})', '(p_{uniform},q_{strat a
   })', '(p_{uniform},q_{strat b})',...
64 , sprintf('1-e^{%.5f*x}', fit_bb1.b), sprintf('1-e^{%.5f*x}
   ', fit_bb2.b), ...
65 , sprintf('1-e^{%.5f*x}', fit_bb3.b), 'location', 'best')
66 print('Bayes_Uni_Fit', '-dpng')
67
68 [~,~,~,Pd_ls_uni,k] = discrete_search2(P_long_strat,q_uni
   ,500, 1);
69 [~,~,~,Pd_ls_ls,k] = discrete_search2(P_long_strat,
   q_long_strat,500, 1);
70 [~,~,~,Pd_ls_bs,k] = discrete_search2(P_long_strat,
   q_both_strat,500, 1);
71 figure
72 step1=10:round(.05*length(Pd_ls_uni)):length(Pd_ls_uni);
73 step2=10:round(.05*length(Pd_ls_ls)):length(Pd_ls_ls);

```

```

74 step3=10:round(.05*length(Pd_ls_bs)):length(Pd_ls_bs);
75 plot(step1,Pd_ls_uni(step1),'xk',step2,Pd_ls_ls(step2),'xr',
76 step3,Pd_ls_bs(step3),'xg')
77 [fit_bb4,gof_bb4]=fit(step1',Pd_ls_uni(step1)',g,'StartPoint'
78 ,[0],'Upper',[0]);
78 [fit_bb5,gof_bb5]=fit(step2',Pd_ls_ls(step2)',g,'StartPoint'
79 ,[0],'Upper',[0]);
79 [fit_bb6,gof_bb6]=fit(step3',Pd_ls_bs(step3)',g,'StartPoint'
80 ,[0],'Upper',[0]);
80 hold on
81 plot(fit_bb4,'k')
82 plot(fit_bb5,'r')
83 plot(fit_bb6,'g')
84 xlabel('n')
84 ylabel('$P_{\{D\}}$')
85 title('$P_{\{D\}}$ vs Number of Glimpses and Exponential Fitted
86 Curves')
86 legend('(p_{strat a},q_{uniform})',(p_{strat a},q_{strat a
87 }),(p_{strat a},q_{strat b})',...
87 ,sprintf('1-e^{%.5f*x}',fit_bb4.b),sprintf('1-e^{%.5f*x}'
88 ,fit_bb5.b),...
88 sprintf('1-e^{%.5f*x}',fit_bb6.b),'location','best')
89 print('Bayes_Long_Fit',' -dpng')
90
91 [~,~,~,Pd_bs_uni,k] = discrete_search2(P_both_strat,q_uni
92 ,500, 1 );
92 [~,~,~,Pd_bs_ls ,k] = discrete_search2(P_both_strat ,
93 q_long_strat,500, 1 );
93 [~,~,~,Pd_bs_bs ,k] = discrete_search2(P_both_strat ,
94 q_both_strat,500, 1 );
94 figure
95 step1=10:round(.05*length(Pd_bs_uni)):length(Pd_bs_uni);
96 step2=10:round(.05*length(Pd_bs_ls)):length(Pd_bs_ls);

```

```

97 step3=10:round(.05*length(Pd_bs_bs)):length(Pd_bs_bs);
98 plot(step1,Pd_bs_uni(step1),'xk',step2,Pd_bs_ls(step2),'xr',
99 step3,Pd_bs_bs(step3),'xg')
100 [fit_bb7,gof_bb7]=fit(step1',Pd_bs_uni(step1)',g,'StartPoint'
101 ,[0],'Upper',[0]);
102 [fit_bb8,gof_bb8]=fit(step2',Pd_bs_ls(step2)',g,'StartPoint'
103 ,[0],'Upper',[0]);
104 [fit_bb9,gof_bb9]=fit(step3',Pd_bs_bs(step3)',g,'StartPoint'
105 ,[0],'Upper',[0]);
106 hold on
107 plot(fit_bb7,'k')
108 plot(fit_bb8,'r')
109 plot(fit_bb9,'g')
110 xlabel('n')
111 ylabel('$P_{\{D\}}$')
112 title('$P_{\{D\}}$ vs Number of Glimpses and Exponential Fitted
113 Curves')
114 legend('(p_{strat b},q_{uniform})',(p_{strat b},q_{strat a}
115 ),'(p_{strat b},q_{strat b})',...
116 ,sprintf('1-e^{%.5f*x}',fit_bb7.b),sprintf('1-e^{%.5f*x}'
117 ,fit_bb8.b),...
118 ,sprintf('1-e^{%.5f*x}',fit_bb9.b),'location','best')
119 print('Bayes_Both_Fit','-dpng')
120
121 [~,~,~,Pd_nw_uni,k]=discrete_search2(P_norm_wide,q_uni
122 ,500,1);
123 [~,~,~,Pd_nw_ls,k]=discrete_search2(P_norm_wide,
124 q_long_strat,500,1);
125 [~,~,~,Pd_nw_bs,k]=discrete_search2(P_norm_wide,
126 q_both_strat,500,1);
127 figure
128 step1=10:round(.05*length(Pd_nw_uni)):length(Pd_nw_uni);
129 step2=10:round(.05*length(Pd_nw_ls)):length(Pd_nw_ls);

```

```

120 step3=10:round(.05*length(Pd_nw_bs)):length(Pd_nw_bs);
121 plot(step1,Pd_nw_uni(step1),'xk',step2,Pd_nw_ls(step2),'xr',
122 step3,Pd_nw_bs(step3),'xg')
123 [fit_bb10,gof_bb10]=fit(step1',Pd_nw_uni(step1)',g,'
124 StartPoint',[0],'Upper',[0]);
125 [fit_bb11,gof_bb11]=fit(step2',Pd_nw_ls(step2)',g,'
126 StartPoint',[0],'Upper',[0]);
127 [fit_bb12,gof_bb12]=fit(step3',Pd_nw_bs(step3)',g,'
128 StartPoint',[0],'Upper',[0]);
129 hold on
130 plot(fit_bb10,'k')
131 plot(fit_bb11,'r')
132 plot(fit_bb12,'g')
133 xlabel('n')
134 ylabel('$P_{\{D\}}$')
135 title('$P_{\{D\}}$ vs Number of Glimpses and Exponential Fitted
136 Curves')
137 legend('(p_{norm},q_{uniform})',(p_{norm},q_{strat a}),(p_{norm},q_{strat b})',...
138 ,sprintf('1-e^{%.5f*x}',fit_bb10.b),sprintf('1-e^{%.5f*x
139 }',fit_bb11.b),...
140 sprintf('1-e^{%.5f*x}',fit_bb12.b),'location','best')
141 print('Bayes_Norm_Fit','-dpng')
142
143 [~,~,~,Pd_ns_uni,k]=discrete_search2(P_norm_side,q_uni
144 ,500,1);
145 [~,~,~,Pd_ns_ls,k]=discrete_search2(P_norm_side,
146 q_long_strat,500,1);
147 [~,~,~,Pd_ns_bs,k]=discrete_search2(P_norm_side,
148 q_both_strat,500,1);
149 figure
150 step1=10:round(.05*length(Pd_ns_uni)):length(Pd_ns_uni);
151 step2=10:round(.05*length(Pd_ns_ls)):length(Pd_ns_ls);

```

```

143 step3=10:round(.05*length(Pd_ns_bs)):length(Pd_ns_bs);
144 plot(step1,Pd_ns_uni(step1),'xk',step2,Pd_ns_ls(step2),'xr',
145 step3,Pd_ns_bs(step3),'xg')
146 [fit_bb13,gof_bb13]=fit(step1',Pd_ns_uni(step1)',g,'
147 StartPoint',[0],'Upper',[0]);
148 [fit_bb14,gof_bb14]=fit(step2',Pd_ns_ls(step2)',g,'
149 StartPoint',[0],'Upper',[0]);
150 [fit_bb15,gof_bb15]=fit(step3',Pd_ns_bs(step3)',g,'
151 StartPoint',[0],'Upper',[0]);
152 hold on
153 plot(fit_bb13,'k')
154 plot(fit_bb14,'r')
155 plot(fit_bb15,'g')
156 xlabel('n')
157 ylabel('$P_{\{D\}}$')
158 title('$P_{\{D\}}$ vs Number of Glimpses and Exponential Fitted
159 Curves')
160 legend('(p_{norm b},q_{uniform})',(p_{norm b},q_{strat a})',
161 '(p_{norm b},q_{strat b})',...
162 ,sprintf('1-e^{%.5f*x}',fit_bb13.b),sprintf('1-e^{%.5f*x
163 }',fit_bb14.b),...
164 sprintf('1-e^{%.5f*x}',fit_bb15.b),'location','best')
165 print('Bayes_NormSide_Fit','-dpng')
166
167 [~,~,~,Pd_nc_uni,k] = discrete_search2(P_norm_corner,q_uni
168 ,500, 1 );
169 [~,~,~,Pd_nc_ls ,k] = discrete_search2(P_norm_corner,
170 q_long_strat,500, 1 );
171 [~,~,~,Pd_nc_bs ,k] = discrete_search2(P_norm_corner,
172 q_both_strat,500, 1 );
173 figure
174 step1=10:round(.05*length(Pd_nc_uni)):length(Pd_nc_uni);
175 step2=10:round(.05*length(Pd_nc_ls)):length(Pd_nc_ls);

```

```

166 step3=10:round(.05*length(Pd_nc_bs)):length(Pd_nc_bs)
167 plot(step1,Pd_nc_uni(step1),'xk',step2,Pd_nc_ls(step2),'xr',
168 step3,Pd_nc_bs(step3),'xg')
169 [fit_bb16,gof_bb16]=fit(step1',Pd_nc_uni(step1)',g,'
170 StartPoint',[0],'Upper',[0]);
171 [fit_bb17,gof_bb17]=fit(step2',Pd_nc_ls(step2)',g,'
172 StartPoint',[0],'Upper',[0]);
173 [fit_bb18,gof_bb18]=fit(step3',Pd_nc_bs(step3)',g,'
174 StartPoint',[0],'Upper',[0]);
175 hold on
176 plot(fit_bb16,'k')
177 plot(fit_bb17,'r')
178 plot(fit_bb18,'g')
179 xlabel('n')
180 ylabel('$P_{\{D\}}$')
181 title('$P_{\{D\}}$ vs Number of Glimpses and Exponential Fitted
182 Curves')
183 legend('(p_{norm c},q_{uniform})',(p_{norm c},q_{strat a})',
184 ,(p_{norm c},q_{strat b})...
185 ,sprintf('1-e^{%.5f*x}',fit_bb16.b),sprintf('1-e^{%.5f*x
186 }',fit_bb17.b),...
187 sprintf('1-e^{%.5f*x}',fit_bb18.b),'location','best')
188 print('Bayes_NormCorner_Fit','dpng')
189 format shortG
190 b=[fit_bb1.b,fit_bb2.b,fit_bb3.b,fit_bb4.b,fit_bb5.b,fit_bb6
191 .b,fit_bb7.b, ...
192 fit_bb8.b,fit_bb9.b,fit_bb10.b,fit_bb11.b,fit_bb12.b,
193 fit_bb13.b,fit_bb14.b, ...
194 fit_bb15.b,fit_bb16.b,fit_bb17.b,fit_bb18.b]
195 r2=[gof_bb1.rsquare,gof_bb2.rsquare,gof_bb3.rsquare,gof_bb4.
196 rsquare, ...
197 gof_bb5.rsquare,gof_bb6.rsquare,gof_bb7.rsquare,gof_bb8.
198 rsquare, ...

```

```

188      gof_bb9.rsquare, gof_bb10.rsquare, gof_bb11.rsquare,
189      gof_bb12.rsquare, ...
190      gof_bb13.rsquare, gof_bb14.rsquare, gof_bb15.rsquare,
191      gof_bb16.rsquare, ...
192      gof_bb17.rsquare, gof_bb18.rsquare]

```

### C.1.2 Search Probabilities $q_i$

```

1 format short
2 format compact
3 Area=1000;
4 x=-5:5;y=x;
5 [X,Y]=meshgrid(x,y);
6 %
7 q0=.3;
8 q_uni=q0*ones(10,10);
9
10 figure
11 subplot(1,3,1)
12 imagesc([-4.5,4.5],[-4.5,4.5],q_uni,[0,1])
13 set(gca, 'YDir', 'normal')
14 cb_q=colorbar;
15 title(cb_q,'q_{i}')
16 hold on
17 plot(X,Y,'k');
18 plot(Y,X,'k');
19 title('Uniform')
20 xlabel('Long')
21 ylabel('Lat')
22 %
23 %
24 q0=.07;
25 qx=1:10;
26 qy=ones(10,1);

```

```

27 q_long_strat=qy*qx*q0*.9;
28 subplot(1,3,2)
29 imagesc([-4.5,4.5],[-4.5,4.5],q_long_strat,[0,max(
    q_long_strat(:))])
30 set(gca, 'YDir', 'normal')
31 cb_q=colorbar;
32 title(cb_q,'q_{i}')
33 hold on
34 plot(X,Y,'k');
35 plot(Y,X,'k');
36 title('Strat. a')
37 xlabel('Long')
38 ylabel('Lat')
39 %
40 q0=.01;
41 qx=(1:10);
42 qy=(1:10)';
43 q_both_strat=qy*qx*q0*.9;
44 subplot(1,3,3)
45 imagesc([-4.5,4.5],[-4.5,4.5],q_both_strat,[0,max(
    q_both_strat(:))])
46 set(gca, 'YDir', 'normal')
47 cb_q=colorbar;
48 title(cb_q,'q_{i}')
49 hold on
50 plot(X,Y,'k');
51 plot(Y,X,'k');
52 title('Strat. b')
53 xlabel('Long')
54 ylabel('Lat')
55 %
56 print('Discrete_Miss_Probablities','-dpng')

```

### C.1.3 Location Probabilities $p_i$

```
1 % clear
2 close all
3 clc
4 format short
5 format compact
6 Area=1000;
7 %create the grid positions
8 p0=0;
9 x=-5:5;y=x;
10 [X,Y]=meshgrid(x,y);
11 %
12 P_uni=ones(10,10);
13 P_uni=(1-p0)*P_uni/sum(P_uni(:));
14 figure
15 subplot(2,3,1)
16 imagesc([-4.5,4.5],[-4.5,4.5],P_uni,[0,1])
17 set(gca, 'YDir', 'normal')
18 cb_p=colorbar;
19 title(cb_p, 'p_{i}')
20 hold on
21 plot(X,Y, 'k');
22 plot(Y,X, 'k');
23 title('Uniform')
24 xlabel('Long.')
25 ylabel('Lat')
26 %
27 Px=1:10;
28 Py=ones(10,1);
29 P_long_strat=Py*Px;
30 P_long_strat=(1-p0)*P_long_strat/sum(P_long_strat(:));
31 subplot(2,3,2)
32 imagesc([-4.5,4.5],[-4.5,4.5],P_long_strat,[0,max(
```

```

        P_long_strat(:)))])
33 set(gca, 'YDir', 'normal')
34 cb_p=colorbar;
35 title(cb_p, 'p_{i}')
36 hold on
37 plot(X,Y, 'k');
38 plot(Y,X, 'k');
39 title('Stratified a')
40 xlabel('Long.')
41 ylabel('Lat')
42 %%
43 Px=1:10;
44 Py=(1:10)';
45 P_both_strat=Py*Px;
46 P_both_strat=(1-p0)*P_both_strat/sum(P_both_strat(:));
47 % figure
48 subplot(2,3,3)
49 imagesc([-4.5,4.5],[-4.5,4.5],P_both_strat,[0,max(
    P_both_strat(:))])
50 set(gca, 'YDir', 'normal')
51 cb_p=colorbar;
52 title(cb_p, 'p_{i}')
53 hold on
54 plot(X,Y, 'k');
55 plot(Y,X, 'k');
56 title('Stratified b')
57 xlabel('Long.')
58 ylabel('Lat')
59 %%
60 Px = normpdf([-4.5:1:4.5],0,2);
61 Py = normpdf([-4.5:1:4.5],0,5)';
62 P_norm_wide=Py*Px;
63 P_norm_wide=(1-p0)*P_norm_wide/sum(P_norm_wide(:));

```

```

64 subplot(2,3,4)
65 imagesc([-4.5,4.5],[-4.5,4.5],P_norm_wide,[0,max(P_norm_wide
   (:))])
66 set(gca, 'YDir', 'normal')
67 cb_p=colorbar;
68 title(cb_p,'p_{i}')
69 hold on
70 plot(X,Y,'k');
71 plot(Y,X,'k');
72 title('Normal a')
73 xlabel('Long.')
74 ylabel('Lat')
75 %%%
76 Px = normpdf([-4.5:1:4.5],3,2);
77 Py = normpdf([-4.5:1:4.5],3,2)';
78 P_norm_corner=Py*Px;
79 P_norm_corner=(1-p0)*P_norm_corner/sum(P_norm_corner(:));
80 subplot(2,3,5)
81 imagesc([-4.5,4.5],[-4.5,4.5],P_norm_corner,[0,max(
   P_norm_corner(:))])
82 set(gca, 'YDir', 'normal')
83 cb_p=colorbar;
84 title(cb_p,'p_{i}')
85 hold on
86 plot(X,Y,'k');
87 plot(Y,X,'k');
88 title('Normal b')
89 xlabel('Long.')
90 ylabel('Lat')
91 %%
92 Px = normpdf([-4.5:1:4.5],3,2);
93 Py = normpdf([-4.5:1:4.5],0,2)';
94 P_norm_side=Py*Px;

```

```

95 P_norm_side=(1-p0)*P_norm_side / sum( P_norm_side (:)) ;
96 subplot(2,3,6)
97 imagesc([-4.5,4.5],[-4.5,4.5],P_norm_side,[0,max(P_norm_side
98 (:))])
99 set(gca, 'YDir', 'normal')
100 cb_p=colorbar;
101 title(cb_p, 'p_{i}')
102 hold on
103 plot(X,Y, 'k');
104 plot(Y,X, 'k');
105 title('Normal c')
106 xlabel('Long.')
107 ylabel('Lat')
108 %%%
109 print('Discrete_Location_Probablities', '-dpng')

```

#### C.1.4 Sample code for the Uniform/Uniform case

```

1 [ pc ,n ,imax ,Pd_uni_uni ,k] = discrete_search2(P_uni ,q_uni
2 ,50 , .95 );
3 figure
4 subplot(2,2,1)
5 imagesc([-4.5,4.5],[-4.5,4.5],pc,[0,max(pc(:))])
6 set(gca, 'YDir', 'normal')
7 cb_p=colorbar;
8 title(cb_p, 'p_{i}')
9 hold on
10 plot(X,Y, 'k');
11 plot(Y,X, 'k');
12 title('Remainder Prob.')
13 xlabel('Long.')
14 ylabel('Lat.')
15 subplot(2,2,2)

```

```

16 imagesc([-4.5,4.5],[-4.5,4.5],n,[0,max(n(:))])
17 set(gca, 'YDir', 'normal')
18 cb_p=colorbar;
19 title(cb_p,'N')
20 hold on
21 plot(X,Y,'k');
22 plot(Y,X,'k');
23 title('Actual Search')
24 xlabel('Long.')
25 ylabel('Lat.')
26 subplot(2,2,3)
27 imagesc([-4.5,4.5],[-4.5,4.5],P_uni,[0,max(P_uni(:))])
28 set(gca, 'YDir', 'normal')
29 cb_p=colorbar;
30 title(cb_p,'p_{i}')
31 hold on
32 plot(X,Y,'k');
33 plot(Y,X,'k');
34 title('Uniform')
35 xlabel('Long.')
36 ylabel('Lat.')
37
38
39 subplot(2,2,4)
40 imagesc([-4.5,4.5],[-4.5,4.5],q_uni,[0,max(q_uni(:))])
41 set(gca, 'YDir', 'normal')
42 cb_q=colorbar;
43 title(cb_q,'q_{i}')
44 hold on
45 plot(X,Y,'k');
46 plot(Y,X,'k');
47 title('Uniform')
48 xlabel('Long.')

```

```

49 ylabel('Lat.')
50 print('Discrete_uni_uni', '-dpng')

```

### C.1.5 Discrete Search Function

```

1 function [ pd ,n ,imax ,Pd_cum2 ,k ] = discrete_search2 ( p ,q ,nT ,
    stop )
2 %Conducts a Discrete Search base on inputs .
3 %p=Location probability density
4 %q=Conditional miss probabilit density
5 %nT=max number of glimpses
6 %stop=Probability of Detection to stop algorithm
7 %Outputs
8 %pd=updated location probability density
9 %n=search matrix
10 %imax=Last box searched
11 %Pd_cum=total probabiliy of detection
12 %k=number of glimpses
13 if ~exist('stop')
14     stop=sum(p(:));
15 elseif stop>sum(p(:))
16     stop=sum(p(:));
17 end
18 p0=1-sum(p(:));
19 n=zeros( size(p) );%initialize look counter
20 Pd_cum=0;
21 Pd_cum2=Pd_cum;
22 pc=p;
23 pc_0=p0;
24 k=1;
25 while k<=nT && Pd_cum2( end )<stop
26     pd=pc .*(1-q) .*q.^n;
27     [pmax ,imax(k) ]=max( pd( : ) );
28     pmax;

```

```

29      [ irow , icol ]=ind2sub( size( pd ) , imax( k ) );
30      n( irow , icol )=n( irow , icol )+1;
31      Pd_cum2( k+1 )=Pd_cum2( k )+pmax ;
32      k=k+1 ;
33  end
34  pd=pc .*( 1 - q ) .* q .^ n ;
35  end

```

## C.2 Bayesian

### C.2.1 Main Code

```

1  clc
2  clear all
3  bayes_search_prob
4  bayes_loc_probs2
5  close all
6  %p-uniform
7  bayes_pUni_qUni
8  p0_uni_uni=pc_0
9  bayes_pUni_qlongstrat
10 p0_uni_ls=pc_0
11 bayes_pUni_qbothstrat
12 p0_uni_bs=pc_0
13 %p-long stratified
14 bayes_plongstrat_qUni
15 p0_ls_uni=pc_0
16 bayes_plongstrat_qlongstrat
17 p0_ls_ls=pc_0
18 bayes_plongstrat_qbothstrat
19 p0_ls_bs=pc_0
20 %p-both stratified
21 bayes_pbothstrat_qUni

```

```

22 p0_bs_uni=pc_0
23 bayes_pbothstrat_qlongstrat
24 p0_bs_ls=pc_0
25 bayes_pbothstrat_qbothstrat
26 p0_bs_bs=pc_0
27 %p-normal center
28 bayes_pnormwide_qUni
29 p0_nw_uni=pc_0
30 bayes_normwide_qlongstrat
31 p0_nw_ls=pc_0
32 bayes_normwide_qbothstrat
33 p0_nw_bs=pc_0
34 %p-normal side
35 bayes_pnormside_qUni
36 p0_ns_uni=pc_0
37 bayes_normside_qlongstrat
38 p0_ns_ls=pc_0
39 bayes_normside_qbothstrat
40 p0_ns_bs=pc_0
41 %p-normal corner
42 bayes_pnormc_qUni
43 p0_nc_uni=pc_0
44 bayes_normc_qlongstrat
45 p0_nc_uni=pc_0
46 bayes_normc_qbothstrat
47 p0_nc_bs=pc_0
48
49 pd=[ Pd_uni_uni(end) ,Pd_uni_ls(end) ,Pd_uni_bs(end) ,Pd_ls_uni(end) ,...
50 Pd_ls_ls(end) ,Pd_ls_bs(end) ,Pd_bs_uni(end) ,Pd_bs_ls(end)
51 ,Pd_bs_bs(end) ...
51 ,Pd_nw_uni(end) ,Pd_nw_ls(end) ,Pd_nw_bs(end) ,Pd_ns_uni(end) ,...

```

```

52      Pd_ns_ls(end),Pd_ns_bs(end),Pd_nc_uni(end),Pd_nc_ls(end)
53      ,...
53      Pd_nc_bs(end)]
54 %%%
55
56 close all
57 set(0,'defaulttextinterpreter','latex')
58 [ ~,~,~,Pd_uni_uni,k,~,~] = bayes_search(P_uni,q_uni,500,
59 .95 );
59 [ ~,~,~,Pd_uni_ls,k,~,~] = bayes_search(P_uni,q_long_strat
60 ,500,.95 );
60 [ ~,~,~,Pd_uni_bs,k,~,~] = bayes_search(P_uni,q_both_strat
61 ,500,.95 );
61 figure
62 step1=10:round(.05*length(Pd_uni_uni)):length(Pd_uni_uni);
63 step2=10:round(.05*length(Pd_uni_ls)):length(Pd_uni_ls);
64 step3=10:round(.05*length(Pd_uni_bs)):length(Pd_uni_bs);
65 plot(step1,Pd_uni_uni(step1),'xk',step2,Pd_uni_ls(step2),'xr
66 ',step3,Pd_uni_bs(step3),'xg')
66 g = fittype('a-exp(b*x)','dependent','Prob','independent','x
67 ');
67
68 [fit_bb1,gof_bb1]=fit(step1',Pd_uni_uni(step1)',g,'
69 StartPoint',[0,0],'Upper',[1,0]);
69 [fit_bb2,gof_bb2]=fit(step2',Pd_uni_ls(step2)',g,'StartPoint
70',[0,0],'Upper',[1,0]);
70 [fit_bb3,gof_bb3]=fit(step3',Pd_uni_bs(step3)',g,'StartPoint
71',[0,0],'Upper',[1,0]);
71 hold on
72 plot(fit_bb1,'k')
73 plot(fit_bb2,'r')
74 plot(fit_bb3,'g')
75 xlabel('n')

```

```

76 ylabel('$P_{\{D\}}$')
77 ylim([0,1])
78 title('$P_{\{D\}}$ vs Number of Glimpses and Exponential Fitted
    Curves')
79 legend('(p_{uniform},q_{uniform})',(p_{uniform},q_{strat a
    }),'(p_{uniform},q_{strat b})',...
80 ,sprintf('%.5f-e^{%.5f*x}',fit_bb1.a,fit_bb1.b),sprintf(
    '%.5f-e^{%.5f*x}',fit_bb2.a,fit_bb2.b),...
81 sprintf('%.5f-e^{%.5f*x}',fit_bb3.a,fit_bb3.b),'location
    ','best')
82
83 print('Bayes_Uni_Fit','-dpng')
84 %%%
85
86 [ ~,~,~,Pd_ls_uni,k,~,~] = bayes_search(P_long_strat,q_uni
    ,500,.95);
87 [ ~,~,~,Pd_ls_ls,k,~,~] = bayes_search(P_long_strat,
    q_long_strat,500,.95);
88 [ ~,~,~,Pd_ls_bs,k,~,~] = bayes_search(P_long_strat,
    q_both_strat,500,.95);
89 figure
90 step1=10:round(.05*length(Pd_ls_uni)):length(Pd_ls_uni);
91 step2=10:round(.05*length(Pd_ls_ls)):length(Pd_ls_ls);
92 step3=10:round(.05*length(Pd_ls_bs)):length(Pd_ls_bs);
93 plot(step1,Pd_ls_uni(step1),'xk',step2,Pd_ls_ls(step2),'xr',
    step3,Pd_ls_bs(step3),'xg')
94 [fit_bb4,gof_bb4]=fit(step1',Pd_ls_uni(step1)',g,'StartPoint
   ',[1,0],'Upper',[1,0]);
95 [fit_bb5,gof_bb5]=fit(step2',Pd_ls_ls(step2)',g,'StartPoint'
    ,[1,0],'Upper',[1,0]);
96 [fit_bb6,gof_bb6]=fit(step3',Pd_ls_bs(step3)',g,'StartPoint'
    ,[1,0],'Upper',[1,0]);
97 hold on

```

```

98 plot(fit_bb4 , 'k')
99 plot(fit_bb5 , 'r')
100 plot(fit_bb6 , 'g')
101 xlabel('n')
102 ylabel('$P_{\{D\}}$')
103 title('$P_{\{D\}}$ vs Number of Glimpses and Exponential Fitted
Curves')
104 legend( '(p_{strat a},q_{uniform})' , '(p_{strat a},q_{strat a
})' , '(p_{strat a},q_{strat b})' ...
105 , sprintf( '%.5f-e^{%.5f*x}' , fit_bb4.a , fit_bb4.b) , sprintf(
'%.5f-e^{%.5f*x}' , fit_bb5.a , fit_bb5.b) , ...
106 sprintf( '%.5f-e^{%.5f*x}' , fit_bb6.a , fit_bb6.b) , 'location
' , 'best' )
107 print('Bayes_Long_Fit' , '-dpng')
108
109 [ ~,~,~,Pd_bs_uni ,k,~,~] = bayes_search(P_both_strat ,q_uni
,500 , .95 );
110 [ ~,~,~,Pd_bs_ls ,k,~,~] = bayes_search(P_both_strat ,
q_long_strat ,500 , .95 );
111 [ ~,~,~,Pd_bs_bs ,k,~,~] = bayes_search(P_both_strat ,
q_both_strat ,500 , .95 );
112 figure
113 step1=10:round(.05*length(Pd_bs_uni)):length(Pd_bs_uni);
114 step2=10:round(.05*length(Pd_bs_ls)):length(Pd_bs_ls);
115 step3=10:round(.05*length(Pd_bs_bs)):length(Pd_bs_bs);
116 plot(step1 ,Pd_bs_uni(step1) , 'xk' ,step2 ,Pd_bs_ls(step2) , 'xr' ,
step3 ,Pd_bs_bs(step3) , 'xg' )
117 [fit_bb7 ,gof_bb7]=fit(step1' ,Pd_bs_uni(step1)' ,g , 'StartPoint
',[1 ,0] , 'Upper' ,[1 ,0]);
118 [fit_bb8 ,gof_bb8]=fit(step2' ,Pd_bs_ls(step2)' ,g , 'StartPoint'
,[1 ,0] , 'Upper' ,[1 ,0]);
119 [fit_bb9 ,gof_bb9]=fit(step3' ,Pd_bs_bs(step3)' ,g , 'StartPoint'
,[1 ,0] , 'Upper' ,[1 ,0]);

```

```

120 hold on
121 plot(fit_bb7 , 'k')
122 plot(fit_bb8 , 'r')
123 plot(fit_bb9 , 'g')
124 xlabel('n')
125 ylabel('$P_{\{D\}}$')
126 title('$P_{\{D\}}$ vs Number of Glimpses and Exponential Fitted
Curves')
127 legend( '(p_{strat b},q_{uniform})' , '(p_{strat b},q_{strat a
}))' , '(p_{strat b},q_{strat b})' ...
128 , sprintf( '%.5f-e^{%.5f*x}' , fit_bb7.a , fit_bb7.b) , sprintf(
'%.5f-e^{%.5f*x}' , fit_bb8.a , fit_bb8.b) , ...
129 sprintf( '%.5f-e^{%.5f*x}' , fit_bb9.a , fit_bb9.b) , 'location
' , 'best')
130 print('Bayes_Both_Fit' , '-dpng')
131
132 [ ~,~,~,Pd_nw_uni ,k,~,~] = bayes_search(P_norm_wide ,q_uni
,500 , .95 );
133 [ ~,~,~,Pd_nw_ls ,k,~,~] = bayes_search(P_norm_wide ,
q_long_strat ,500 , .95 );
134 [ ~,~,~,Pd_nw_bs ,k,~,~] = bayes_search(P_norm_wide ,
q_both_strat ,500 , .95 );
135 figure
136 step1=10:round(.05*length(Pd_nw_uni)):length(Pd_nw_uni);
137 step2=10:round(.05*length(Pd_nw_ls)):length(Pd_nw_ls);
138 step3=10:round(.05*length(Pd_nw_bs)):length(Pd_nw_bs);
139 plot(step1 ,Pd_nw_uni(step1) , 'xk' ,step2 ,Pd_nw_ls(step2) , 'xr' ,
step3 ,Pd_nw_bs(step3) , 'xg')
140 [ fit_bb10 ,gof_bb10]=fit(step1' ,Pd_nw_uni(step1)' ,g , '
StartPoint' ,[1 ,0] , 'Upper' ,[1 ,0]);
141 [ fit_bb11 ,gof_bb11]=fit(step2' ,Pd_nw_ls(step2)' ,g , '
StartPoint' ,[1 ,0] , 'Upper' ,[1 ,0]);
142 [ fit_bb12 ,gof_bb12]=fit(step3' ,Pd_nw_bs(step3)' ,g , '

```

```

        StartPoint',[1,0], 'Upper',[1,0]);
143 hold on
144 plot(fit_bb10,'k')
145 plot(fit_bb11,'r')
146 plot(fit_bb12,'g')
147 xlabel('n')
148 ylabel('$P_{\{D\}}$')
149 title('$P_{\{D\}}$ vs Number of Glimpses and Exponential Fitted
Curves')
150 legend('(p_{norm},q_{uniform})',(p_{norm},q_{strat a}),(
p_{norm},q_{strat b})',...
151 , sprintf('%.5f-e^{%.5f*x}',fit_bb10.a,fit_bb10.b),
152 sprintf('%.5f-e^{%.5f*x}',fit_bb11.a,fit_bb11.b),...
153 sprintf('%.5f-e^{%.5f*x}',fit_bb12.a,fit_bb12.b),'
154 location,'best')
155 print('Bayes_Norm_Fit','-dpng')

156
155 [~,~,~,Pd_ns_uni,k,~,~] = bayes_search(P_norm_side,q_uni
,500,.95);
156 [~,~,~,Pd_ns_ls,k,~,~] = bayes_search(P_norm_side,
q_long_strat,500,.95);
157 [~,~,~,Pd_ns_bs,k,~,~] = bayes_search(P_norm_side,
q_both_strat,500,.95);
158 figure
159 step1=10:round(.05*length(Pd_ns_uni)):length(Pd_ns_uni);
160 step2=10:round(.05*length(Pd_ns_ls)):length(Pd_ns_ls);
161 step3=10:round(.05*length(Pd_ns_bs)):length(Pd_ns_bs);
162 plot(step1,Pd_ns_uni(step1),'xk',step2,Pd_ns_ls(step2),'xr',
step3,Pd_ns_bs(step3),'xg')
163 [fit_bb13,gof_bb13]=fit(step1',Pd_ns_uni(step1)',g,
164 StartPoint',[1,0], 'Upper',[1,0]);
164 [fit_bb14,gof_bb14]=fit(step2',Pd_ns_ls(step2)',g,
164 StartPoint',[1,0], 'Upper',[1,0]);

```

```

165 [ fit_bb15 ,gof_bb15]= fit( step3 ',Pd_ns_bs( step3 ) ',g ,'
166     StartPoint',[1,0],[ 'Upper',[1,0]);
167 hold on
168 plot(fit_bb13 , 'k')
169 plot(fit_bb14 , 'r')
170 xlabel('n')
171 ylabel('$P_{\{D\}}$')
172 title('$P_{\{D\}}$ vs Number of Glimpses and Exponential Fitted
173 Curves')
174 legend( '(p_{norm b},q_{uniform})' , '(p_{norm b},q_{strat a})',
175     '(p_{norm b},q_{strat b})' ...
176     , sprintf( '%.5f-e^{%.5f*x}' , fit_bb13 .a , fit_bb13 .b) ,
177     sprintf( '%.5f-e^{%.5f*x}' , fit_bb14 .a , fit_bb14 .b) ,...
178     sprintf( '%.5f-e^{%.5f*x}' , fit_bb15 .a , fit_bb15 .b) ,
179     'location','best')
180 print('Bayes_NormSide_Fit' ,'-dpng')
181
182 [ ~,~,~,Pd_nc_uni ,k,~,~] = bayes_search(P_norm_corner ,q_uni
183 ,500 , .95 );
184 [ ~,~,~,Pd_nc_ls ,k,~,~] = bayes_search(P_norm_corner ,
185 q_long_strat ,500 , .95 );
186 [ ~,~,~,Pd_nc_bs ,k,~,~] = bayes_search(P_norm_corner ,
187 q_both_strat ,500 , .95 );
188 figure
189 step1=10:round(.05*length(Pd_nc_uni)):length(Pd_nc_uni);
190 step2=10:round(.05*length(Pd_nc_ls)):length(Pd_nc_ls);
191 step3=10:round(.05*length(Pd_nc_bs)):length(Pd_nc_bs);
192 plot(step1 ,Pd_nc_uni(step1) , 'xk' ,step2 ,Pd_nc_ls(step2) , 'xr' ,
193     step3 ,Pd_nc_bs(step3) , 'xg' )
194 [ fit_bb16 ,gof_bb16]= fit( step1 ',Pd_nc_uni(step1) ',g ,'
195     StartPoint',[1,0],[ 'Upper',[1,0]);
196 [ fit_bb17 ,gof_bb17]= fit( step2 ',Pd_nc_ls(step2) ',g ,'

```

```

    StartPoint',[1,0], 'Upper',[1,0]);
188 [fit_bb18,gof_bb18]=fit(step3',Pd_nc_bs(step3)',g,'
    StartPoint',[1,0], 'Upper',[1,0]);
189 hold on
190 plot(fit_bb16,'k')
191 plot(fit_bb17,'r')
192 plot(fit_bb18,'g')
193 xlabel('n')
194 ylabel('$P_{\{D\}}$')
195 title('$P_{\{D\}}$ vs Number of Glimpses and Exponential Fitted
    Curves')
196 legend('(p_{norm c},q_{uniform})',(p_{norm c},q_{strat a}),
    ,(p_{norm c},q_{strat b})...
197 ,sprintf('%.5f-e^{%.5f*x}',fit_bb16.a,fit_bb16.b),
    sprintf('%.5f-e^{%.5f*x}',fit_bb17.a,fit_bb17.b),...
198 sprintf('%.5f-e^{%.5f*x}',fit_bb18.a,fit_bb18.b),'%
    location','best')
199 print('Bayes_NormCorner_Fit',' -dpng')
200 format shortG
201 b=[fit_bb1.b,fit_bb2.b,fit_bb3.b,fit_bb4.b,fit_bb5.b,fit_bb6
    .b,fit_bb7.b, ...
202 fit_bb8.b,fit_bb9.b,fit_bb10.b,fit_bb11.b,fit_bb12.b,
    fit_bb13.b,fit_bb14.b, ...
203 fit_bb15.b,fit_bb16.b,fit_bb17.b,fit_bb18.b]
204 r2=[gof_bb1.rsquare,gof_bb2.rsquare,gof_bb3.rsquare,gof_bb4.
    rsquare, ...
205 gof_bb5.rsquare,gof_bb6.rsquare,gof_bb7.rsquare,gof_bb8.
    rsquare, ...
206 gof_bb9.rsquare,gof_bb10.rsquare,gof_bb11.rsquare,
    gof_bb12.rsquare, ...
207 gof_bb13.rsquare,gof_bb14.rsquare,gof_bb15.rsquare,
    gof_bb16.rsquare, ...
208 gof_bb17.rsquare,gof_bb18.rsquare]

```

```

209
210  %%%
211  [ ~,~,~,Pd_uni_bs ,k,~,~] = bayes_search(P_uni ,q_both_strat
212  ,500 , 1);
213  [ ~,~,~,Pd_uni_bs_r ,k] = bayes_search_random(P_uni ,
214  q_both_strat ,500 , 1 );
215  figure
216  plot(1:length(Pd_uni_bs ),Pd_uni_bs , 'xr' ,1:length(Pd_uni_bs_r
217  ),Pd_uni_bs_r , 'r' )
218  %%%
219  for i=1:9
220  p0=.1*i;
221  P_uni=ones(10,10);
222  P_uni=(1-p0)*P_uni/sum(P_uni (:));
223  [ ~,~,~,Pd_uni_bs_p0(i ,:) ,k,~,~] = bayes_search(P_uni ,
224  q_both_strat ,500 , 1);
225  end
226  figure
227  for i=1:9
228  hold on
229  plot(1:25:500 ,Pd_uni_bs_p0(i ,1:25:500))
230  end

```

### C.2.2 Sample code for the Uniform/Uniform case

```

1 [ pc ,n ,imax ,Pd_uni_uni ,k ,pc_0 ,k_check] = bayes_search(P_uni
2  ,q_uni ,50 , .95 );
3 figure
4 subplot(2,2,1)
5 imagesc([-4.5 ,4.5 ],[-4.5 ,4.5 ],pc ,[0 ,max(pc (:))])
6 set(gca , 'YDir' , 'normal')
7 cb_p=colorbar;
8 title(cb_p , 'p_{ i }')
9 hold on

```

```

9  plot(X,Y, 'k');
10 plot(Y,X, 'k');
11 title('Remainder Prob.')
12 xlabel('Long.')
13 ylabel('Lat.')
14
15 subplot(2,2,2)
16 imagesc([-4.5,4.5],[-4.5,4.5],n,[0,max(n(:))])
17 set(gca, 'YDir', 'normal')
18 cb_p=colorbar;
19 title(cb_p, 'N')
20 hold on
21 plot(X,Y, 'k');
22 plot(Y,X, 'k');
23 title('Actual Search')
24 xlabel('Long.')
25 ylabel('Lat.')
26 subplot(2,2,3)
27 imagesc([-4.5,4.5],[-4.5,4.5],P_uni,[0,max(P_uni(:))])
28 set(gca, 'YDir', 'normal')
29 cb_p=colorbar;
30 title(cb_p, 'p_{i}')
31 hold on
32 plot(X,Y, 'k');
33 plot(Y,X, 'k');
34 title('Uniform')
35 xlabel('Long.')
36 ylabel('Lat.')
37
38
39 subplot(2,2,4)
40 imagesc([-4.5,4.5],[-4.5,4.5],q_uni,[0,max(q_uni(:))])
41 set(gca, 'YDir', 'normal')

```

```

42 cb_q=colorbar;
43 title(cb_q,'q_{i}');
44 hold on
45 plot(X,Y,'k');
46 plot(Y,X,'k');
47 title('Uniform')
48 xlabel('Long.')
49 ylabel('Lat.')
50 print('Bayes_uni_uni',' -dpng')

```

### C.2.3 Bayes Search Function

```

1 function [ pd,n,imax,Pd_cum ,k,pc_0,k_check] = bayes_search(
2 p,q,nT, stop )
3 %Conducts a Bayesian Search base on inputs .
4 %p=Location probability density
5 %q=Conditional miss probabilit density
6 %nT=max number of glimpses
7 %stop=Probability of Detection to stop algorithm
8 %Outputs
9 %pd=updated location probability density
10 %n=search matrix
11 %imax=Last box searched
12 %k=number of glimpses
13 %pc_0=final probability tgt is not in search area
14 %k_check=game theory detection glimpse
15 if ~exist('stop')
16     stop=sum(p(:));
17 elseif stop>sum(p(:))
18     stop=sum(p(:));
19 end
20 check=0;
21 n=zeros(size(p));%initialize look counter
22 Pd_cum=0;

```

```

22 pc_0=1-sum(p(:));
23 pc=p;
24 k=1;
25 tgt_pos=randsample(1:101,1,true,[p(:);pc_0]);
26 [tgt_row,tgt_col]=ind2sub(size(p),tgt_pos);
27 while k<=nT && Pd_cum(end)<stop
28     pd=pc.* (1-q);
29     [pmax,imax(k)]=max(pd(:));
30     [irow,icol]=ind2sub(size(pd),imax(k));
31     if check==0
32         if irow==tgt_row && icol==tgt_col
33             check=binornd(1,1-q(irow,icol));
34         if check==1
35             k_check=k;
36         end
37     end
38     end
39     pc(irow,icol)=pc(irow,icol).*q(irow,icol);
40     pc=pc/(1-pmax);
41     pc_0=pc_0/(1-pmax);
42     n(irow,icol)=n(irow,icol)+1;
43     Pd_cum(k+1)=Pd_cum(k)+(1-Pd_cum(k))*pmax;
44     k=k+1;
45 end
46 if check==0
47     k_check=NaN;
48 end
49 pd=pc;
50 end

```

### C.3 Continuous

Code is as in [6] with the exception that  $R_1 = R_0 e^{-v_1/V}$  and  $R_2 = R_0 e^{-v_2/V}$

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